Calculating bedrock level from ice surface elevation using the shallow ice approximation

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Introduction

In glaciated regions where detailed radar studies or seismic surveys have not been made, ice thickness and location of the bedrock often remain unknown. We propose a model determining the thickness of a glacier solely from surface elevation and mass balance data, assuming that the shallow ice approximation (SIA) holds.

We pay particular attention to
1) keeping the model computationally inexpensive, allowing large areas to be processed quickly;
2) making the model easy to use, avoiding steps that require manual data processing.

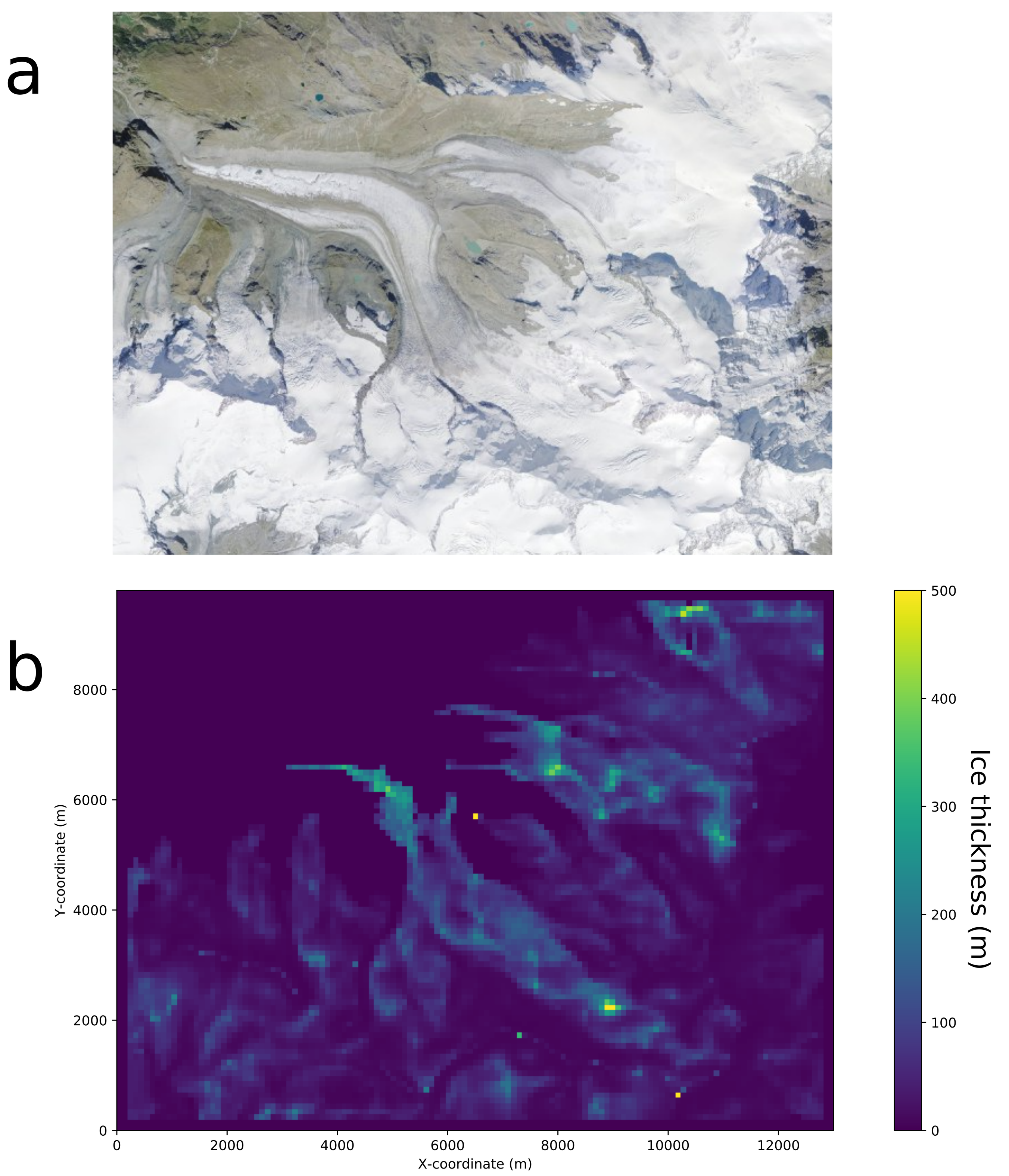


Fig. 1 Gorner glacier and the surrounding area near Zermatt, Switzerland. **(a)** Satellite image. **(b)** Ice thickness in the corresponding area as determined by the model. Note that this model run assumed that the glacier should be in steady state, which might explain why the modelled extent is a slight underestimate.

