

1. Introduction

What numbers characterise topography? How can we reduce a complex landscape to a few numbers that let us easily and robustly compare different regions at different scales, perhaps with different data coverage available? Mean and variance might be an obvious starting point, but this implies some rather strong assumptions — that the topography is a stationary, white, and Gaussian process. A common alternative assumption is that the covariance is takes an exponential, rather than Gaussian, form — an equally strong, if different, assumption.

Here, we instead use a Matérn parameterisation to let us solve for, rather than assume, the shape of the covariance function. This form uses three parameters to characterise the topography: **(1) σ , the variance;** **(2) ν , the smoothness or differentiability;** and **(3) ρ , the correlation length or range.** Building on Simons & Olhede (2013), we use a spectral domain ‘Whittle’ maximum-likelihood estimation procedure to estimate these three parameters across sample regions of the Atlantic and Pacific Oceans. The results highlight the need to explicitly account for anisotropy in analysis of oceanic structure, even in fast-spreading areas and focusing on mid- to long-wavelength features (data were low-pass filtered to 20 km).

2. Sample Topography Analyses

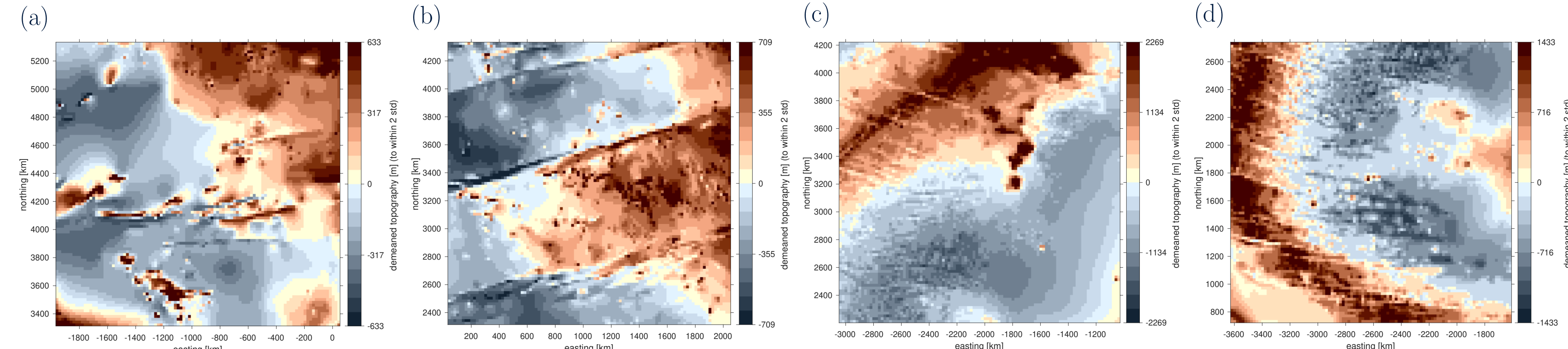


Fig. 1: Sample regions of de-measured topography from (a) and (b) the Pacific and (c) and (d) the Atlantic.

