Non-Hydrostatic Depth-Averaged Modeling of Free Surface Flow Driven Sediment Transport

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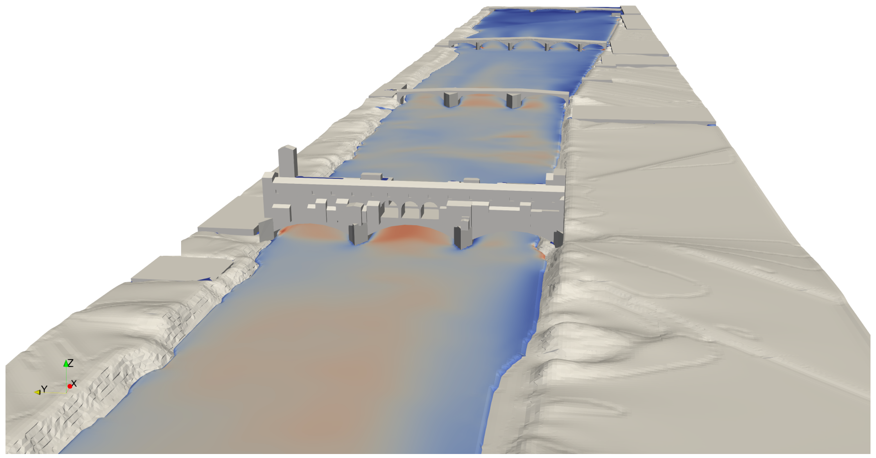
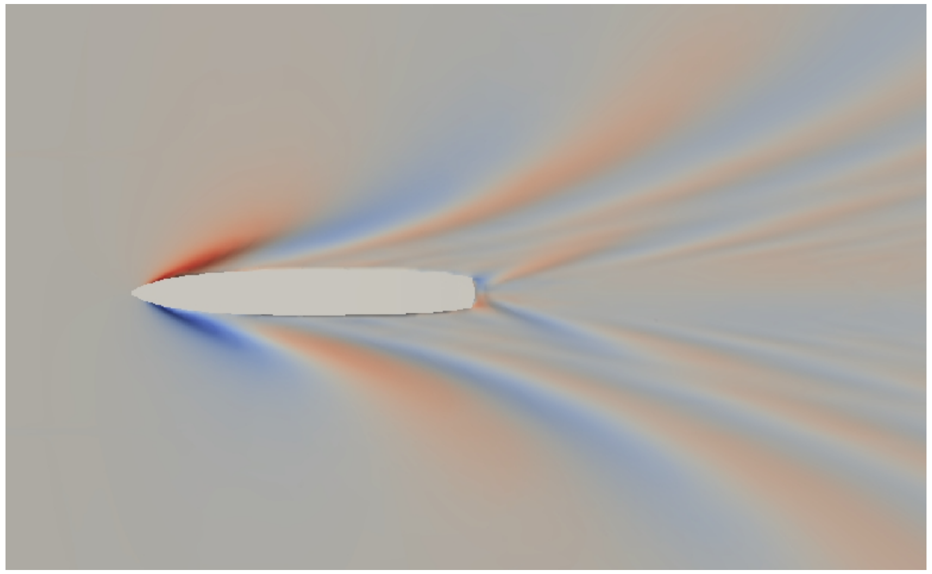
REEF3D : Open-Source Hydrodynamics

- Developed at the Department of Civil and Environmental Engineering, NTNU Trondheim



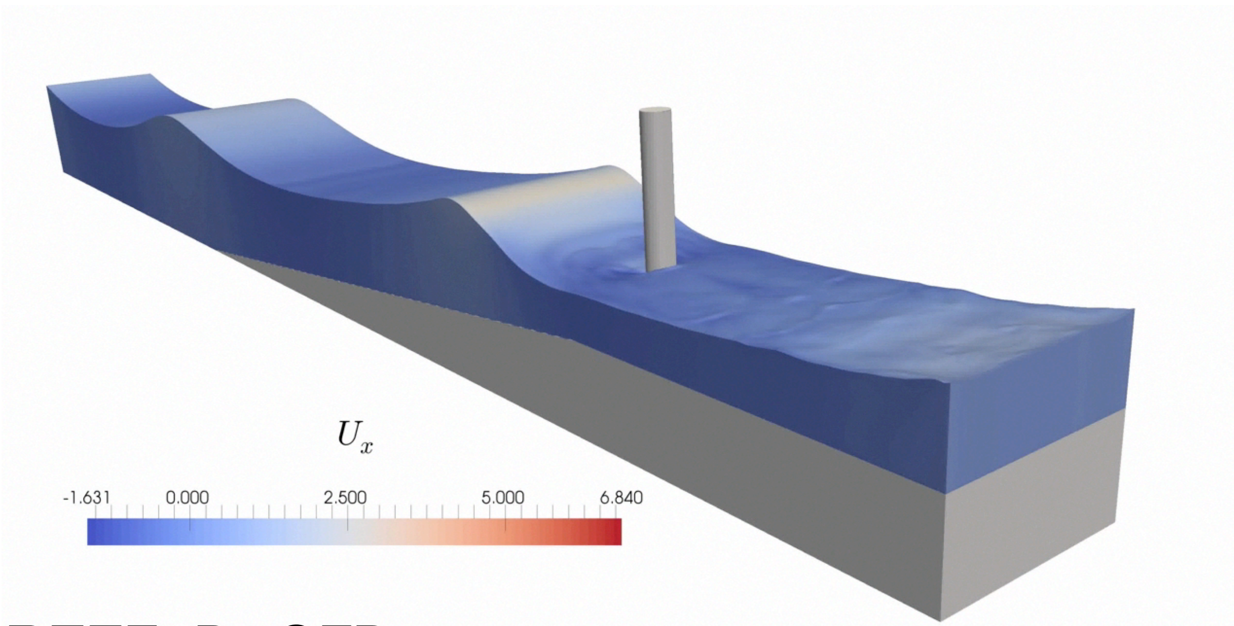
- **Focus**:
  - Free Surface Flows
  - Wave Hydrodynamics
  - Wave Structure Interaction
  - Floating Structures
  - Open Channel Flow
  - Sediment Transport