Synthetic example

After LeMeur et al. (2002)

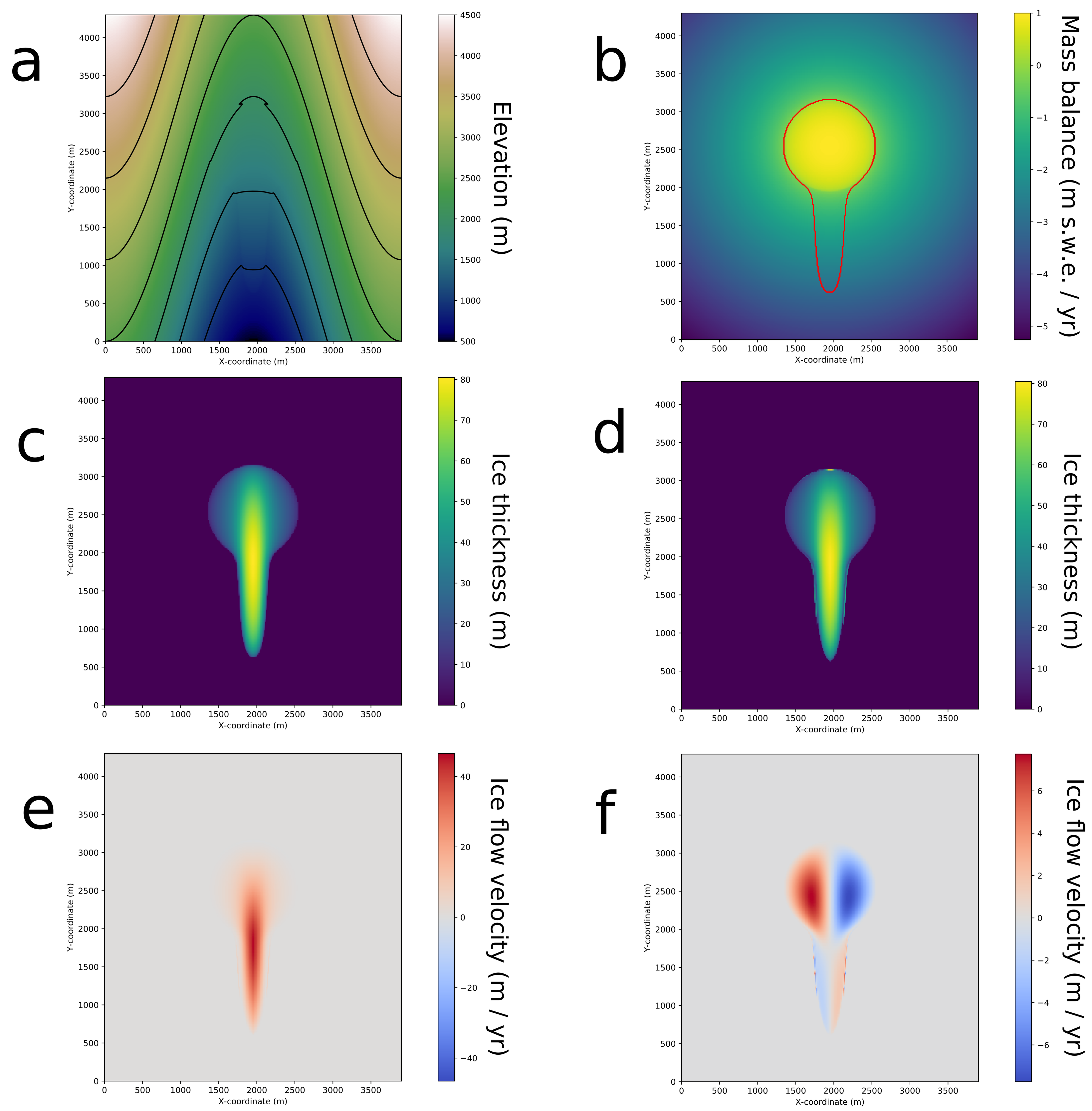


Fig. 2 Using a fictitious surface elevation **(a)** and mass balance field **(b)** from LeMeur et al. (2002) to test the model. The smooth sloping valley surface is only disturbed by a glacier **(c)** produced by running a forward model based on SIA to its steady state (glacier outline also shown in red on (b) for clarity). Ice thickness **(d)** and vertically averaged flow velocity components **(e)** and **(f)** (down the valley and across it, respectively) were calculated with the backward model taking (a) and (b) as inputs. This example includes only deformational ice flow, no sliding.

Potential future applications and benchmarking

Given the computational cheapness and ease of use, the model can benefit:

1) calculations over large spatial scales such as ice sheets (which adhere better to SIA anyway)
2) inversions for mass balance (and ice thickness) corresponding to given glacial extent
3) interactive displays of glacial dynamics

To analyse the performance of the model, we intend to test it on a variety of glacial environments – especially where SIA holds well (such as in ice caps). Benchmarking against models involved in the recent Ice Thickness Models Intercomparison Experiment (Farinotti et al., 2017) will indicate its standing relative to earlier attempts.