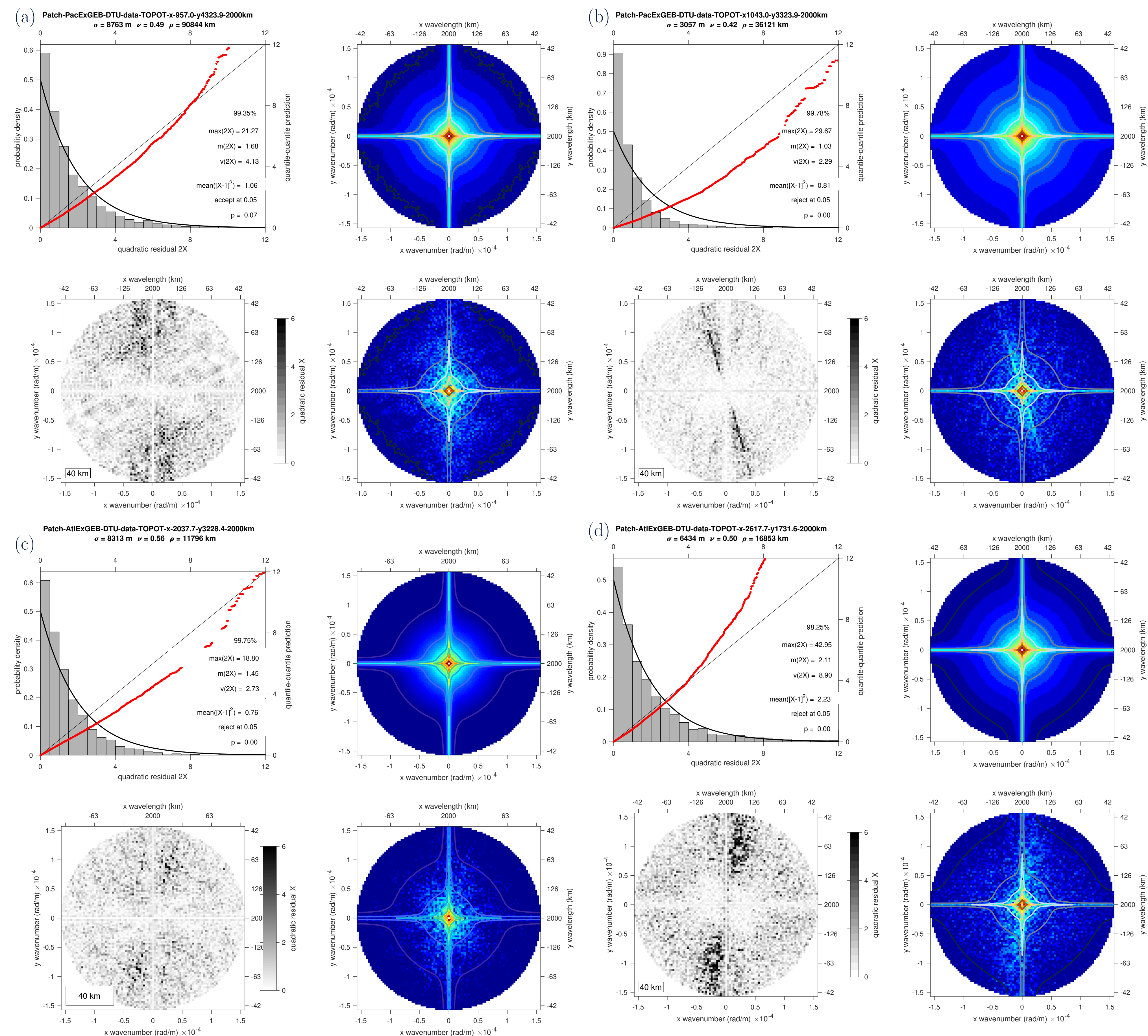


Fig. 2: Maximum likelihood estimates of the four topographic regions in Figure 1. Each set of four panels shows: *Top left:* Observed histogram of the residuals (grey bars) and its prediction (thick black line) together with the quantile-quantile plot (thick red line). *Top right:* Predicted blurred spectral density. *Bottom left:* Spectral map of the residuals. *Bottom right:* Observed periodogram, with contours from the prediction above.

2. Sample Topography Analyses, continued

- The fit between the observed histogram of residuals and their predicted probability density often appears relatively good, but there is evidence of a consistent shift towards lower numbers (marked in Figure 2b, but present in all four examples).
- Quantile-quantile plots show varying amounts of divergence from the 1:1 line, and can fall either above or below it, but are generally concave-upward to straight.
- Spectral maps of the residuals for all four examples show an imprint of anisotropy, e.g., in Figure 2b, orthogonal to the strong fracture zones of Figure 1b. There is a suggestion that stronger anisotropy may correlate with stronger divergence.
- An example centred over Hawaii (Figure 3), where the large volcanic chain might dominate the background signature of the seafloor spreading fabric, shows a fundamentally similar pattern to the other examples. The trend of the anisotropy, however, is different to that of Figures 1b and 2b, and appears orthogonal to the island chain.
- This is consistent with results of Kalhins & Simons (2017) on the anisotropy in the relationship between gravity and topography in the oceans, and its potential relationship to anisotropy in the long-term mechanical strength of oceanic lithosphere. Unlike in the continents, where little robust anisotropy in this relationship could be detected, the oceans showed widespread anisotropy with a first-order correlation with spreading direction. However, this form of anisotropy vanished around large ocean islands like Hawaii.