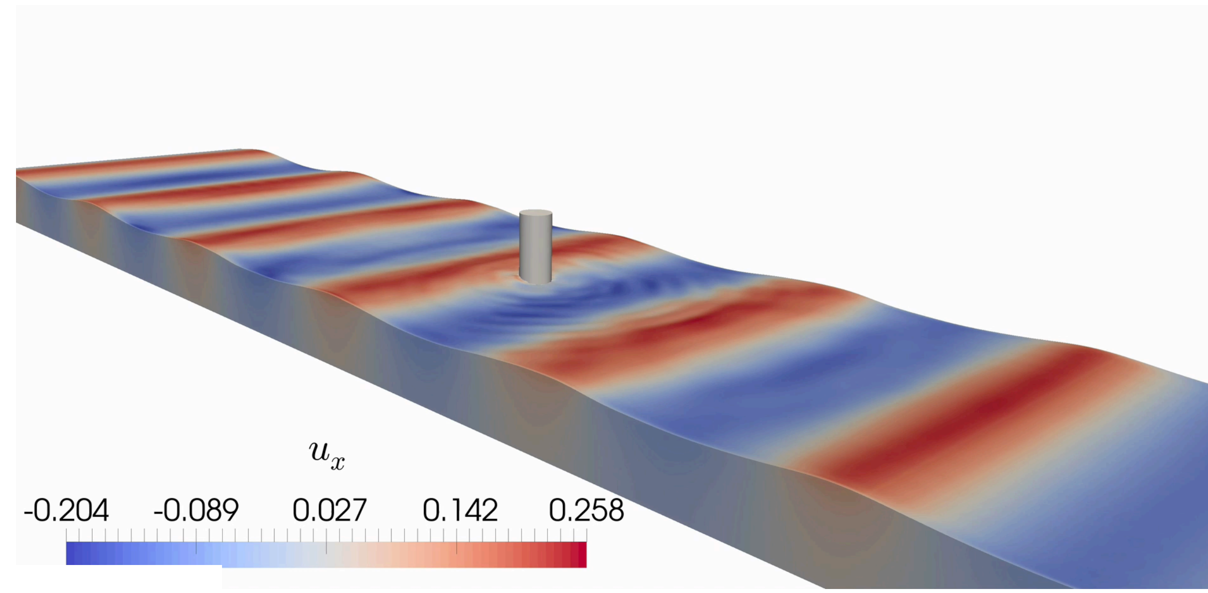
- Code written in C++
- modular code structure
- Published under GNU GPL v3

