Geometrical Model of the Earth's Geocenter Based on GRACE Gravity Field Maps

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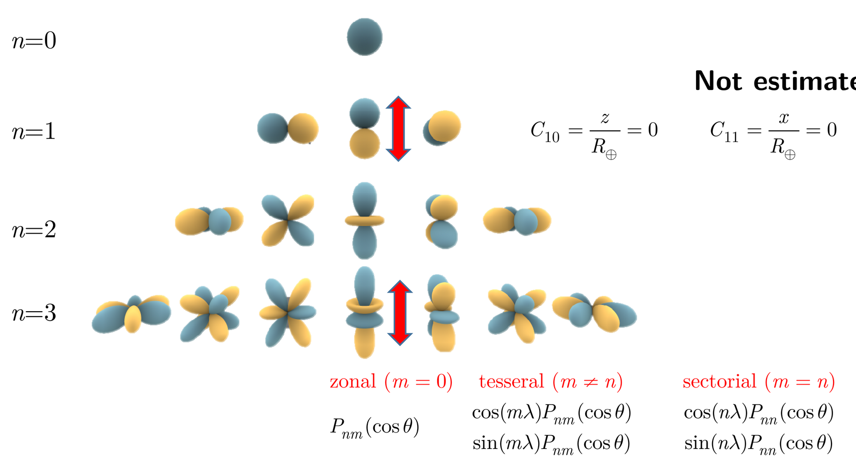
Summary:

We demonstrate that GRACE gravity field maps could be used to derive annual amplitudes and secular rates in the geocenter z-coordinate from the low-degree odd coefficients ("pear-shaped"), i.e., from the C_{30}, C_{50} and C_{70}, although degree 1 gravity field coefficients are not estimated. This is because "pear-shaped" coefficients are not symmetrical with the equator like even zonnals C_{20}, C_{40} and C_{60}, and they are big enough relative to other low-degree "pear-shaped" coefficients to absorb any translation rate present when degree 1 gravity field coefficients are not estimated. If degree 1 gravity field coefficients are derived together with all other gravity field coefficients, degree 1 absorbs systematic effects associated to space geodesy techniques and reference frame realization. Therefore, when degree 1 coefficients are not estimated, any rate in the geocenter z-coordinate is reflected in the translation of the "pear-shaped" harmonics. This also follows from the translation of spherical harmonics. We derived the secular rate and annual amplitudes in geocenter z-coordinate from the low-degree odd coefficients ("pear-shaped") over the last 10 years (GRACE RL05) and compared it with results from the global GPS and SLR solutions, tide-gauge records over the last 100 years and the limited data set of geocenter z-coordinates estimated from the combined orbit determination for the Jason-2 satellite and the GPS constellation. We confirm the initial assumption that temporal gravity field maps provided by the GRACE mission contain an information on the geocenter z-coordinates and estimated annual amplitudes are very close to results from GPS/SLR/LEO solutions. In addition, this approach reveals an interesting information that the asymmetrical mean sea lever rise between the Northern and the Southern hemispheres could be reflected in the rate of asymmetric surface spherical harmonics ("pear-shaped"). Following (Cazenave and Llovel 2010), satellite altimetry observations suggest that the mean sea level has been rising faster over the Southern than over the Northern Hemisphere, whereas recently (Wöppelmann et al. 2014) using selected tide-gauges measurements corrected with the glacial isostatic adjustment (GIA) and GPS velocities report the opposite sign, i.e. the mean sea level rise of $2.0{\pm}0.2$ mm/yr for the Northern hemisphere and $1.1{\pm}0.2$ mm/yr for the Southern hemisphere. Based on the 10 years of GRACE gravity field models (GRACE RL05), we can draw the conclusion that difference in the mean sea level rise between the Northern and the Southern hemispheres is reflected in the rate of the z-coordinate of the geocenter and that the mean sea level has been rising faster over the Southern than over the Northern hemisphere (confirmed Church priv. com.). At the end we derive similar approach from the rates in the even-degree zonal spherical harmonics and derive a rate in the scale of GRACE gravity fields of -0.5 ppb/10 yr. This shows that GRACE gravity field maps represented by spherical harmonics contain a scale and one can use temporal gravity field maps to monitor its variations over time.

