Ocean response to varying wind in models with time and depth dependent eddy viscosity

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Abstract

The work examines response of the upper ocean to time-varying winds. In the Ekman paradigm the effect of wind is considered as time-varying horizontally uniform tangential wind stress applied to the ocean surface and the turbulent diffusion of momentum is described employing the Boussinesq closure hypothesis via a single scalar eddy viscosity. Here, in contrast to all other works based upon the Ekman model and its extensions, we assume the eddy viscosity to be both time and depth dependent. Under the assumptions of linear dependence of eddy viscosity on depth and arbitrary time dependence of wind we find exact general solution to the Navier-Stokes equations which describes dynamics of the Ekman boundary layer in terms of the Green’s function. These three basic scenarios are examined in detail: (a) An increase of wind ending up with a plateau; (b) Strictly sinusoidal wind; (c) Switch off of the wind.

The theoretical model

We describe ocean response to varying wind starting with the Navier-Stokes equations for stratified viscous and horizontally uniform flow on the non-traditional $f$-plane under the Boussinesq approximation. In the Cartesian frame with $x$ directed eastward, $y$-northward and $z$-downward, with the origin at the free surface of the ocean the Reynolds averaged Navier-Stokes equations for the eastward and northward velocities $u,v$ caused by a time-varying horizontally uniform wind stress take the form

$$ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu(z) \nabla^2 \mathbf{u} $$

(1)

where $\mathbf{u} = u \mathbf{i} + v \mathbf{j}$ is the complex horizontal velocity, $\rho$ is density, $\nu(z)\equiv\nu(z,t)$ is the eddy viscosity coefficient, $\mathbf{T} = T_x \mathbf{i} + T_y \mathbf{j}$ the Coriolis parameter $(f \equiv f_0 \sin \phi)$ and $\phi$ is the Earth’s rotation rate. The equation (5) represents an exact reduction of the Navier-Stokes equations for the horizontally uniform viscous flows on the $f$-plane with time and depth dependent viscosity. In the present work the effects of stratification are not considered.

The motion has to satisfy the continuity of the shear stress boundary condition at the free surface: a horizontally uniform time dependent wind produces tangential stress $\tau_y$ at the free surface, so that

$$ \nu(z,t) \frac{\partial u}{\partial z} |_{z=H} = -\frac{\tau_y}{\rho} $$

(2)

The velocity should vanish at the bottom $z = H$, however, throughout this paper we will consider only deep fluid, which implies

$$ \frac{\partial u}{\partial z} \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty. $$

Solvability model

Here we consider a particular class of eddy viscosity depth and time dependence $\nu(z,t)$ to be separable form, i.e. $\nu(z,t) = \nu_0(z)\nu_0(t)$ Then the substitution

$$ \mathbf{u}(z,t) = e^{cH(t)} \mathbf{W}(t; z); \quad T = t - cH(t) $$

(4)

turns the governing equation (1) into the diffusion equation with a given vertical dependence of the diffusion coefficient $g(z)$:

$$ \frac{\partial \mathbf{W}}{\partial T} = \frac{\partial}{\partial z} \left( g(z) \frac{\partial \mathbf{W}}{\partial z} \right) $$

(5a)

The boundary and initial conditions take the form:

$$ \nu(z,t) \frac{\partial W}{\partial z} |_{z=0} = F(T) \quad \text{at} \quad z = 0, $$

(5b)

$$ \mathbf{W}(z,t) |_{z=H} = \mathbf{U}(z), \quad \mathbf{U}(z) \sim \mathbf{u}(z), $$

(5c)

$$ \mathbf{W}(z,t) \rightarrow 0 \quad \text{as} \quad z \rightarrow \infty, $$

(5d)

Under the assumption of the separable form of the eddy viscosity its vertical dependence $g(z) = g_0 + g_1 z$ does not vary with time.

Time-dependent viscosity model with $\nu_0 = 0$

Often, the linear growth of viscosity with depth is so strong that when our main interest is in processes in the first meters or tens of meters below the surface the eddy viscosity nonzero value at the surface is so insignificant that it can be neglected. When $\nu_0 = 0$, the solution takes the form

$$ \mathbf{W}(z,t) = e^{-cH(t)} \mathbf{W}(z; T), \quad T = t - cH(t) $$

(6)

where

$$ \mathbf{W}(z; T) = \frac{1}{\nu_1(T)} \frac{1}{\rho} \mathbf{F}(T) $$

(7)

The motion has to satisfy the continuity of the shear stress boundary condition at the free ocean surface, so that

$$ \nu_1(T) \frac{\partial W}{\partial z} |_{z=0} = F(T) \quad \text{at} \quad z = 0. $$

(8)

The second derivative of the solution of the classical Ekman model and time-dependent viscosity model when $T = t$ and at different times $T = \frac{t}{f} + \frac{1}{2f}$ is denoted by $\eta$-component (left). The second derivative of $y$-component (right). $f = 10^{-4}$ s$^{-1}$, $v = 10^{-4}$ m$^{-1}$.s$^{-1}$. We adopt Zikanov’s parametrization of eddy viscosity we found also considerable dependence of the response on latitude. We also report a severe limitation of the Ekman type models and their all known generalizations employed in modelling of the oceanic surface boundary layer. The Ekman current caused by a growing wind quickly becomes unstable with respect to inviscid infinitesimal instability. These instabilities are fast, which suggests spikes of dramatically enhanced mixing in the corresponding parts of the water column. The instabilities have certain preferred spatial scales, which violates the assumed spatial homogeneity. This picture is incompatible with the existing paradigm which assumes a smooth diffusion of momentum and spatial homogeneity of the flow.

References