www.reef3d.com

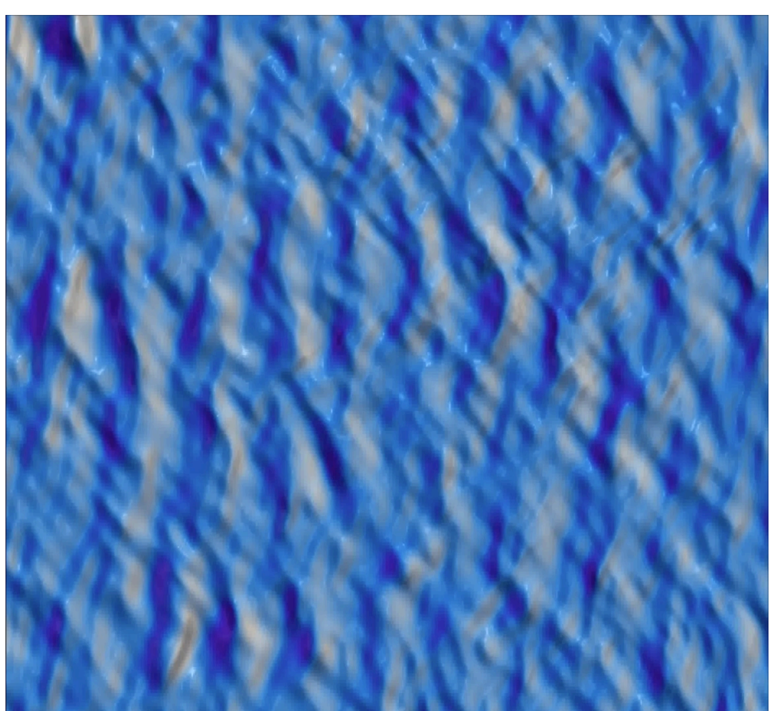
REEF3D : Open-Source Hydrodynamics



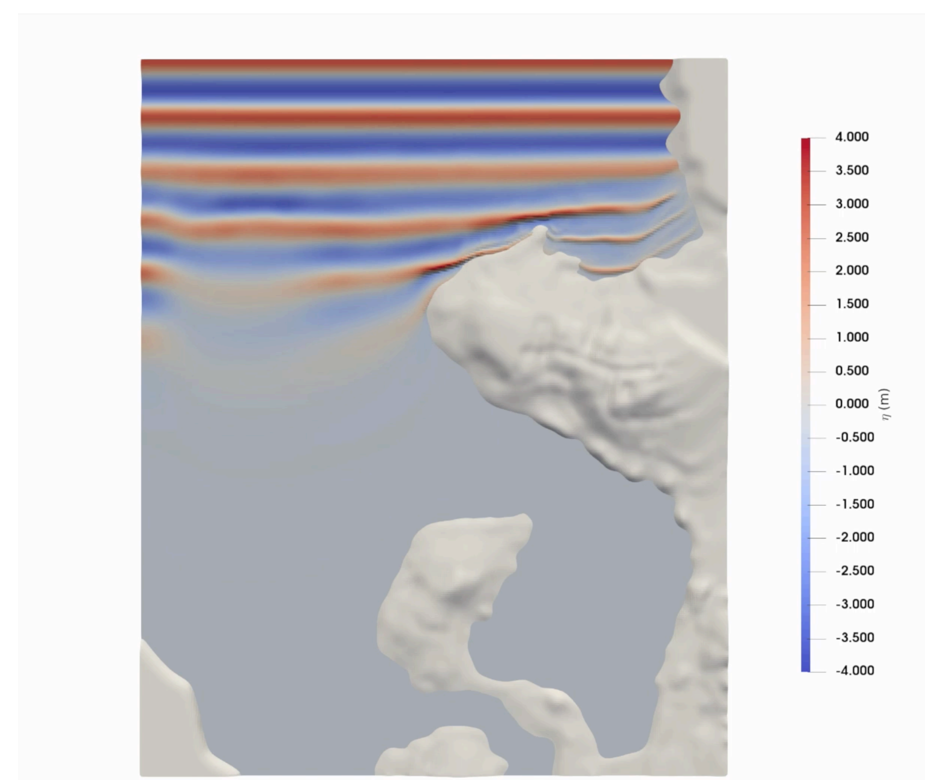
REEF3D::CFD



REEF3D::NSEWAVE



REEF3D::FNPF



REEF3D::SFLOW

REEF3D : Open-Source Hydrodynamics

high

Detail

low

Model	Dimensions	Turbulence	Br. Waves
REEF3D : CFD	3D	yes	yes
REEF3D : NSEWAVE	3D	yes	no
REEF3D : FNPF	3D	no	no
REEF3D : SFLOW	2D	yes	no

low

Speed

high

Motivation for Non-Hydrostatic SWE

Utvik [NVE, 2016]

Gudbrandsdal [NTB, 1995]

Longyearbyen

Motivation:
- large length scales
- extreme events: complex free surface
- large bed gradients
- sediment transport