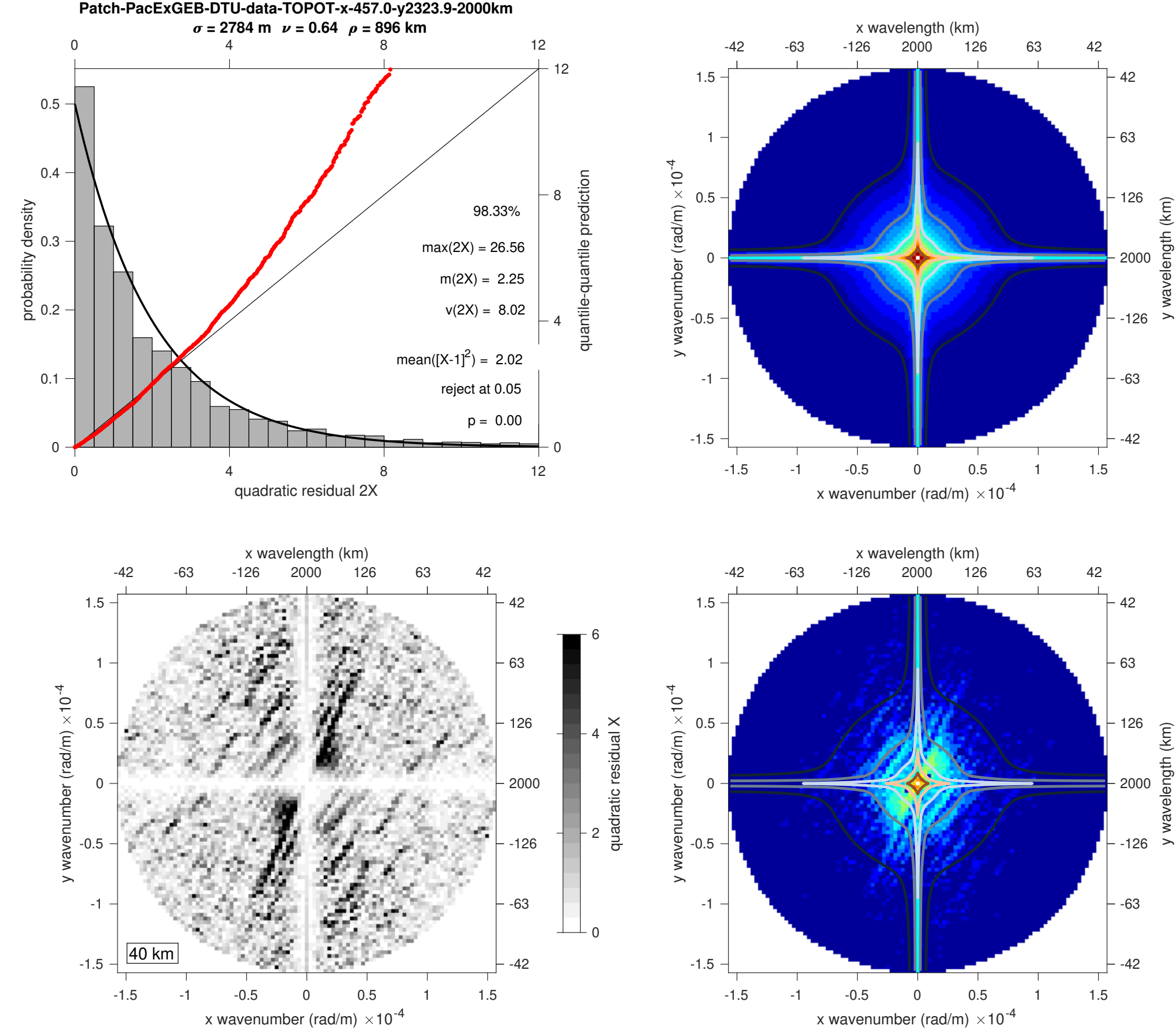


Fig. 3: Analysis for a region centred on Hawaii

3. Stability of the Estimates

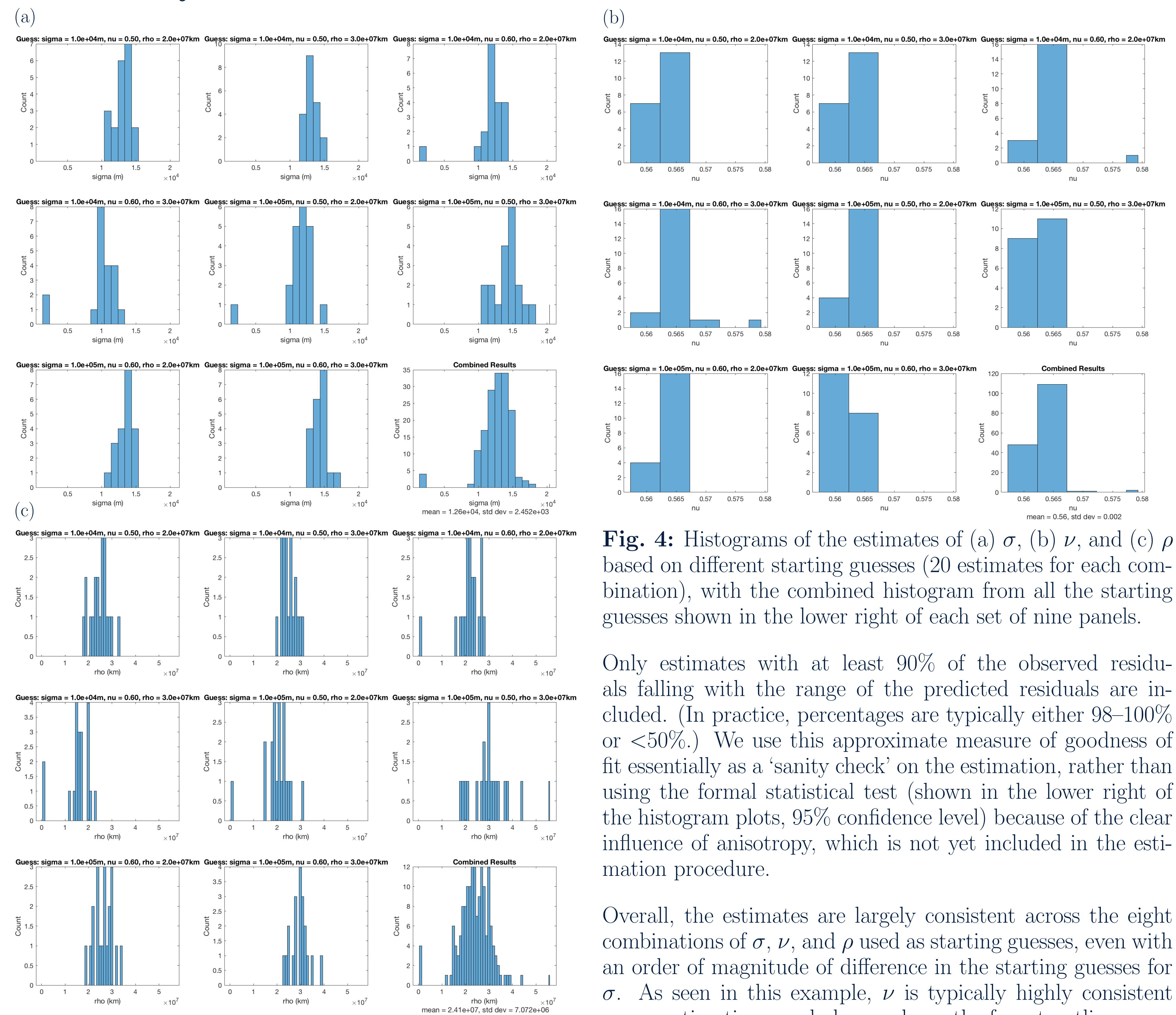


Fig. 4: Histograms of the estimates of (a) σ , (b) ν , and (c) ρ based on different starting guesses (20 estimates for each combination), with the combined histogram from all the starting guesses shown in the lower right of each set of nine panels.

Only estimates with at least 90% of the observed residuals falling within the range of the predicted residuals are included. (In practice, percentages are typically either 98–100% or <50%.) We use this approximate measure of goodness of fit essentially as a ‘sanity check’ on the estimation, rather than using the formal statistical test (shown in the lower right of the histogram plots, 95% confidence level) because of the clear influence of anisotropy, which is not yet included in the estimation procedure.

Overall, the estimates are largely consistent across the eight combinations of σ , ν , and ρ used as starting guesses, even with an order of magnitude of difference in the starting guesses for σ . As seen in this example, ν is typically highly consistent across estimations, and also produces the fewest outliers.

The variance, σ , and correlation length, ρ , are less tightly constrained, but still produce a relatively normal distribution, and there is no indication that the final estimate is significantly determined by the starting guess or of a bi- or multi-model distribution.