Method by real example

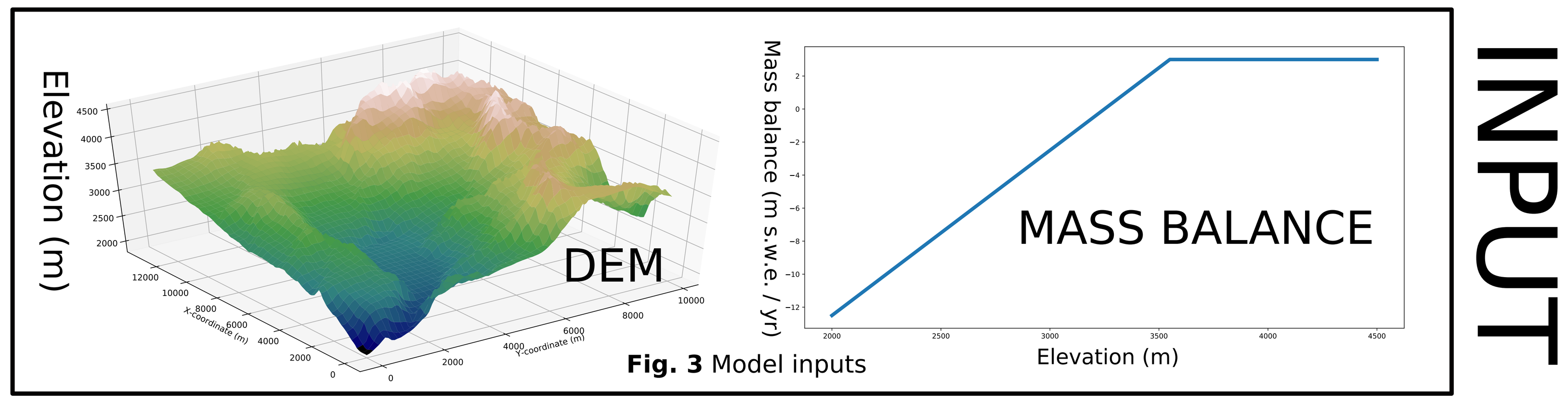


Fig. 3 Model inputs

Mass conservation (see legend at bottom right of the poster):

$$\frac{\partial h}{\partial t} = \nabla \cdot \vec{F} + M$$

With SIA (flow velocity broken into deformation and sliding):

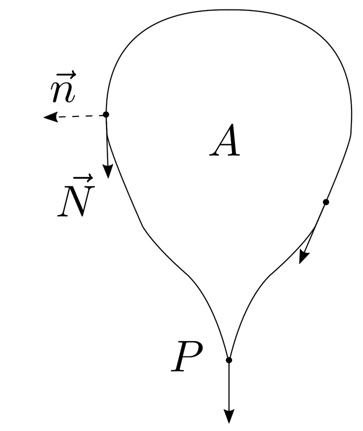
$$\frac{\partial h}{\partial t} = \nabla \cdot [-(f_d h^{n+2} + f_s h^n)(\rho g)^n |\nabla S|^{n-1} \nabla S] + M$$

Two substitutions:

$$d = (\rho g)^n (f_d h^{n+2} + f_s h^n)$$
$$\vec{N} = |\nabla S|^{n-1} \nabla S$$

Equation to solve (for steady state):

$$\nabla \cdot (d\vec{N}) = M$$



Integrating the equation over the catchment area of a point on the glacier:

$$\int_A \nabla \cdot (d\vec{N})\, da = \int_A M\, da$$

Gauss's theorem applied to the left side:

$$\oint_L d\vec{N} \cdot \vec{n}\, dL = \int_A M\, da$$

At the boundary of catchment area, flow follows the boundary, so for a given side length of model cell:

