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Turbulence can cause anisotropy

SA and BM anisotropies in 3 cases $(h,\ p,\ f)$

Main equations and parameters

Comparison of different anisotropies' effects

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- All kinds of anisotropy with greater diffusivity in vertical direction facilitate convection
- There are also anisotropies which inhibit convection in comparison with isotropic case
- Anisotropies, in particular the strong ones, allow the preference (or/and existence) of some modes, what is not possible in the isotropic case

anisotropy = anisotropic diffusion, anisotropic diffusive coefficients, anisotropic diffusivities

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There is strong belief that the Earth's Core is in turbulent state

The Earth's core is driven into motion by buoyancy forces so strong that the flow and field are turbulent, fluctuating on every length and time scale, as it is accepted by the most of geophysicists.Traditional approach to turbulence (see, e.g., Krause and Rädler, 1980)

$$\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}', \;\; \mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}'$$

the mean IE is:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \eta_0 \nabla^2 \overline{\mathbf{B}} + \boldsymbol{\nabla} \times (\overline{\mathbf{u}} \times \overline{\mathbf{B}}) + \boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}}, \quad \boldsymbol{\mathcal{E}} = \overline{\mathbf{u}' \times \mathbf{B}'}$$

$\boldsymbol{\mathcal{E}} = \overline{\mathbf{u}' \times \mathbf{B}'}$
within suitable approximations and conditions we can say that $\boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}} = -\boldsymbol{\nabla} \times (\beta \boldsymbol{\nabla} \times \overline{\mathbf{B}})$, therefore

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \eta_0 \nabla^2 \overline{\mathbf{B}} + \boldsymbol{\nabla} \times (\overline{\mathbf{u}} \times \overline{\mathbf{B}}) + \beta \nabla^2 \overline{\mathbf{B}}, \quad \text{"beta-effect"}$$

In general, due to turbulence the diffusive coefficients are anisotropic; for instance the buoyancy has a preferred direction $\Rightarrow$ local turbulence may be significantly anisotropic with respect to gravity direction

$$\boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}} = -\boldsymbol{\nabla} \times (\beta \boldsymbol{\nabla} \times \overline{\mathbf{B}}) \rightarrow \boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}} = -\boldsymbol{\nabla} \times (\boldsymbol{\beta} \boldsymbol{\nabla} \times \overline{\mathbf{B}}) = \boldsymbol{\nabla} \cdot (\boldsymbol{\beta} \boldsymbol{\nabla} \overline{\mathbf{B}})$$

where $\boldsymbol{\beta}$ is a tensor quantity, therefore we can speak about an "anisotropic beta-effect" or anisotropic magnetic diffusivity, anisotropic $\boldsymbol{\eta}$ tensor $(\boldsymbol{\eta} = \boldsymbol{\eta}_0 + \boldsymbol{\beta})$

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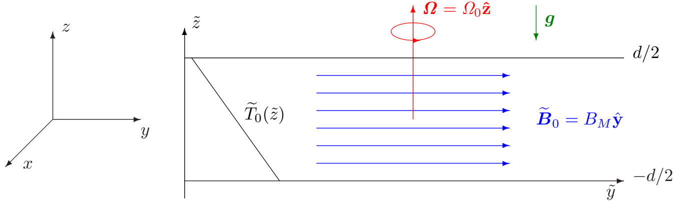
Anisotropy in viscosity and thermal diffusivity

Turbulence can cause anisotropy also in viscosity and thermal diffusivity (see, e.g., Fearn and Roberts, 2007), therefore it is worth to introduce anisotropy in the rotating magnetoconvection models; this was done for the first time in the model by Šoltis and Brestenský (2010) and recently it was advanced by Filippi et al. (2019).
These models deals about Rotating Magnetoconvection in horizontal plane layer rotating about vertical axis and permeated by homogeneous horizontal magnetic field (Roberts and Jones, 2000), influenced by anisotropic diffusivities, viscosity and thermal diffusivity.



Model of rotating magnetoconvection with homogeneous horizontal basic magnetic field in the infinite horizontal unstable stratified fluid layer with temperature profile, $T_0(\tilde{z})$.