4. Spatial Variation in Roughness and Covariance

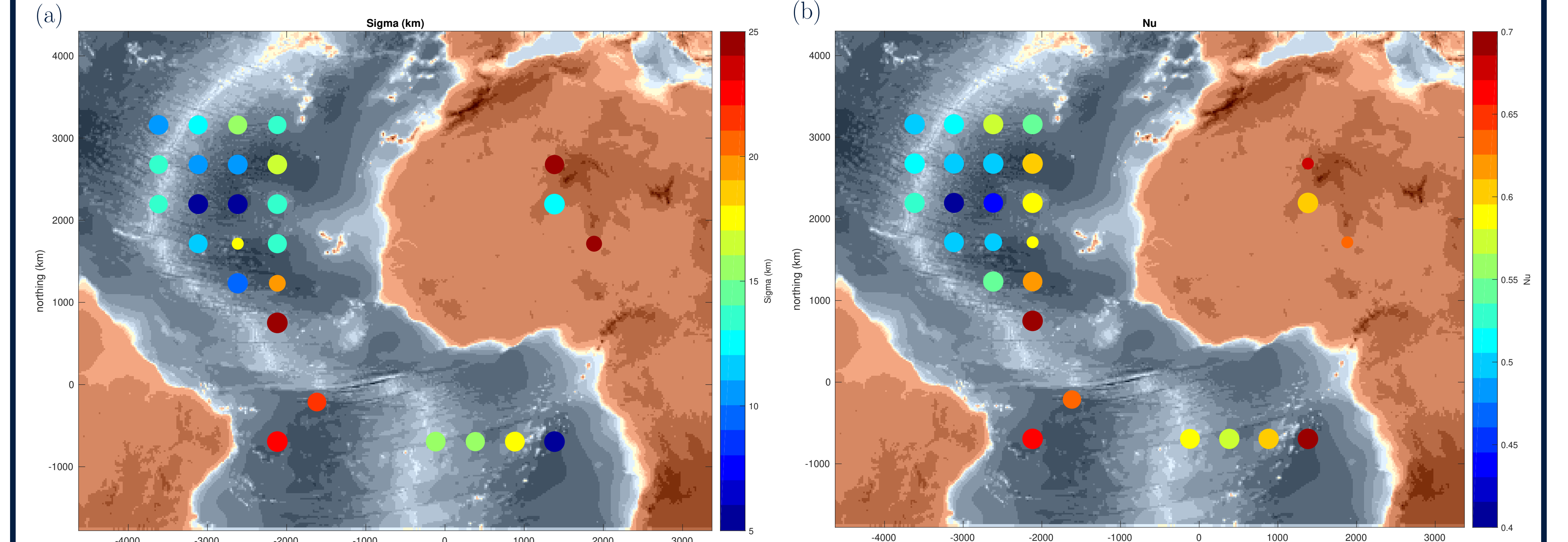
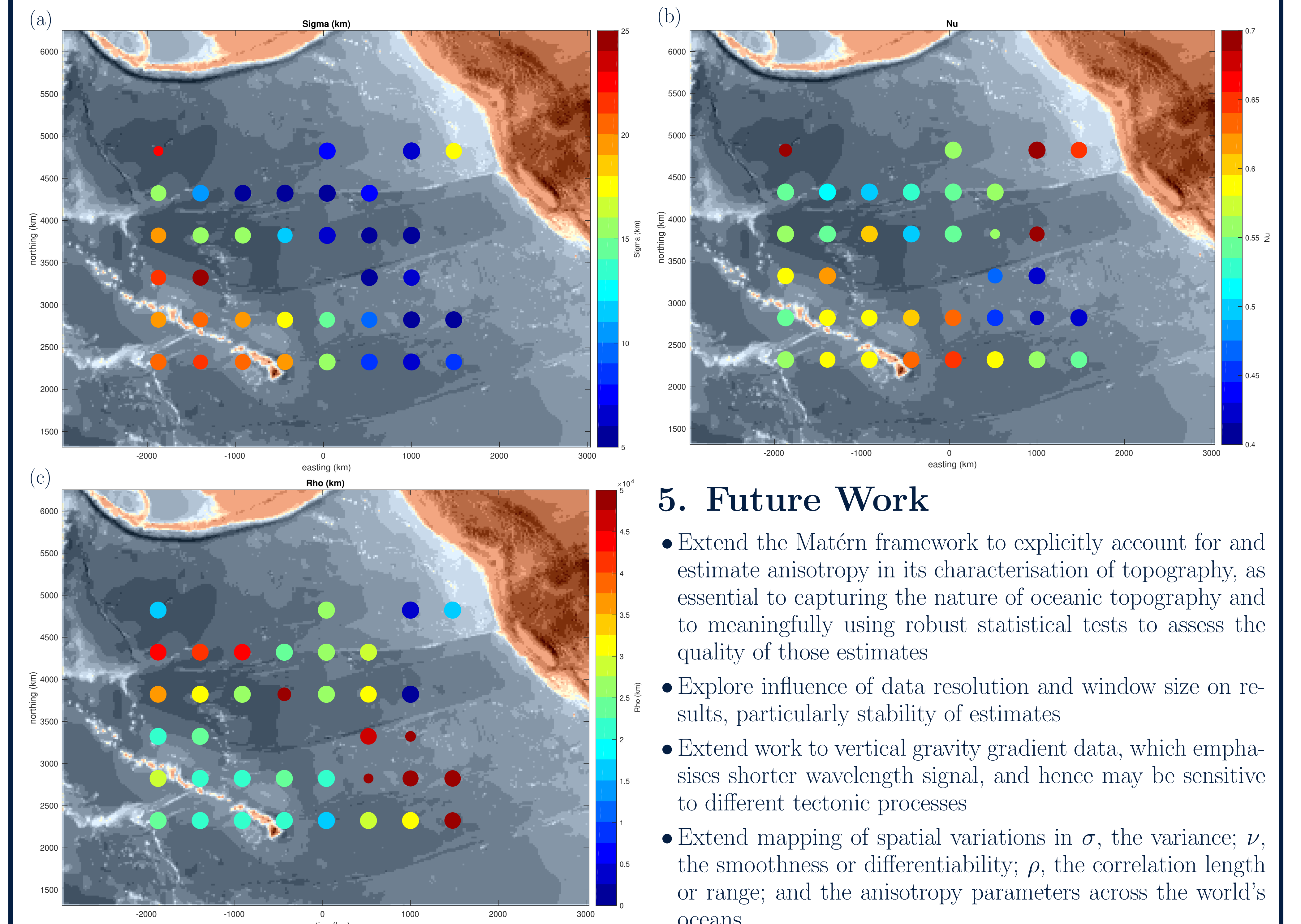