$$\oint_L d\vec{N} \cdot \vec{n}\, dL = d_P |\vec{N}|_P\, d\ell$$

$$d_P = \frac{1}{|\vec{N}|_P\, d\ell} \int_A M\, da$$

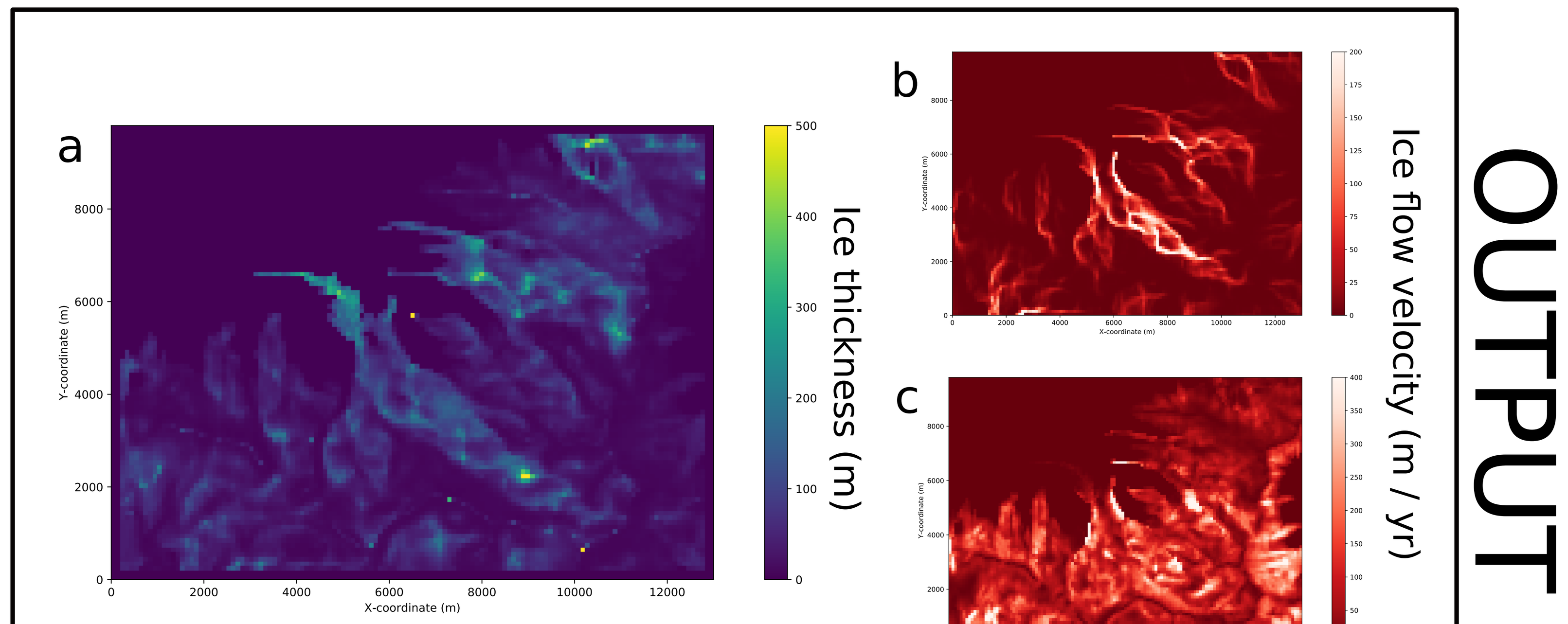
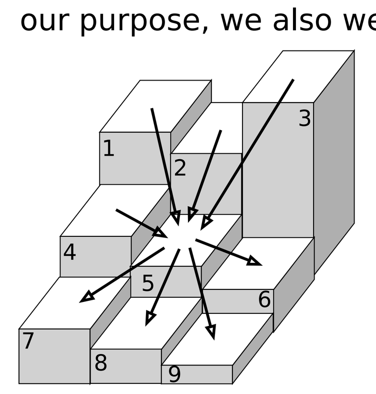


Fig. 4 Model outputs, in this case ice thickness **(a)** and deformational velocity **(b)** as well as sliding velocity **(c)**, for Gorner glacier in Switzerland.

Integration over catchment area by FastScape

To integrate mass balance over the the catchment area, we use **FastScape** (Braun and Willett, 2013) – a method originally developed to solve the stream power law for fluvial incision. As such, it finds all grid cells contributing to the flow through a point and adds together their areas to get the catchment area. For our purpose, we also weigh the sum by mass balance in each cell.



In this model, we let flow from a grid cell contribute to multiple cells immediately downstream (see figure on the left) in proportion to the slope defined by each pair. We find that this more accurately reproduces the real environment, where flow does not have to follow a grid pattern.

Legend

h – ice thickness
$\vec{F}$ – ice flux
M – surface mass balance
f_d – deformational velocity coefficient
f_s – sliding velocity coefficient
∇S – surface gradient
A – catchment area of point P
L – catchment area boundary
$\vec{n}$ – outward pointing normal vector of L
$d\ell$ – side length of model cell

References

Jean Braun and Sean D. Willett. "A very efficient O(n), implicit and parallel method to solve the stream power equation governing fluvial incision and landscape evolution." In: *Geomorphology* 180-181 (2013), pp. 170-179.

Daniel Farinotti et al. "How accurate are estimates of glacer ice thickness? Results from ITMIX, the Ice Thicness Models Intercomparison eXperiment." In: *The Cryosphere* 11 (2017), pp. 949-970.

Emmanuel Le Meur et al. “Glacier flow modelling: a comparison of the Shallow Ice Approximation and the full-Stokes solution”. In: *C. R. Physique* 5 (2004), pp. 709–722.