Anomalous Diffusions of Suspended Sediment Transport by Two-particle Stochastic Diffusion Particle Tracking Model

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Objectives

1. To model the random behaviors of suspended sediment particles in turbulent flow.
2. To propose a state-of-the-art two-particle stochastic diffusion particle tracking model (two-particle SD-PTM) which can
   - Simulate suspended sediment transport in the turbulence flows
   - Take particle correlation into consideration
   - Describe the anomalous diffusions of sediment motions

Methodology

In the two-particle SD-PTM, particles can be separated by the molecular diffusion when they are in the immediate neighborhood.

\[ dX_i(t) = \left[ (\overline{u} + \xi) \frac{\Delta x}{\Delta t} \right] dt + \sqrt{2D_0(1 - \beta^2)} dB_i + \beta dB_i' \]

**Governing equation of two-particle SD-PTM**

- \( r_i \) : representation of particles, \( i = 1, 2 \)
- \( \xi_i \) : position of particle \((x(t), y(t), z(t))\)
- \( \overline{u} \) : mean flow velocity in different direction
- \( D_0 \) : turbulence diffusion coefficient
- \( \beta \) : the diffusion effect which can be chosen between 0 to 1

**Deterministic term**

- \( db_i \) : the small scale turbulence and molecular diffusion
- \( db_i' \) : the large scale turbulence which has spatial correlation

Re-suspension Mechanism

- Threshold of suspended load
- The expansion of probability density function of \( w_i \):

**Turbulent Diffusion**

- \( \xi \) : turbulence diffusion coefficient
- \( \rho_i \) : bulk density
- \( \xi_i \) : particle density
- \( \xi' \) : eddy viscosity which is determined by SST turbulence model

**Spatial Correlation**

- By taking the spatial correlation of particles into consideration, the proposed model can account for the influence of turbulence of various scales on sediment movement.
- Dependent Brownian motion (\( \xi_i \)) can be used to simulate spatial correlation of particles constrained by large eddies.

**Conclusions**

A state-of-the-art two-particle stochastic diffusion particle tracking model (two-particle SD-PTM) is proposed to

- To simulate the suspended particle transport in turbulent flows
- To take particle correlation into consideration
- To incorporate a more sophisticated turbulent diffusivity formula and a recently developed re-suspension mechanism
- To verify the proposed model by comparing the quantified sediment concentrations against experimental data
- To verify the Markovian property after a sufficiently long time
- To describe the transition of anomalous diffusions of suspended sediment transport

**Model Hypotheses and Validations**

**Model Hypothesis — Markovian Property**

It is hypothesized that movement of sediment particles satisfies the Markovian property.

**Model Hypothesis — Fickian Law**

It is hypothesized that suspended sediment particle movement follows the Fickian law.

**Hypothesis Failed**

The change of the trend in the ensemble variances can be attributed to particle deposition and resuspension.