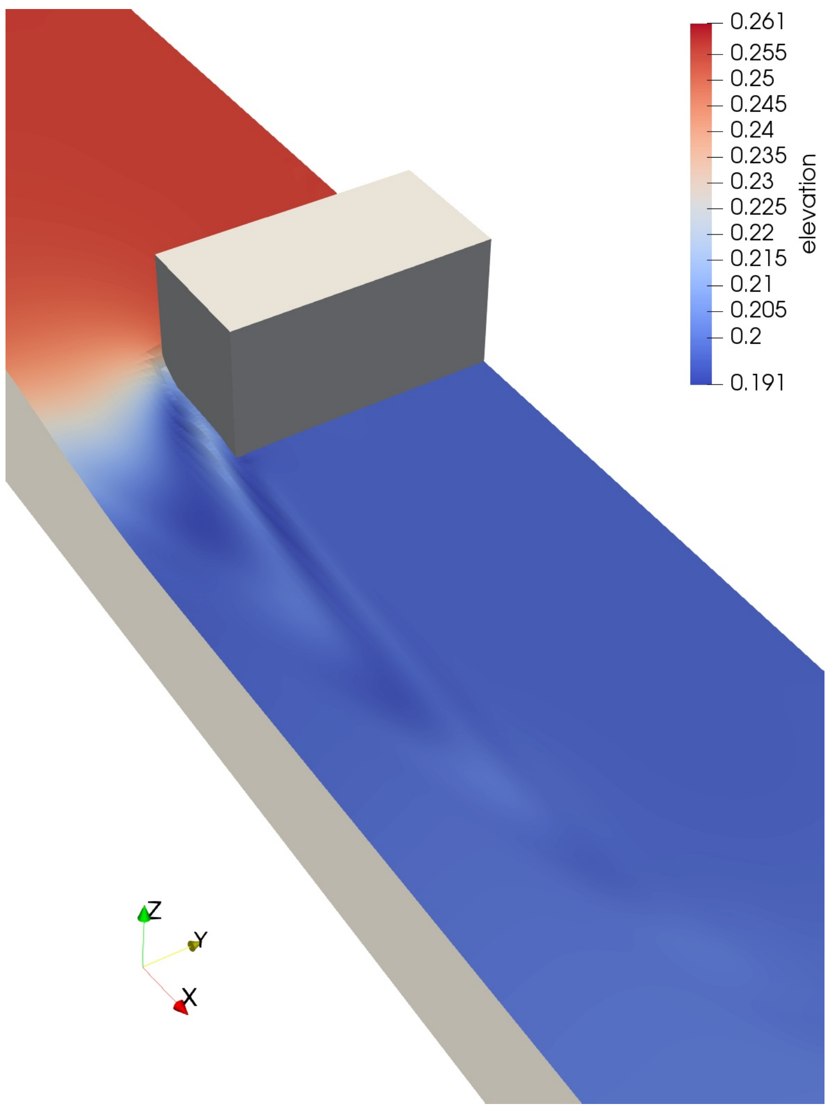
SWE: why non-hydrostatic?

Waves: d = 0.5m; H = 0.05m; L = 4.0m

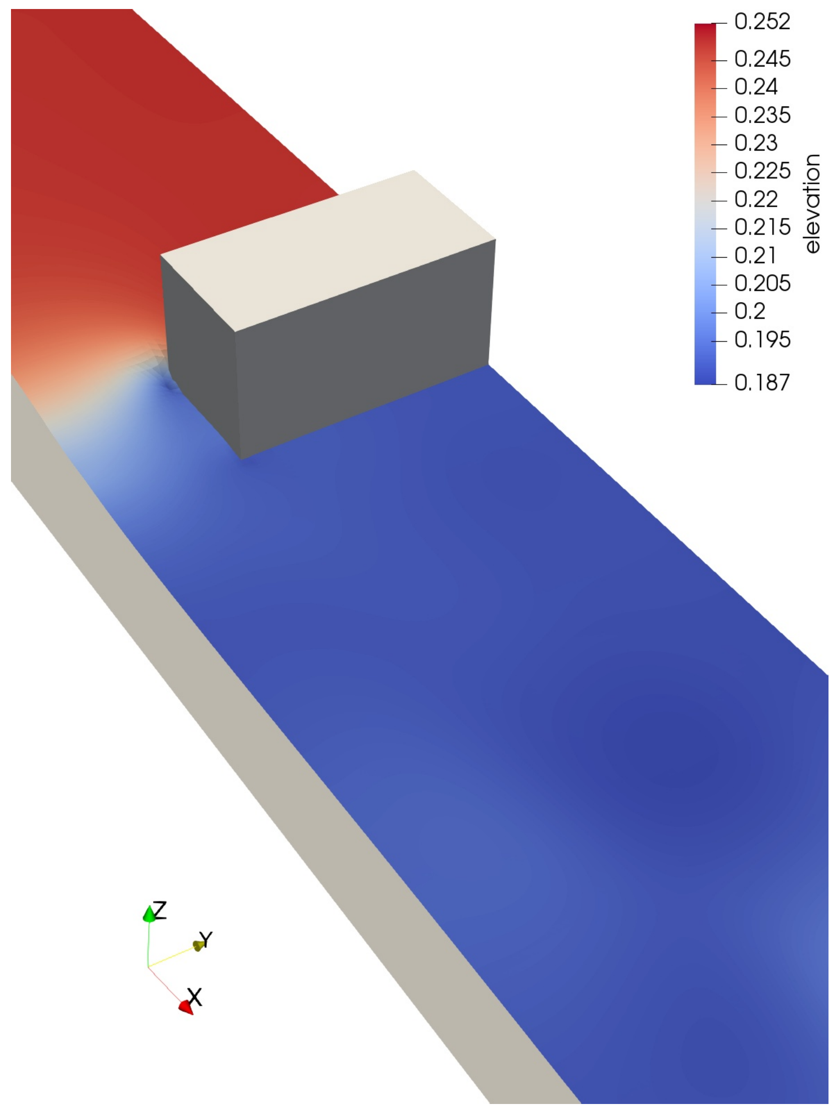
Hydrostatic

Non-Hydrostatic

SWE: why non-hydrostatic?



