Anomalous Diffusions of Suspended Sediment Transport by Two-particle Stochastic Diffusion Particle Tracking Model

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Objectives

1. To model the random behaviors of suspended sediment particles in turbulent flow
2. To propose a state-of-the-art two-particle stochastic diffusion particle tracking model (two-particle SD-PTM) which can
   - Simulate suspended sediment transport in the turbulence flows
   - Take **particle correlation** into consideration
   - Describe the **anomalous diffusions** of sediment motions

Methodology

In the two-particle SD-PTM, particles can be separated by the molecular diffusion when they are in the immediate neighborhood.

Governing equation of two-particle SD-PTM

small scale turbulence / large scale turbulence

$$dX_t^j(t) = \underbrace{\left(\overline{U} + \frac{\partial \epsilon_s}{\partial x_t^j}\right)dt}_{\textit{deterministic term}} + \underbrace{\sqrt{2\epsilon_s}\left(\sqrt{(1-\beta^2)}d\boldsymbol{B_t} + \beta dB_t'\right)}_{\textit{random term}}$$

- j : representation of particles, j = 1,2
- X_t : position of a particle $[X(t), Y(t), Z(t)]$
- $\overline{U}$: main flow velocity in different direction
- ε_s : turbulence diffusion coefficient
- dB_t : the small scale turbulence and molecular diffusion
- dB'_t : the large scale turbulence which has spatial correlation
- β : the diffusion effect which can be chosen between 0 to 1

Re-suspension Mechanism

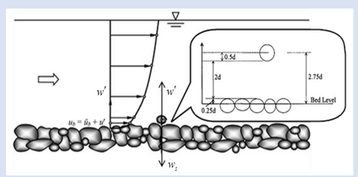
Threshold of suspended load

The expansion of probability density function of w':

$$P_{w'}(w' \geq 0) = \frac{1}{16}(17 + \hat{w} - \hat{w}^2)\exp(-\hat{w}),$$
$$P_{w'}(w' < 0) = 0, \quad \hat{w} = w'/\sigma_z \approx w'/u_*$$

(Bose and Dey, 2013)

$w' > w_s$ → **Re-suspend**



(modified from Chen and Chew (1999) and Bose and Dey (2013)

Turbulent Diffusion

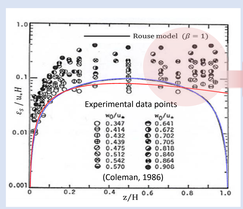
Turbulent diffusion

$$\varepsilon_s = \left(\frac{St}{\left(1-\rho_f/\rho_s\right)} + \frac{1}{1+St}\right)^{-1} \nu_t$$

Schmidt number

(Absi et al. 2011)

- ε_s : turbulence diffusion coefficient
- ρ_f : fluid density
- ρ_s : particle density
- St : stokes number
- ν_t : eddy viscosity which is calibrated by DNS data

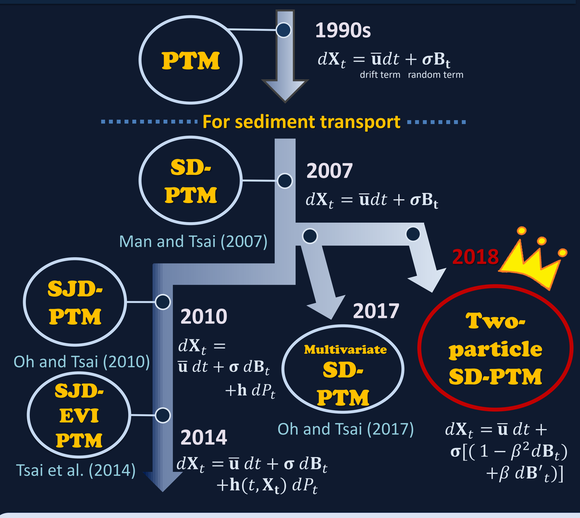


Absi et al.(2011) model v.s. Rouse model

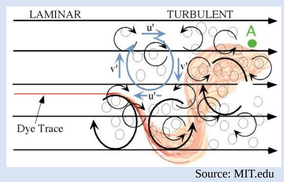
Suspended sediment diffusivity in the model provides non-zero estimate of turbulent diffusivity at the water surface

Absi et al. (2011)
Rouse

PTM Development



Spatial Correlation



Source: MIT.edu

By taking the **spatial correlation of particles** into consideration, the proposed model can account for the influence of turbulence of various scales on sediment movement .

Dependent Brownian motion (B_t') can be used to simulate spatial correlation of particles constrained by large eddies.