Idea: Geocenter Rate from the "Pear-Shaped" Gravity Harmonics

$$V(r,\theta,\lambda)=\frac{GM}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{R_{\oplus}^{n}}{r^{n}}P_{nm}(\cos\theta)\Big(C_{nm}\cos m\lambda+S_{nm}\sin m\lambda\Big)$$ gravitational potential in the form of spherical harmonics

$$Y_{nm}(\theta,\lambda)=P_{nm}(\cos\theta)e^{im\lambda}$$ surface spherical harmonics



Not estimated:

$$C_{10}=\frac{z}{R_{\oplus}}=0 \qquad C_{11}=\frac{x}{R_{\oplus}}=0 \qquad S_{11}=\frac{y}{R_{\oplus}}=0$$

symmetrical with equator

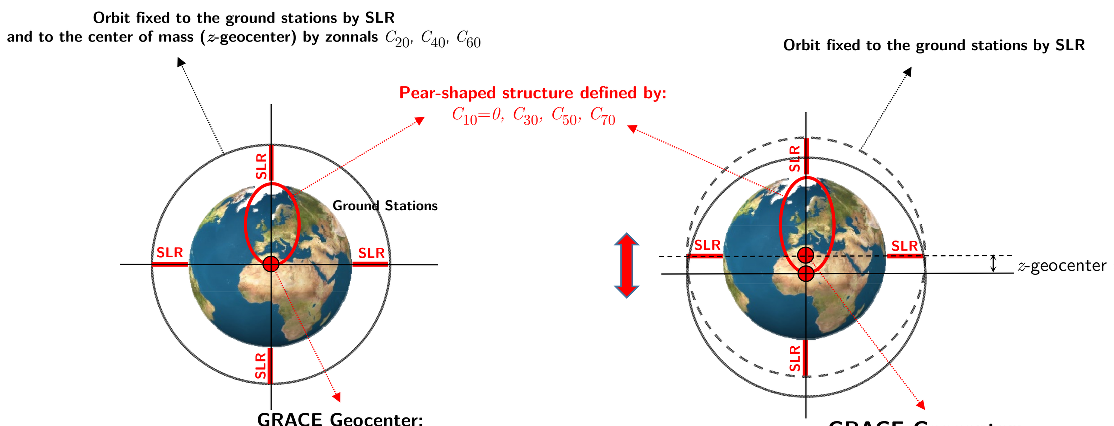
$$Y_{20}(\cos\theta)=P_{20}(\cos\theta)\cos 0\lambda=\frac{3}{2}\frac{z^2}{R_{\oplus}^2}-\frac{1}{2}$$
$$Y_{40}(\cos\theta)=P_{60}(\cos\theta)\cos 0\lambda=\frac{35}{8}\frac{z^4}{R_{\oplus}^4}-\frac{30}{8}\frac{z^2}{R_{\oplus}^2}+\frac{3}{8}$$
$$Y_{60}(\cos\theta)=P_{60}(\cos\theta)\cos 0\lambda=\frac{231}{16}\frac{z^6}{R_{\oplus}^6}-\frac{315}{16}\frac{z^4}{R_{\oplus}^4}+\frac{105}{16}\frac{z^2}{R_{\oplus}^2}-\frac{5}{16}$$

asymmetrical with equator
"pear-shaped"

$$Y_{30}(\cos\theta)=P_{30}(\cos\theta)\cos 0\lambda=\frac{5}{2}\frac{z^3}{R_{\oplus}^3}-\frac{3}{2}\frac{z}{R_{\oplus}}$$
$$Y_{50}(\cos\theta)=P_{50}(\cos\theta)\cos 0\lambda=\frac{63}{8}\frac{z^5}{R_{\oplus}^5}-\frac{70}{8}\frac{z^3}{R_{\oplus}^3}+\frac{15}{8}\frac{z}{R_{\oplus}}$$
$$Y_{70}(\cos\theta)=P_{70}(\cos\theta)\cos 0\lambda=\frac{429}{16}\frac{z^7}{R_{\oplus}^7}-\frac{693}{16}\frac{z^5}{R_{\oplus}^5}+\frac{315}{16}\frac{z^3}{R_{\oplus}^3}-\frac{35}{16}\frac{z}{R_{\oplus}}$$

Any rate in the geocenter z-coordinate is a function of "pear shaped" harmonics only (asymmetrical with equator)

Geocenter, Gravity and Orbits



GRACE Geocenter:
Shape of the orbit and the shape of the rotational ellipsoid are driven by the even zonals: C_{20}, C_{40}, C_{60}, C_{80} that cannot measure an offset/rate in the z-coordinate of the geocenter since they are fully symmetrical with the equator.