Non-Hydrostatic
free surface elevation



Hydrostatic
free surface elevation

[Haun]

Setup

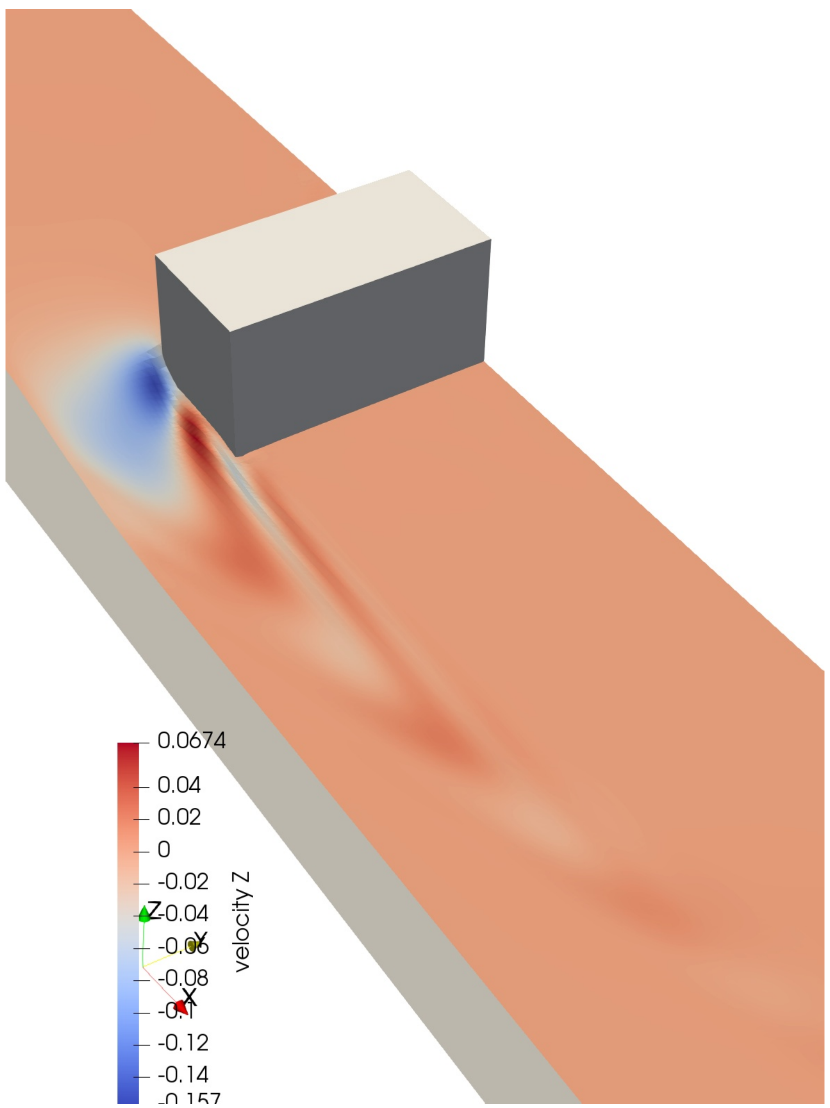
Q = 0.0257 m^3/s

h_{out} = 0.1985 m

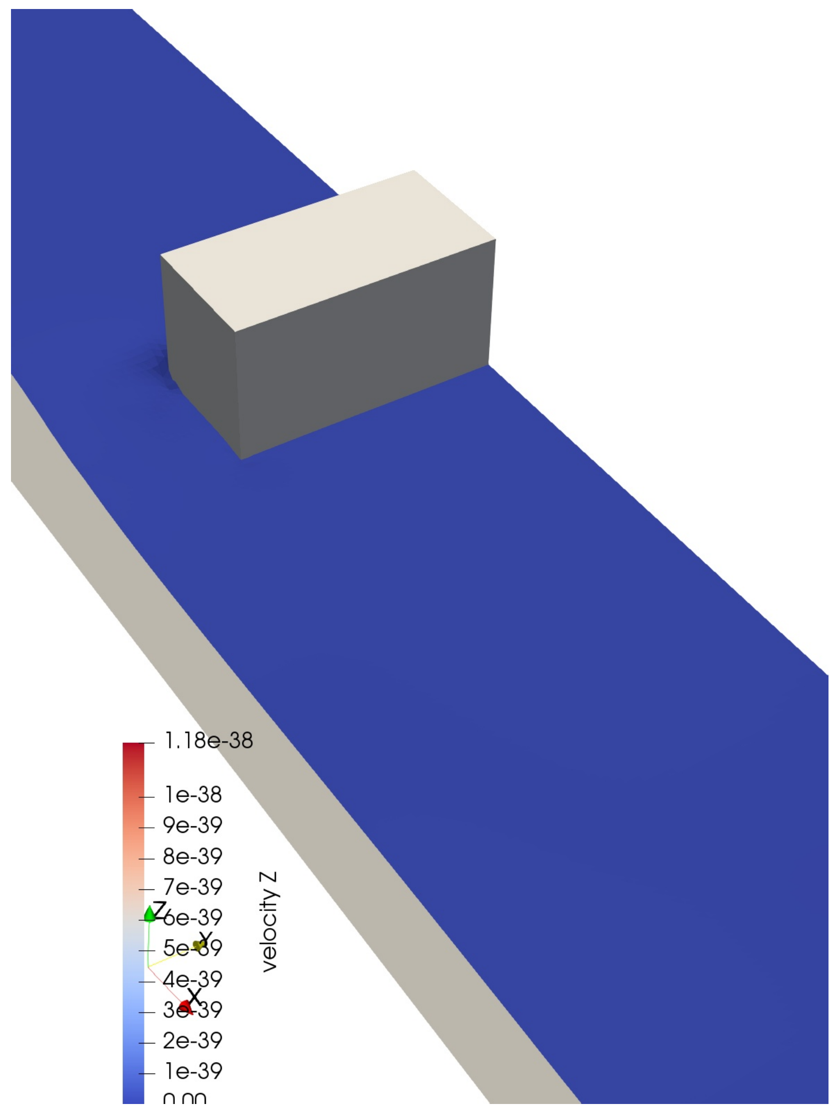
b = 0.6 m

$B_{Abutment}$ = 0.4 m

SWE: why non-hydrostatic?



Non-Hydrostatic
vertical velocity



Hydrostatic
vertical velocity

[Haun]

Setup

$Q = 0.0257\ m^3/s$

$h_{out} = 0.1985$ m

b = 0.6 m

$B_{Abutment} = 0.4$ m

Non-Hydrostatic Shallow Water Equations

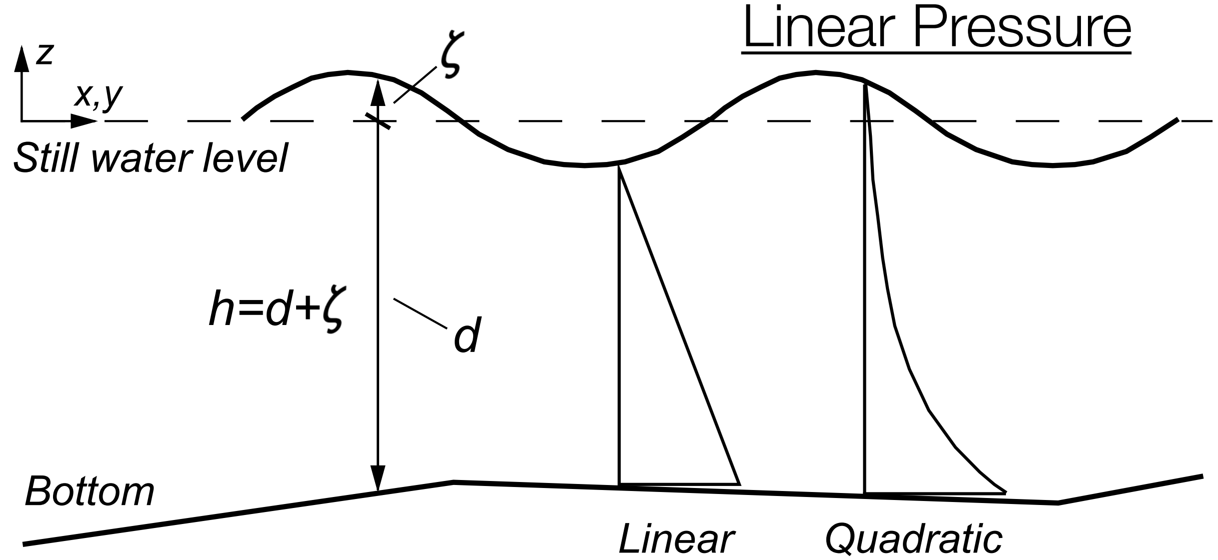
$$\frac{\partial \zeta}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0,$$ Free Surface

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -g\frac{\partial \zeta}{\partial x} - \frac{1}{\rho h}\left(\frac{\partial hq}{\partial x} - 2q\frac{\partial d}{\partial x}\right),$$ Velocity, x-dir

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\frac{\partial \zeta}{\partial y} - \frac{1}{\rho h}\left(\frac{\partial hq}{\partial y} - 2q\frac{\partial d}{\partial y}\right),$$ Velocity, y-dir

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = -\frac{2q}{\rho h}.$$ vertical velocity

