Ocean response to varying wind in models with time and depth dependent eddy viscosity

Rema Almelah and Victor Shrira

r.b.almelah@keele.ac.uk v.i.shrira@keele.ac.uk

Keele University, School of Computing and Mathematics

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Abstract

The work examines response of the upper ocean to time-varying winds. In the Ekman paradigm the effect of wind is considered as time-varying horizontally uniform tangential wind stress applied to the ocean surface and the turbulent diffusion of momentum is described employing the Boussinesq closure hypothesis via a single scalar eddy viscosity. Here, in contrast to all other works based upon the Ekman model and its extensions, we assume the eddy viscosity to be both time and depth dependent. Under the assumptions of linear dependence of eddy viscosity on depth and arbitrary time dependence of wind we find exact general solution to the Navier-Stokes equations which describes dynamics of the Ekman boundary layer in terms of the Green's function. Three basic scenarios are examined in detail: (a) An increase of wind ending up with a plateau; (b) Strictly sinusoidal wind; (c) Switch off of the wind.

The mathematical model

We describe ocean response to varying wind starting with the Navier-Stokes equations for stratified viscous and horizontally uniform flow on the non-traditional f-plane under the Boussinesq approximation. In the Cartesian frame with x directed eastward, y-northward and z-downward, with the origin at the free surface of the ocean the Reynolds averaged Navier-Stokes equations for the the eastward and northward velocities u, v caused by a time-varying horizontally uniform wind stress take the form

$$\frac{\partial \boldsymbol{U}}{\partial t} + if\boldsymbol{U} = \frac{\partial}{\partial z}\left(\nu_e(z,t)\frac{\partial \boldsymbol{U}}{\partial z}\right). \tag{1}$$

where $\boldsymbol{U} = u + iv$ is the complex horizontal velocity, ρ is density, $\nu_e(z,t)$ is the eddy viscosity coefficient , $f = 2\Omega \sin\phi$ is the Coriolis parameter (Ω and ϕ are the Earth's rotation frequency and the latitude respectively).

The equation (1) represents an exact reduction of the Navier-Stokes equations for the horizontally uniform viscous flows on the f-plane with time and depth dependent viscosity. In the present work the effects of stratification are not considered.

The motion has to satisfy the continuity of the shear stress boundary condition at the free surface: a horizontally uniform time dependent wind produces tangential stress $\boldsymbol{\tau}(t)$ at the ocean surface, so that

$$\left[\nu_e(z,t)\frac{\partial \boldsymbol{U}}{\partial z}\right]_{z=0} = \frac{-\boldsymbol{\tau}(t)}{\rho}. \tag{2}$$

The velocity should vanish at the bottom $z = H$, however, throughout this paper we will consider only deep fluid, which implies

$$\frac{\partial \boldsymbol{U}}{\partial z} \to 0 \quad \text{as} \quad z \to \infty. \tag{3}$$

Solvable model

Here we consider a particular class of eddy viscosity depth and time dependence assuming $\nu_e(z,t)$ to be in separable form, i.e. $\nu_e(z,t) = \nu_{e1}(t)g(z)$. Then the substitution

$$\boldsymbol{U}(z,t) = e^{-ift}\boldsymbol{W}(T,z); \quad T = \int_0^t \nu_{e1}(\xi)d\xi \tag{4}$$

turns the governing equation (1) into the diffusion equation with a given vertical dependence of the diffusion coefficient $g(z)$:

$$\frac{\partial \boldsymbol{W}}{\partial T} = \frac{\partial}{\partial z}\left(g(z)\frac{\partial \boldsymbol{W}}{\partial z}\right). \tag{5a}$$

The boundary and initial conditions take the form:

$$\frac{\partial \boldsymbol{W}}{\partial z} = \frac{-e^{ift(T)}\boldsymbol{\tau}(t(T))}{\rho\,\nu_e(z,t(T))} \equiv \boldsymbol{F}(t) \quad \text{at} \quad z = 0, \tag{5b}$$

$$\frac{\partial \boldsymbol{W}}{\partial z} \to 0 \quad \text{as} \quad z \to \infty, \tag{5c}$$

$$\boldsymbol{W}(z,t)|_{t=0} = \boldsymbol{U}(z,0). \tag{5d}$$

Under the assumption of the separable form of the eddy viscosity its vertical dependence $g(z) = g_0 + g_1 z$ does not vary with time.

General solution of particular cases

Time-dependent viscosity model with $g_0 = 0$

Often, the linear growth of viscosity with depth is so strong that when our main interest is in processes in the first meters or tens of meters below the surface the eddy viscosity nonzero value at the surface is so insignificant that it can be neglected. When $g_0 = 0$, the solution takes the form

$$\boldsymbol{U}(z,t) = e^{-ift}\boldsymbol{W}(z,T) \tag{6}$$

where

$$\boldsymbol{W}(z,T) = \frac{1}{g_1}\frac{e^{ift(T)}\boldsymbol{\tau}(t(T))}{\rho\,\nu_{e1}(t(T))} *_{(T)} \frac{1}{T}e^{-z/g_1 T},$$

where $\eta *_{(T)} \psi$ is the convolution with respect to T.

$$\eta *_{(T)} \psi = \int_0^T \eta(T-\xi)\psi(\xi)d\xi$$

If we set ν_{e1} to be a constant the solution (6) coincides with that of Madsen (1977).