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Anisotropic diffusive coefficients in SA and BM anisotropy (partial and full)

SA and BM anisotropic diffusions modelling

Partial anisotropy model by Šoltis and Brestenský (2010) with $\boldsymbol{\nu}$, $\boldsymbol{\kappa}$ anisotropic tensors and $\boldsymbol{\eta}_0$ isotropic tensor, where

$$\boldsymbol{\nu} = \begin{pmatrix} \nu_{xx} & 0 & 0 \\ 0 & \nu_{yy} & 0 \\ 0 & 0 & \nu_{zz} \end{pmatrix}, \quad \boldsymbol{\kappa} = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}, \quad \boldsymbol{\eta}_0 = \begin{pmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_0 & 0 \\ 0 & 0 & \eta_0 \end{pmatrix}.$$

Magnetic diffusion also due to turbulence

inspired a full anisotropy model (Filippi et al., 2019) with anisotropic $\boldsymbol{\nu}$, $\boldsymbol{\kappa}$, $\boldsymbol{\eta}$, where $\eta_{xx} = \eta_0 + \beta_{xx}$, $\eta_{yy} = \eta_0 + \beta_{yy}$, $\eta_{zz} = \eta_0 + \beta_{zz}$.

Later, we will speak about another kind of anisotropy, heat transport anisotropy, a partial anisotropy with $\boldsymbol{\nu}$ also isotropic.

SA, Stratification anisotropy:

$\nu_{xx} = \nu_{yy} \neq \nu_{zz}$, $\kappa_{xx} = \kappa_{yy} \neq \kappa_{zz}$, $\eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0$ and $\eta_{xx} = \eta_{yy} \neq \eta_{zz}$.

Gravity or/and the Archimedean buoyancy force are dominant

BM by Braginsky and Meytlis (1990):

$\nu_{xx} < \nu_{yy} = \nu_{zz}$, $\kappa_{xx} < \kappa_{yy} = \kappa_{zz}$, $\eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0$ and $\eta_{xx} < \eta_{yy} = \eta_{zz}$.

Rotation and magnetic field are dominant

There is "horizontal isotropy" in SA, but not in BM

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Heat transport anisotropy

Donald and Roberts (2004) developed a dynamo model which introduces anisotropy only in the thermal diffusivity. Inspired by them we began to study also another case of anisotropy in rotating magnetoconvection, we call it Heat transport anisotropy.

$$\boldsymbol{\nu} = \begin{pmatrix} \nu_0 & 0 & 0 \\ 0 & \nu_0 & 0 \\ 0 & 0 & \nu_0 \end{pmatrix}, \quad \boldsymbol{\kappa} = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}, \quad \boldsymbol{\eta}_0 = \begin{pmatrix} \eta_0 & 0 & 0 \\ 0 & \eta_0 & 0 \\ 0 & 0 & \eta_0 \end{pmatrix}.$$

This new simplified model is developed for wider chance to compare the effects of more anisotropic models.

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Dimensionless governing equations and parameters (partial and full) 1/2

So, in case of partial and full anisotropy we get the following main dimensionless equations, after using standard procedures

$$R_o \partial_t \mathbf{u} + \hat{\mathbf{z}} \times \mathbf{u} = -\nabla p + \Lambda(\nabla \times \mathbf{b}) \times \hat{\mathbf{y}} + R\vartheta \hat{\mathbf{z}} + E_z \nabla_\nu^2 \mathbf{u}$$

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \hat{\mathbf{y}}) + \nabla^2 \mathbf{b}, \quad \partial_t \mathbf{b} = \nabla \times (\mathbf{u} \times \hat{\mathbf{y}}) + \nabla_\eta^2 \mathbf{b}$$

$$\frac{1}{q_z} \partial_t \vartheta = \hat{\mathbf{z}} \cdot \mathbf{u} + \nabla_\kappa^2 \vartheta,$$