GRACE Geocenter:
Since the main pear-shaped term is C_{10}=0, geocenter rate measured by the satellite orbit (fixed to the ground stations by e.g. SLR) can only be adjusted by the remaining pear-shaped low-degree terms C_{30}, C_{50} and C_{70} that are asymmetrical with the equator.

Figure 1 A very simple relation between the rate in the geocenter z-coordinate and "pear-shaped" spherical harmonics with satellite orbits defined by the Earth's geocenter and the network of ground station coordinates.

GRACE "Pear-Shaped" Gravity Maps (RL05)

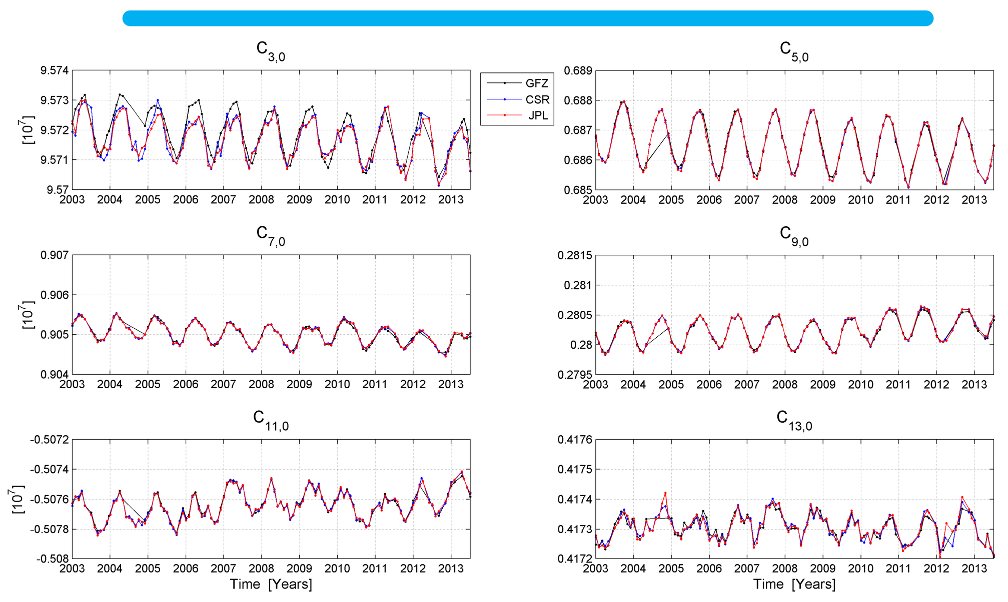


Figure 2 Normalized odd zonal degree coefficients ("pear-shaped") from GRACE monthly gravity fields, RL05. One can clearly see an annual period and a very strong rate in all odd zonal degree coefficients up to degree C_{70}. For higher degrees this rate is smaller and lost in noise. Notice the 10 times higher amplitude of C_{30} compared to other odd coefficients.

Mathematical Model of Geocenter Rates

$$V(r,\theta,\lambda)=\frac{GM}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{R_{\oplus}^{n}}{r^{n}}P_{nm}(\cos\theta)\Big(C_{nm}\cos m\lambda+S_{nm}\sin m\lambda\Big)$$

$$Y_{nm}(\theta,\lambda)=P_{nm}(\cos\theta)e^{im\lambda}$$ surface spherical harmonics

Mathematical Model

$$\frac{dY_{n0}(\cos\theta)}{dt}=\frac{dY_{n0}(\cos\theta)}{dz}\frac{dz}{dt}=\frac{dC_{n0}}{dt} \quad\rightarrow\quad \frac{dz}{dt}=\sqrt{2n+1}\,\frac{d\overline{C}_{n0}}{dt}\left(\frac{dY_{n0}(\cos\theta_{mean})}{dz}\right)^{-1}$$

$$\frac{dY_{n0}(\cos\theta_{mean})}{dz}=\frac{1}{2R_{\oplus}}\int_{-R_{\oplus}}^{R_{\oplus}}\frac{dY_{n0}(\cos\theta)}{dz}dz$$