Linear Pressure



Solution of the Dynamic Pressure

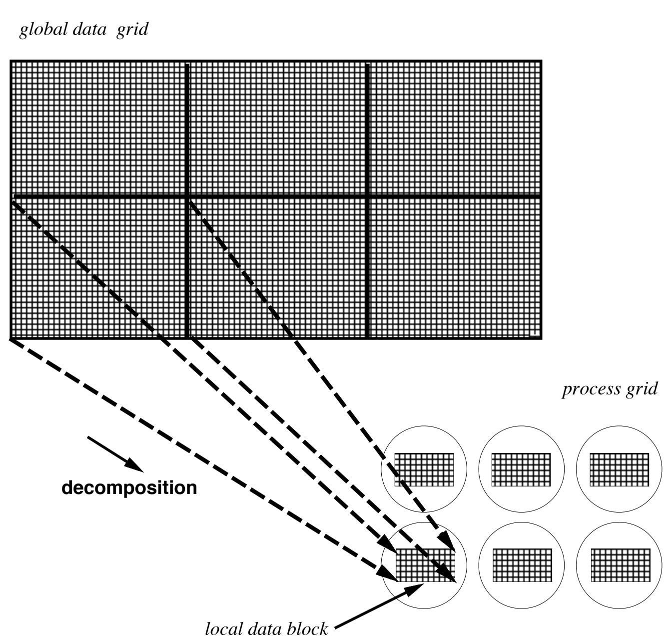
Poisson Eq. for dynamic pressure

$$\frac{h_p}{\rho}\left(\frac{\partial^2 q}{\partial x^2}+\frac{\partial^2 q}{\partial y^2}\right)+\frac{2q}{\rho h_p}=\frac{1}{\partial x \partial t}\left(-h_p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-2w-u\frac{\partial d}{\partial x}-v\frac{\partial d}{\partial y}\right)$$