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{B} = 0$$

where $\tilde{z} = dz$, $\tilde{\mathbf{b}} = B_M \mathbf{b}$ and

$$\tilde{\mathbf{u}} = U\mathbf{u} = \frac{\eta_0}{d}\mathbf{u}, \qquad \tilde{t} = \frac{d}{U} = \frac{d^2}{\eta_0} t, \qquad \tilde{p} = 2\Omega_0 \eta_0 \rho_0 p, \quad \tilde{\vartheta} = \frac{\eta_0 \Delta T}{\kappa_{zz}} \vartheta,$$

$$\tilde{\mathbf{u}} = U\mathbf{u} = \frac{\eta_{zz}}{d}\mathbf{u}, \qquad \tilde{t} = \frac{d}{U} t = \frac{d^2}{\eta_{zz}} t, \qquad \tilde{p} = 2\Omega_0 \eta_{zz} \rho_0 p, \quad \tilde{\vartheta} = \frac{\eta_{zz} \Delta T}{\kappa_{zz}} \vartheta$$

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Dimensionless governing equations and parameters (partial and full) 2/2

$$R_o = \frac{\eta_0}{2\Omega_0 d^2},\ \Lambda = \frac{B_M^2}{2\Omega_0\rho_0\mu_0\eta_0},\ E_z = \frac{\nu_{zz}}{2\Omega_0 d^2},\ R = \frac{\alpha_T g\Delta T d}{2\Omega_0\kappa_{zz}},\ q_z = \frac{\kappa_{zz}}{\eta_0},$$

$$R_o = \frac{\eta_{zz}}{2\Omega_0 d^2},\quad \Lambda = \frac{B_M^2}{2\Omega_0\rho_0\mu_0\eta_{zz}},\quad q_z = \frac{\kappa_{zz}}{\eta_{zz}}$$

$$\alpha_\nu = \frac{\nu_{xx}}{\nu_{zz}},\quad \alpha_\kappa = \frac{\kappa_{xx}}{\kappa_{zz}},\quad \alpha_\eta = \frac{\eta_{xx}}{\eta_{zz}}$$

SA anisotropy:

$$E_z\nabla_\nu^2\mathbf{u} = E_z[(1-\alpha_\nu)\partial_{zz} + \alpha_\nu\nabla^2]\mathbf{u},\quad \nabla_\kappa^2\vartheta = [(1-\alpha_\kappa)\partial_{zz} + \alpha_\kappa\nabla^2]\vartheta,$$

$$\nabla_\eta^2\mathbf{b} = [(1-\alpha_\eta)\partial_{zz} + \alpha_\eta\nabla^2]\mathbf{b}$$

BM anisotropy:

$$E_z\nabla_\nu^2\mathbf{u} = E_z[(\alpha_\nu-1)\partial_{xx} + \nabla^2]\mathbf{u},\quad \nabla_\kappa^2\vartheta = [(\alpha_\kappa-1)\partial_{xx} + \nabla^2]\vartheta,$$

$$\nabla_\eta^2\mathbf{b} = [(\alpha_\eta-1)\partial_{xx} + \nabla^2]\mathbf{b}$$

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Method of solution in partial and full anisotropy

Further, for simplicity we assume that in partial and full anisotropy:

$$\alpha_\kappa = \alpha_\nu = \alpha, \qquad \alpha_\kappa = \alpha_\nu = \alpha_\eta = \alpha$$

solution by some methods developed, e.g., in Chandrasekhar (1961):

$$\nabla\cdot\mathbf{v} = \nabla\cdot\mathbf{B} = 0 \Rightarrow \mathbf{u} = a^{-2}[\nabla\times(\nabla\times w\hat{\mathbf{z}})+\nabla\times\omega\hat{\mathbf{z}}] \quad \text{and} \quad \mathbf{b} = a^{-2}[\nabla\times(\nabla\times b\hat{\mathbf{z}})+\nabla\times j\hat{\mathbf{z}}]$$

All perturbations (w, ω, b, j and ϑ) have a form like
$f(x,y,z,t) = \Re e[F(z)\exp(ilx+imy)\exp(\lambda t)]$ with $a = \sqrt{l^2+m^2}$, $\lambda = i\sigma \in \mathbb{C}$,
$F(z) = W(z)$, $\Omega(z)$, $B(z)$, $J(z)$, and $\Theta(z)$