Following (Heiskanen and Moritz 1967) the first three odd spherical harmonics ("pear-shaped") can be defined as:

$$Y_{30}(\cos\theta)=P_{30}(\cos\theta)\cos 0\lambda=\frac{5}{2}\frac{z^3}{R_{\oplus}^3}-\frac{3}{2}\frac{z}{R_{\oplus}}$$
$$Y_{50}(\cos\theta)=P_{50}(\cos\theta)\cos 0\lambda=\frac{63}{8}\frac{z^5}{R_{\oplus}^5}-\frac{70}{8}\frac{z^3}{R_{\oplus}^3}+\frac{15}{8}\frac{z}{R_{\oplus}}$$
$$Y_{70}(\cos\theta)=P_{70}(\cos\theta)\cos 0\lambda=\frac{429}{16}\frac{z^7}{R_{\oplus}^7}-\frac{693}{16}\frac{z^5}{R_{\oplus}^5}+\frac{315}{16}\frac{z^3}{R_{\oplus}^3}-\frac{35}{16}\frac{z}{R_{\oplus}}$$

Model of the Geocenter z-coordinate

Geocenter Model from GRACE Gravity Maps

SECULAR RATE

$$\frac{dz}{dt}(\overline{C}_{30})=R_{\oplus}\sqrt{7}\times(-0.60\cdot10^{-10})\,/\,10\text{ yr}=-1.0\text{ mm/10 yr}\quad\pm0.13\text{ mm/10 yr}$$
$$\frac{dz}{dt}(\overline{C}_{50})=R_{\oplus}\sqrt{11}\times(-0.62\cdot10^{-10})\,/\,10\text{ yr}=-1.3\text{ mm/10 yr}\quad\pm0.07\text{ mm/10 yr}$$
$$\frac{dz}{dt}(\overline{C}_{70})=R_{\oplus}\sqrt{15}\times(-0.31\cdot10^{-10})\,/\,10\text{ yr}=-0.8\text{ mm/10 yr}\quad\pm0.07\text{ mm/10 yr}$$

ANNUAL AMPLITUDE

$$A_c(\overline{C}_{30})=R_{\oplus}\sqrt{7}\times(0.78\cdot10^{-10})\cos(\omega t)=1.3\text{ mm}\times\cos(\omega t)\quad\pm0.05\text{ mm}$$
$$A_c(\overline{C}_{50})=R_{\oplus}\sqrt{11}\times(1.04\cdot10^{-10})\cos(\omega t)=2.2\text{ mm}\times\cos(\omega t)\quad\pm0.03\text{ mm}$$
$$A_c(\overline{C}_{70})=R_{\oplus}\sqrt{15}\times(0.29\cdot10^{-10})\cos(\omega t)=0.7\text{ mm}\times\cos(\omega t)\quad\pm0.03\text{ mm}$$

Figure 3 Secular rates dz/dt and annual amplitudes A_C of the geocenter z-coordinate derived from the GRACE gravity field maps (RL05) with estimated formal errors. This model is based on the empirical model of low-degree coefficients ("pear-shaped") from Table 1.

Empirical Model of "Pear-Shaped" Gravity Coefficients

Secular **Annual**

$$\overline{C}_{n0}(t)=\overline{C}_{n0}(t_0)+\dot{\overline{C}}_{n0}t+A_c\cos(\omega t+A_{0c}) \qquad t=\text{time in days}$$
$$\overline{S}_{n0}(t)=\overline{S}_{n0}(t_0)+\dot{\overline{S}}_{n0}t+A_s\cos(\omega t+A_{0s}) \qquad \omega=2\pi\,/\,365.25\text{ annual frequency}$$

Secular Rate

$$\dot{\overline{C}}_{30}=(-0.60\pm0.07)\cdot10^{-10}/10\text{ yr}$$
$$\dot{\overline{C}}_{50}=(-0.62\pm0.03)\cdot10^{-10}/10\text{ yr}$$
$$\dot{\overline{C}}_{70}=(-0.31\pm0.03)\cdot10^{-10}/10\text{ yr}$$

Annual Amplitude

$$A_C(\overline{C}_{30})=(0.78\pm0.03)\cdot10^{-10}\cos(\omega t)$$
$$A_C(\overline{C}_{50})=(1.04\pm0.01)\cdot10^{-10}\cos(\omega t)$$
$$A_C(\overline{C}_{70})=(0.29\pm0.01)\cdot10^{-10}\cos(\omega t)$$

Table 1 Empirical model of secular rates and annual amplitudes A_C of the odd zonal degree coefficients C_{n0} ("pear-shaped") from the GRACE monthly gravity field maps in Figure 1, given relative to the nominal epoch 2003.0.