Spatial Correlation function

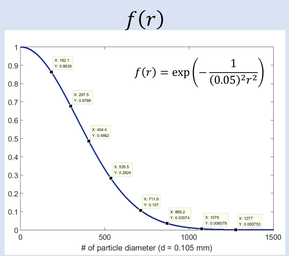
The inter-particle correlation is dependent on the distance between two particles r.

$f(r) = \exp(-\alpha r^{-2})$

$\alpha = 1/DX^2$

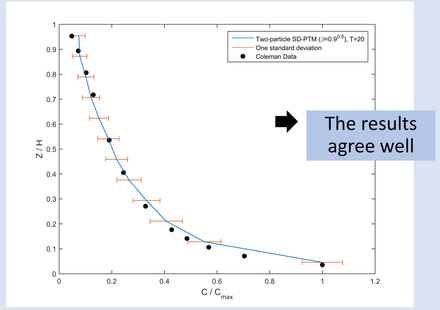
DX : related to the turbulence length scale

(Spivakovskaya et al., 2007)



Model Hypotheses and Validations

Model Validation



The results agree well

Validate **sediment concentration** against experimental data (Coleman, 1986)

Experimental conditions	C02 (Colman, 1986)
Flow depth (m)	0.171
Reynolds number	179775
Shear velocity (m/s)	0.041
Karman coefficient	0.433
Particle specific gravity	2.65
Particle diameter (mm)	0.15
The diffusion effect, β	$\sqrt{0.9}$
Time step (s) $w_s\Delta t/h < 0.01$ (Argall et al., 2004)	0.05
Realizations	500

Model Hypothesis – Markovian Property

Markovian Property (Memorylessness)

It is hypothesized that movement of sediment particles satisfies the Markovian property.

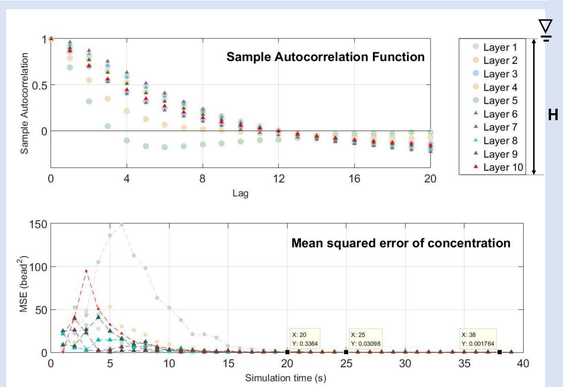
$$p(x_{n+1}, t_{n+1} | x_n, t_n;\ x_{n-1}, t_{n-1}; \ldots) = p(x_{n+1}, t_{n+1} | x_n, t_n)$$

Future state / Present state

After being released for a long time, particle position is independent of its initial position

Ergodicity

The ensemble average concentrations equal the time average concentrations after 20 seconds (t>20s).



Model Hypothesis – Fickian Law

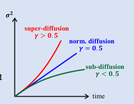
Fickian Law

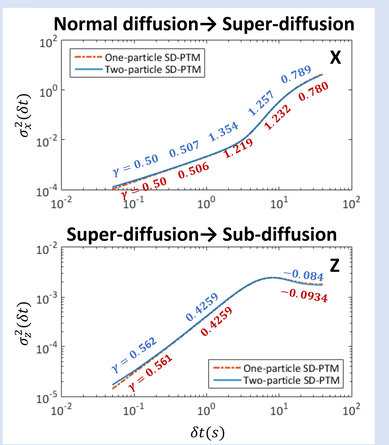
It is hypothesized that suspended sediment particle movement follows the Fickian law.

$$\sigma_x^2 = \langle (x_i - \langle x \rangle)^2 \rangle \propto (t)^{2\gamma}$$

$\langle x \rangle$: the mean of the particle displacement x_i over time t

γ : the scaling diffusion exponent $\begin{cases} \gamma = 0.5, normal\ diffusion \\ \gamma \neq 0.5, anomalous \end{cases}$





Transition of Particle motions

x-dir (σ_x^2) : normal → super-diffusion

z-dir (σ_z^2) : super → sub-diffusion

Hypothesis Falsified

The change of the trend in the ensemble variances can be attributed to **particle deposition and resuspension**.

Conclusions

A state-of-the-art two-particle stochastic diffusion particle tracking model (two-particle SD-PTM) is proposed to

- To simulate the **suspended particle transport** in turbulent flows
- To take particle correlation into consideration
- To incorporate a more sophisticated **turbulent diffusivity** formula and a recently developed **re-suspension mechanism**
- To **verify** the proposed model by comparing the quantified sediment concentrations against experimental data
- To **verify** the Markovian property after a sufficiently long time
- To describe the transition of anomalous diffusions of suspended sediment transport