Time-dependent viscosity model with uniform viscosity ($g_1 = 0$)

This is the most direct generalization of the Ekman model with varying in time depth independent viscosity. It might be appropriate for well mixed shallow mixed layers for relatively short time scales. Applying the Laplace transform with respect to T to the diffusion equation and boundary conditions (eqs.(5)) we find the general solution in the form

$$\boldsymbol{U}(z,t) = e^{-ift}\boldsymbol{W}(T,z); \tag{7}$$

where

$$\boldsymbol{W}(z,T) = \frac{e^{ift(T)}\boldsymbol{\tau}(t(T))}{\rho\,\nu_{e1}(t(T))} *_{(T)} \frac{e^{-z^2/4T}}{\sqrt{\pi T}}$$

Eq.(7) describes the time dependent current in terms of Green's function for an infinite homogeneous ocean when the eddy viscosity varies with time only. When ν_{e1} is constant, the solution coincides with the Ekman solution (1905).

Increasing wind

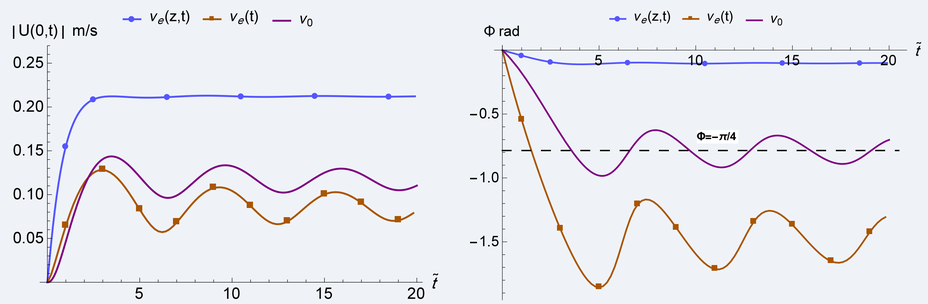

Figure: Evolution of the Ekman current under growing wind with the growth timesscale $\delta = 2h$. The magnitude of the current surface velocity and the angle of its deflection from the wind: Comparison between the models with the time-dependent viscosity, time-depth dependent viscosity and the constant viscosity. $\tilde{t} = ft$. The parameter values are: $f = 10^{-4}s^{-1}$, $\Omega = 0.7f$ rad s^{-1}, $\nu_0 \approx 2\times10^{-2}m^2s^{-1}$, $\rho = 1027\ kg\ m^{-3}$, $\tau_0 = 0.175Nm^{-2}$, $|U_{10}| = 10ms^{-1}$.

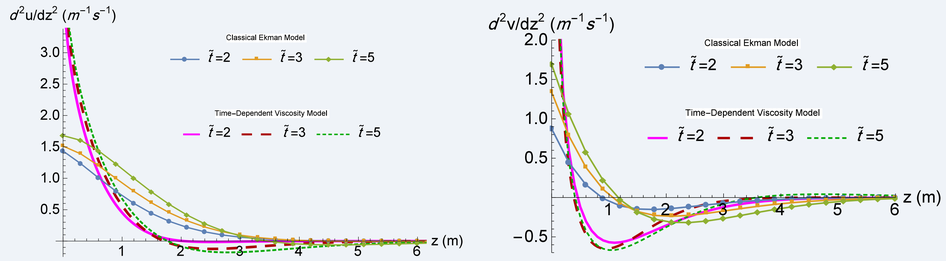

Figure: The second derivative of the solution of the classical Ekman model and time-dependent viscosity model when $\delta = 3h$ and at different times: $\tilde{t} = 2$, $\tilde{t} = 3$ and $\tilde{t} = 5$. The second derivative of x-component (left). The second derivative of y-component (right). $f = 10^{-4}s^{-1}$, $\nu_0 = 10^{-4}m^2s^{-1}$.

Periodic wind

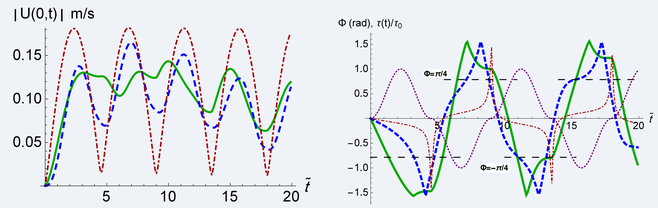

Figure: Evolution of the Ekman current from rest under a suddenly turned on periodic wind in three different models: time-depth dependent viscosity model (6) is shown by dot-dashed line (brown online), the time-dependent viscosity model (7)) is shown by solid line (green online), the classical Ekman solution with constant eddy viscosity by dashed line (blue online). Current magnitude at the surface $|\boldsymbol{U}(0,t)|$. The angle Φ between the wind and the surface current $\boldsymbol{U}(0,t)$. The parameters are the same as in the first Figure.

Conclusion

We showed that the Ekman theory could be easily extended to take into account time dependent eddy viscosity, which is a better reflection of reality. Under the assumptions of horizontal uniformity of the motions and linear dependence of eddy viscosity on depth (which is underpinned by the mixing length closure hypothesis and LES simulations) and arbitrary time dependence of wind we find exact general solution to the Navier-Stokes equations with time and depth dependent viscosity which describes dynamics of the Ekman boundary layer in terms of the Green's function. An examination of the basic scenarios demonstrated that taking into account both time and depth dependence of eddy viscosity leads to substantial changes in the Ekman current response (compared to the model with constant eddy viscosity). Under the adopted Zikanov's parametrization of eddy viscosity we found also considerable dependence of the response on latitude. We also report a severe limitation of the Ekman type models and their all known generalizations employed in modelling of the oceanic surface boundary layer. The Ekman current caused by a growing wind quickly becomes unstable with respect to inviscid inflectional instability. These instabilities are fast, which suggests spikes of dramatically enhanced mixing in the corresponding parts of the water column. The instabilities have certain preferred spatial scales, which violates the assumed spatial homogeneity. This picture is incompatible with the existing paradigm which assumes a smooth diffusion of momentum and spatial homogeneity of the flow.

References

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