If we take some curls of momentum, induction and heat equations, introduce inverted magnetic Prandtl number $p = \eta_{zz}/\nu_{zz} = R_o/E_z$ and we consider the simplest boundary conditions at $z = \pm 1/2$, i.e.:

- stress free boundaries: $W = D^2W = D\Omega = 0$
- perfectly thermally conducting boundaries: $\Theta = 0$
- perfectly electrically conducting boundaries: $B = DJ = 0$.

$$K^2 = \pi^2 + a^2, \quad K_\alpha^2 = \begin{cases} \pi^2 + \alpha l^2 + m^2 \\ \pi^2 + \alpha a^2 \end{cases} \text{in case of} \begin{matrix} \text{BM} \\ \text{SA} \end{matrix} \text{anisotropy}$$

$$R\frac{q_z(K^2+\lambda)}{(q_zK_\alpha^2+\lambda)}a^2 = \frac{K^2[E_z(K^2+\lambda)(K_\alpha^2+p\lambda)+m^2\Lambda]^2+\pi^2(K^2+\lambda)^2}{E_z(K^2+\lambda)(K_\alpha^2+p\lambda)+m^2\Lambda}$$

$$R\frac{q_z(K_\alpha^2+\lambda)}{(q_zK_\alpha^2+\lambda)}a^2 = \frac{K^2[E_z(K_\alpha^2+\lambda)(K_\alpha^2+p\lambda)+m^2\mathit{\Lambda}]^2+\pi^2(K_\alpha^2+\lambda)^2}{E_z(K_\alpha^2+\lambda)(K_\alpha^2+p\lambda)+m^2\mathit{\Lambda}}.$$

"dispersion relations"

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Critical Rayleigh numbers in stationary SA (partial, full) anisotropy

$\lambda = 0,\ 1_\alpha = (K_\alpha/K)^2$

$$R_p^s = \frac{\pi(\pi^2+a^2)^{1/2}K_\alpha^2}{a^2}\left(C_p + \frac{1}{C_p}\right),\ R^s = \frac{\pi(\pi^2+a^2)^{3/2}}{a^2}\left(C_\alpha + \frac{1_\alpha^2}{C_\alpha}\right).$$

where $C_p = \frac{E_z K^2 K_\alpha^2 + \Lambda m^2}{\pi K}$ and $C_\alpha = \frac{E_z K_\alpha^4 + \Lambda m^2}{\pi K}$
in SA $K_\alpha^2 = \pi^2 + \alpha a^2$ and

$$C_p = \frac{E_z(\pi^2+a^2)(\pi^2+\alpha a^2)+\Lambda m^2}{\pi(\pi^2+a^2)^{1/2}},\ C_\alpha = \frac{E_z(\pi^2+\alpha a^2)^2+\Lambda m^2}{\pi(\pi^2+a^2)^{1/2}} \qquad (1)$$

$$\frac{\partial R_p^s}{\partial C_p} = 0,\ \frac{\partial R_p^s}{\partial a^2} = 0 \qquad \text{and} \qquad \frac{\partial R^s}{\partial C_\alpha} = 0,\ \frac{\partial R^s}{\partial a^2} = 0$$

we have critical C_p, a^2 and C_α, a^2 in case of partial and full anisotropy respectively thus

$$C_{pc} = 1,\ a_c^2 = \pi^2 2_\alpha = \pi^2\frac{1+\sqrt{1+8\alpha}}{2\alpha} \quad \text{and} \quad C_{\alpha c} = 1_\alpha, a_c^2 = \pi^2 2_\alpha \qquad (2)$$

by the (1) and (2) it is possible to obtain m_c and l_c

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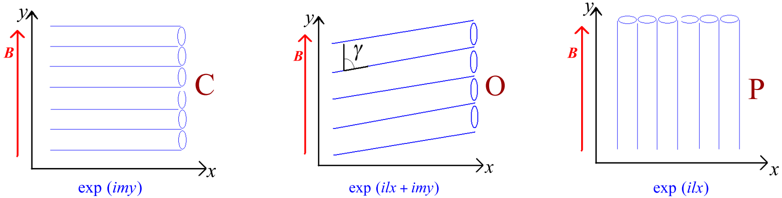
The orientation of the rolls of convection