Scale of the GRACE Gravity Maps

The central term of the gravity field defines the mean gravitational potential of the Earth. In the case of homogeneous sea level rise over all oceans it is expected that only zonal surface spherical harmonics will be affected since they are symmetrical w.r.t. the equator. The mean gravitational potential as well as the shape of the oblate ellipsoid will not be changed under this assumption. Thus, one could expect a scale type effect that will be reflected in a change of the mean sphere in the expansion of the Earth's gravity field in terms of spherical harmonics. The derivative of the radius of the mean sphere of the spherical harmonic expansion can be calculated from the mean value theorem for integrals in the following way

$$\frac{dY_{n0}(\cos\theta_{mean})}{dR_{\oplus}}=\frac{2}{R_{\oplus}}\int_{0}^{R_{\oplus}}\frac{dY_{n0}(\cos\theta)}{dR_{\oplus}}dz$$

and considering

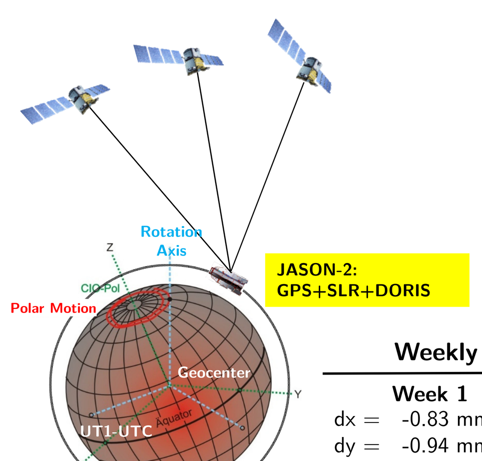
$$Y_{20}(\cos\theta)=P_{20}(\cos\theta)\cos 0\lambda=\frac{3}{2}\frac{z^2}{R_{\oplus}^2}-\frac{1}{2}$$
$$Y_{40}(\cos\theta)=P_{60}(\cos\theta)\cos 0\lambda=\frac{35}{8}\frac{z^4}{R_{\oplus}^4}-\frac{30}{8}\frac{z^2}{R_{\oplus}^2}+\frac{3}{8}$$
$$Y_{60}(\cos\theta)=P_{60}(\cos\theta)\cos 0\lambda=\frac{231}{16}\frac{z^6}{R_{\oplus}^6}-\frac{315}{16}\frac{z^4}{R_{\oplus}^4}+\frac{105}{16}\frac{z^2}{R_{\oplus}^2}-\frac{5}{16}$$

from where we obtain the rate in the scale of the geometrical frame that defines expansion of Earth's gravitational field in terms of spherical harmonics

$$\frac{dR_{\oplus}}{dt}(\overline{C}_{20})=R_{\oplus}\sqrt{5}\times(-1.7\cdot10^{-10})\,/\,10\text{ yr}=-2.4\text{ mm/10 yr}$$
$$\frac{dR_{\oplus}}{dt}(\overline{C}_{40})=R_{\oplus}\sqrt{9}\times(-1.0\cdot10^{-10})\,/\,10\text{ yr}=-1.9\text{ mm/10 yr}$$
$$\frac{dR_{\oplus}}{dt}(\overline{C}_{60})=R_{\oplus}\sqrt{13}\times(-0.9\cdot10^{-10})\,/\,10\text{ yr}=-1.7\text{ mm/10 yr}$$

or the relative rate in the scale of -0.5 ppb/10 yr. The scale of the conventional terrestrial frame is defined by the scale of the station coordinates of the ground networks of space geodesy techniques fixed to the continental Earth's crust. This shows that spherical harmonics also contain an intrinsic scale and one can use temporal gravity field maps to monitor its variations over time.

Geocenter from the LEO+GPS Combination



Weekly Geocenter Coordinates (CONT08)

Week 1	Week 2	Week 3
dx = -0.83 mm	dx = -1.78 mm	dx = -1.72 mm
dy = -0.94 mm	dy = -1.67 mm	dy = -1.22 mm
dz = -5.90 mm	**dz = -5.75 mm**	**dz = -5.60 mm**

Figure 4 Helmert transformation of weekly coordinates solution (after stacking daily NEQs) from the combined GPS/Jason-2 constellation (GPS, DORIS, SLR) – CONT'08 Campaign. GPS-only or combined GPS/Jason-2 solutions of the geocenter z-coordinate tend to give larger oscillations over several weeks, see also (Männel and Rothacher, 2017), that is most likely associated with systematic effects in space geodesy techniques. This means that GRACE gravity field maps could provide more accurate information on the geocenter z-coordinate.