Fig. 5: Maps of the mean (a) σ , (b) ν , and (c) ρ across the central and northern Atlantic Ocean using a range of starting guesses and a 2000 km window. Dot size is inversely scaled with standard deviation. Gaps in the central Atlantic near the Strakhov, Romanche, Charcot, and Ascension Fracture Zones did not produce estimates with a sufficiently close match between the residual histogram and its prediction. There is a strong positive correlation between σ (‘variance’) and ν (‘smoothness’ or ‘differentiability’), which are both then negatively correlated with ρ (‘correlation length’ or ‘range’).

Fig. 6 (below): As for Figure 5, but for the northeastern Pacific Ocean. ν and ρ still appear negatively correlated, but σ is dominated by the contrast between the Hawaiian Chain and the rest of the area. High values of ρ and low values of ν trace the boundary between these two domains.



5. Future Work

- Extend the Matérn framework to explicitly account for and estimate anisotropy in its characterisation of topography, as essential to capturing the nature of oceanic topography and to meaningfully using robust statistical tests to assess the quality of those estimates
- Explore influence of data resolution and window size on results, particularly stability of estimates
- Extend work to vertical gravity gradient data, which emphasises shorter wavelength signal, and hence may be sensitive to different tectonic processes
- Extend mapping of spatial variations in σ , the variance; ν , the smoothness or differentiability; ρ , the correlation length or range; and the anisotropy parameters across the world’s oceans

References: Kalhins & Simons (2017), *Exploring the Tectonic Evolution of the Seafloor using Roughness, Covariance, and Anisotropy in Bathymetry and Marine Gravity*, AGU Fall Meeting, Abstract O522B-03. Simons & Olhede (2013), *Maximum-likelihood estimation of lithospheric flexural rigidity, initial-loading fraction and load correlation, under isotropy*, GJI 193, 1300–1342.