Roberts and Jones (2000)



Cross rolls (stationary convection SC mode)

Oblique roll (stationary convection SO mode)

Parallel rolls (stationary convection P mode)

$\gamma = \arctan\left(\frac{m}{l}\right)$ is the angle between the axis of rolls and the magnetic field $\boldsymbol{B}$.
$l = 0 \Rightarrow \gamma = \pi/2$ rolls $\perp$ $\boldsymbol{B}$, $m = 0 \Rightarrow \gamma = 0$ rolls $\parallel$ $\boldsymbol{B}$

graphical results about partial and full Sa, So and BM anisotropy

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Comparison of three different anisotropic diffusions in SA and BM cases

heat transport (h), partial (p) and full (f) anisotropies

		isotropic	anisotropic
h-case $\Rightarrow$	$R\frac{q_z(K^2+\lambda)}{(q_z K_\alpha^2+\lambda)}a^2 = \frac{K^2N_h^2+\pi^2\left(K^2+\lambda\right)^2}{N_h}$, $N_h = (K^2+\lambda)(K^2+p\lambda)E + \Lambda m^2$	ν, η	κ
p-case $\Rightarrow$	$R\frac{q_z(K^2+\lambda)}{(q_z K_\alpha^2+\lambda)}a^2 = \frac{K^2N_p^2+\pi^2\left(K^2+\lambda\right)^2}{N_p}$, $N_p = (K^2+\lambda)(K_\alpha^2+p\lambda)E_z + \Lambda m^2$	η	ν, κ
f-case $\Rightarrow$	$R\frac{q_z(K_\alpha^2+\lambda)}{(q_z K_\alpha^2+\lambda)}a^2 = \frac{K^2N_f^2+\pi^2\left(K_\alpha^2+\lambda\right)^2}{N_f}$, $N_f = (K_\alpha^2+\lambda)(K_\alpha^2+p\lambda)E_z + \Lambda m^2$, $q_z = \kappa_{zz}/\eta_{zz}$	–	ν, κ, η

some regime diagrams in 3 cases of anisotropy

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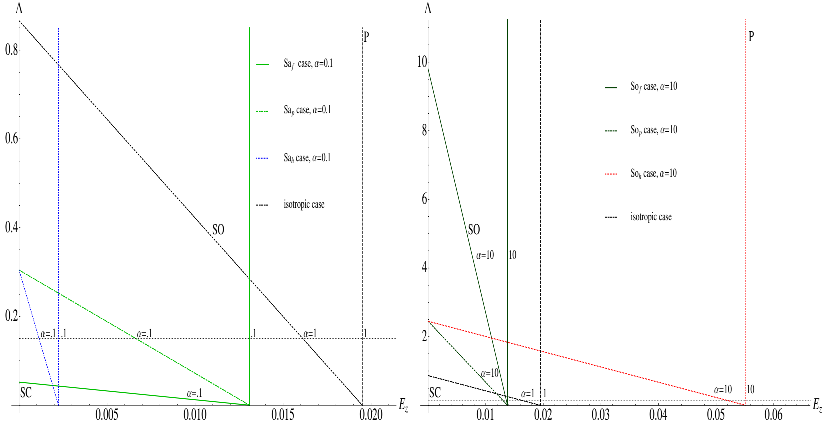
ΛE_z Regime diagrams in SA anisotropy ($\alpha = 0.1, 10$)



The ΛE_z regime diagrams for steady convection in SA anisotropy in h, p and f cases compared with isotropy. In the left figure atmospheric anisotropy (Sa), in the right figure oceanic anisotropy (So).