Comparison with GPS, SLR and Altimetry

ANNUAL AMPLITUDE from GPS and SLR:

Rebischung & Garayt (2013)	**3.0 mm**	GPS (1997-2009)
König et al. (2015)	**2.4 mm ± 0.8 mm**	SLR/LEO (2006-2012)
Männel & Rothacher (2017)	**5.9 mm ± 0.8 mm**	GPS/LEO (2010-2013)

SECULAR RATE from Altimetry:

Derived geocenter model based on GRACE RL05 confirms the hemispherical rate of the mean sea level rise (Cazenave and Llovel 2010) and not the solution from tide-gauges (Wöppelmann, 2014). This confirms that the mean sea level has been rising faster over the Southern than over the Northern hemisphere, (also Church priv. com.).

Figure 5 Comparison of the derived geocenter model with reported rates in the z-coordinate of the geocenter from the global GPS and SLR solutions and altimetry/tide-gauges records over the last 100 years. Comparing the model from Figure 3, one can see that gravity maps give smaller annual amplitudes with significantly smaller error bars. On the other hand, following (Cazenave and Llovel 2010), satellite altimetry observations suggest that the mean sea level has been rising faster over the Southern than over the Northern Hemisphere, whereas recently (Wöppelmann et al. 2014), (Santamaría-Gómez 2014) using selected tide-gauges measurements corrected with the glacial isostatic adjustment (GIA) and GPS velocities report the opposite sign, i.e., the mean sea level rise of $2.0{\pm}0.2$ mm/yr for the Northern hemisphere and $1.1{\pm}0.2$ mm/yr for the Southern hemisphere. Geocenter rate from GRACE RL05 has an opposite sign compared to tide-gauges records that reported the rate of 0.9 mm/yr (Santamaría-Gómez 2014) in the geocenter z-coordinate.

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Conclusions

- We demonstrate that GRACE gravity field maps could be used to derive annual amplitudes and secular rates in the geocenter z-coordinate from the low-degree odd coefficients ("pear-shaped"), i.e. from the C_{30}, C_{50} and C_{70}, although degree 1 gravity field coefficients are not estimated. This is because "pear-shaped" coefficients are not symmetrical with the equator like even zonnals C_{20}, C_{40} and C_{60} and they are big enough relative to other low-degree "pear-shaped" coefficients to absorb any translation rate present when degree 1 gravity field coefficients are not estimated. If degree 1 gravity field coefficients are derived together with all other gravity field coefficients, degree 1 absorbs systematic effects associated to space geodesy techniques and reference frame realization. Therefore, when degree 1 coefficients are not estimated, any rate in the geocenter z-coordinate is reflected in the translation of the "pear-shaped" harmonics. This also follows from the translation of spherical harmonics.
- Estimated secular rates from GRACE gravity field maps in the geocenter z-coordinate are in the order of dz/dt= -1.03 $mm/10$ yr and annual amplitudes of A_C =1.4 mm compared to typical annual amplitude of 3 mm, e.g., (Rebischung and Garayt 2013) derived from space geodesy techniques, such as GPS and SLR or combined GPS/LEO solutions.
- The geocenter model confirms that the mean sea level has been rising faster over the Southern than over the Northern hemisphere. This confirms the sign of the hemispherical sea lever rise (Cazenave and Llovel 2010) compared to an opposite sign of (Wöppelmann et al. 2014), (Santamaría-Gómez 2014), (Church priv. com).
- Derived geocenter model reveals an interesting information that the asymmetrical mean sea lever rise between the Northern and the Southern hemispheres could be reflected in the rate of asymmetric surface spherical harmonics ("pear-shaped") and in the derived geocenter z-coordinate.
- With the similar approach, we derive rates in the even-degree zonal spherical harmonics and derive a rate in the scale of GRACE gravity field of -0.5 ppb/10 yr. This shows that GRACE gravity field maps represented by spherical harmonics contain a scale and one can use temporal gravity field maps to monitor its variations over time.