...back to comparison

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Anisotropic diffusive coefficients in SA and BM anisotropy (partial and full)

Assumption 1: following anisotropy in the viscosity and thermal diffusivity holds

$$\boldsymbol{\nu} = \begin{pmatrix} \nu_{xx} & 0 & 0 \\ 0 & \nu_{yy} & 0 \\ 0 & 0 & \nu_{zz} \end{pmatrix}, \qquad \boldsymbol{\kappa} = \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}.$$

Assumption 2: same diffusion in two directions, but a different diffusion in the third one. This is called *partial anisotropy* for reasons which will be clearer later. However, in a more general case we need to deal also with the anisotropy in the magnetic diffusion also due to turbulence

$$\eta_{xx} = \eta_0 + \beta_{xx}, \; \eta_{yy} = \eta_0 + \beta_{yy}, \; \eta_{zz} = \eta_0 + \beta_{zz}$$

We call this situation *full anisotropy* (Filippi et al., 2019) considering magnetic diffusivity also anisotropic

SA, Stratification anisotropy:

$\nu_{xx} = \nu_{yy} \neq \nu_{zz}, \; \kappa_{xx} = \kappa_{yy} \neq \kappa_{zz}, \eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0$ and $\eta_{xx} = \eta_{yy} \neq \eta_{zz}$.

Gravity or/and the Archimedean buoyancy force lead the dynamics of turbulent eddies

BM by Braginsky and Meytlis (1990):

$\nu_{xx} < \nu_{yy} = \nu_{zz}, \; \kappa_{xx} < \kappa_{yy} = \kappa_{zz}, \; \eta_{xx} = \eta_{yy} = \eta_{zz} = \eta_0$ and $\eta_{xx} < \eta_{yy} = \eta_{zz}$.

Rotation and magnetic field lead the dynamics of turbulent eddies

There is “horizontal isotropy” in SA, but not in BM

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Method of solution in partial and full anisotropy

In the next slides, for simplicity we assume that in partial and full anisotropy:

$$\alpha_\kappa = \alpha_\nu = \alpha, \quad \alpha_\kappa = \alpha_\nu = \alpha_\eta = \alpha$$

We look for a solution by using some methods developed, e.g., in Chandrasekhar (1961): $\mathbf{u}$ and $\mathbf{B}$ are divergenceless, therefore

$$\mathbf{u} = a^{-2}[\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times w\hat{\mathbf{z}}) + \boldsymbol{\nabla} \times \omega\hat{\mathbf{z}}] \quad \text{and} \quad \mathbf{b} = a^{-2}[\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times b\hat{\mathbf{z}}) + \boldsymbol{\nabla} \times j\hat{\mathbf{z}}]$$

All perturbations $(w, \omega, b, j$ and $\vartheta)$ have a form

$$f(x, y, z, t) = \Re e[F(z)\exp(ilx + imy)\exp(\lambda t)]$$

$a = \sqrt{l^2 + m^2}$, $\lambda = i\sigma \in \mathbb{C}$, $F(z) = W(z)$, $\Omega(z)$, $B(z)$, $J(z)$, and $\Theta(z)$

$$[E_z\mathcal{D}_\alpha - R_o\lambda]\Omega + DW + im\Lambda J = 0,$$

$$(D^2 - a^2)[E_z\mathcal{D}_\alpha - R_o\lambda]W - D\Omega + im\Lambda(D^2 - a^2)B = a^2 R\Theta,$$

$$(D^2 - a^2 - \lambda)J + im\Omega = 0, \quad (\mathcal{D}_\alpha - \lambda)J + im\Omega = 0$$

$$(D^2 - a^2 - \lambda)B + imW = 0, \quad (\mathcal{D}_\alpha - \lambda)B + imW = 0$$

$$(\mathcal{D}_\alpha - \zeta\lambda)\Theta + W = 0.$$

where $\zeta = q_z^{-1}$, $D = d/dz$ and $\mathcal{D}_\alpha$ is equal to $D^2 - \alpha l^2 - m^2$ and $D^2 - \alpha l^2 - \alpha m^2$ for BM and SA types of anisotropies, respectively.

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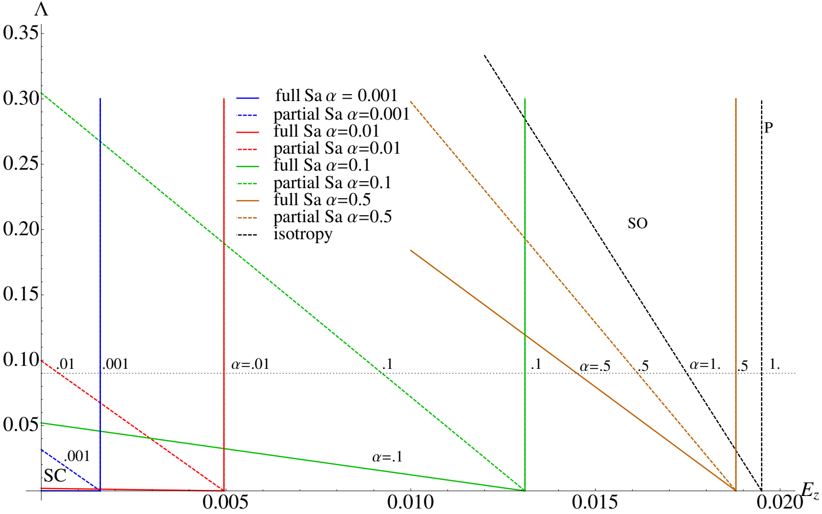
Regime diagrams in Sa anisotropy, $\alpha < 1$

ΛE_z diagrams for several cases of Sa anisotropy compared with isotropic case.



…back to rolls

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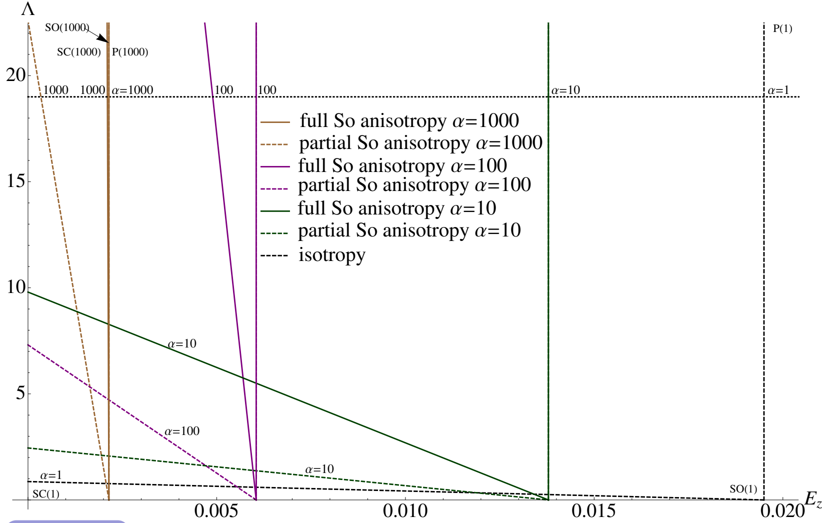
Regime diagrams in So anisotropy, $\alpha > 1$

ΛE_z diagrams for several cases of So anisotropy compared with isotropic case.



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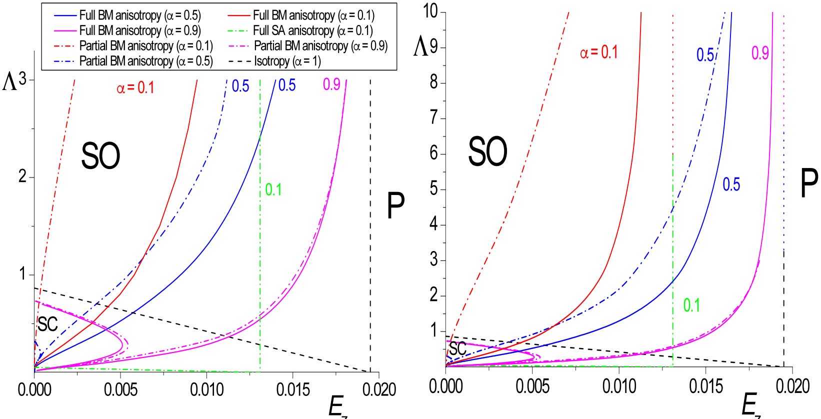
ΛE_z-Regime diagrams in BM anisotropy



The regime diagrams for steady convection in various cases of partial and full BM anisotropy in two different Λ-axis scales. For comparison cases of isotropy, $\alpha = 1$, and of strong Sa, $\alpha = 0.1$, are added; there are three asymptotes at $\Lambda \to \infty$, represented by the dotted vertical lines, for six SO/P lines for α = 0.1, 0.5, 0.9.

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