continuity equation

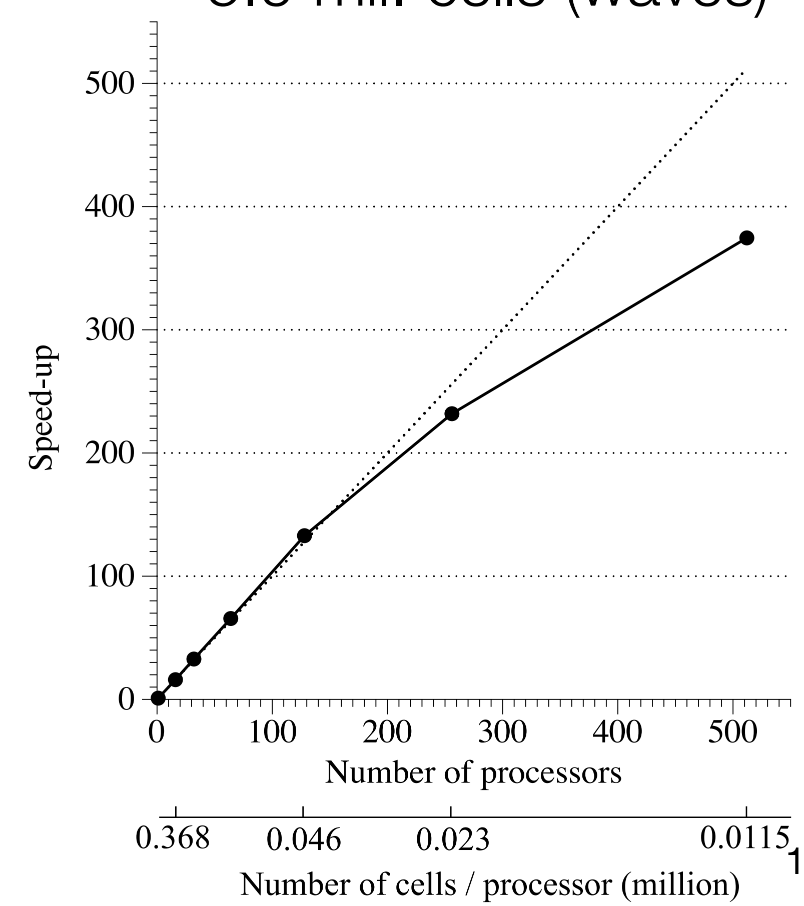
→ $Ax = b$ system of linear Equations

→ **hypre:**
geometric multigrid preconditioned conjugated gradient solver

→ 

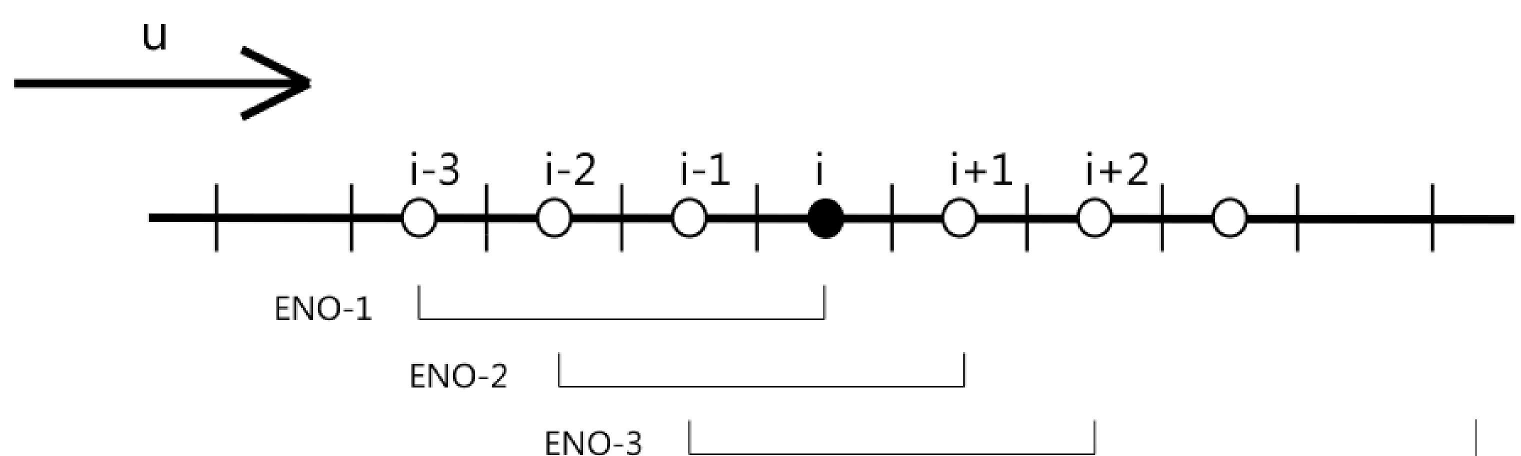
HPC:
domain decomposed parallelization, MPI

5.9 mil. cells (waves)



Spatial Discretization

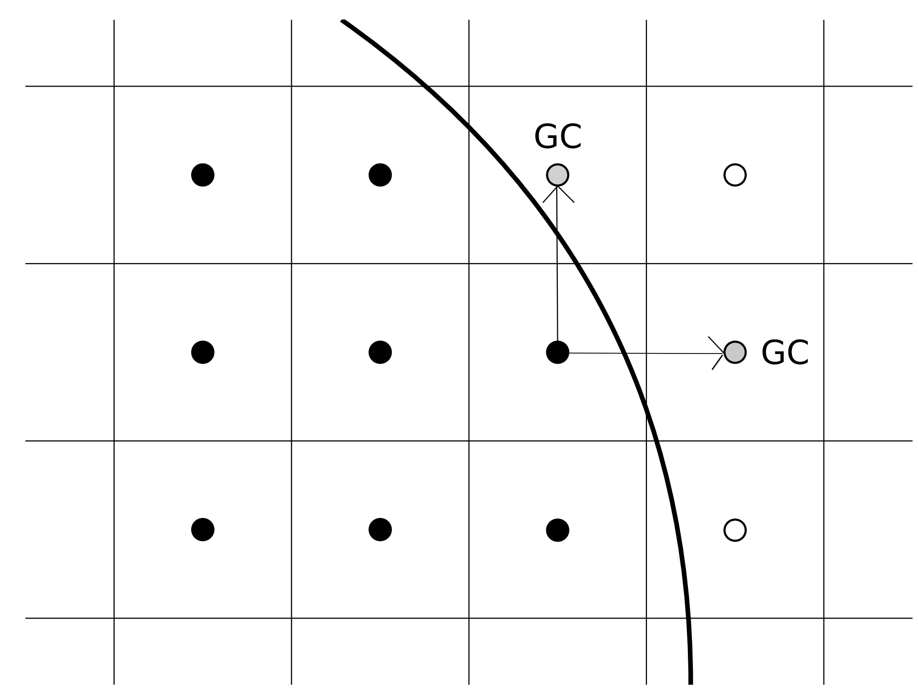
Convection Discretization: Conservative 5th-order WENO



$$U\frac{\partial U}{\partial x} \approx \frac{1}{\Delta x}\left(\tilde{U}_{i+1/2}U_{i+1/2} - \tilde{U}_{i-1/2}U_{i-1/2}\right)$$

$$U^{\pm}_{i+1/2} = \omega_1^{\pm}U^{1\pm}_{i+1/2} + \omega_2^{\pm}U^{2\pm}_{i+1/2} + \omega_3^{\pm}U^{3\pm}_{i+1/2}$$

- can handle large gradient
- high accuracy
- maintains the sharpness of the extrema



- **Ghost cell immersed boundary**
  - implicit enforcing of boundary conditions
  - no negative effect on numerical stability

Time Discretization

3rd-order TVD Runge-Kutta:

$$\phi^{(1)} = \phi^n + \Delta t L\left(\phi^n\right)$$
$$\phi^{(2)} = \frac{3}{4}\phi^n + \frac{1}{4}\phi^{(1)} + \frac{1}{4}\Delta L\left(\phi^{(1)}\right)$$
$$\phi^{n+1} = \frac{1}{3}\phi^n + \frac{2}{3}\phi^{(2)} + \frac{2}{3}\Delta L\left(\phi^{(2)}\right)$$

Adaptive Time-Stepping:

$$\delta t \leq 2\left(\left(\frac{|u|_{max}}{\delta x} + V\right) + \sqrt{\left(\frac{|u|_{max}}{\delta x} + V\right)^2 + \frac{4|g|_{g1}}{\delta x}}\right)^{-1}$$

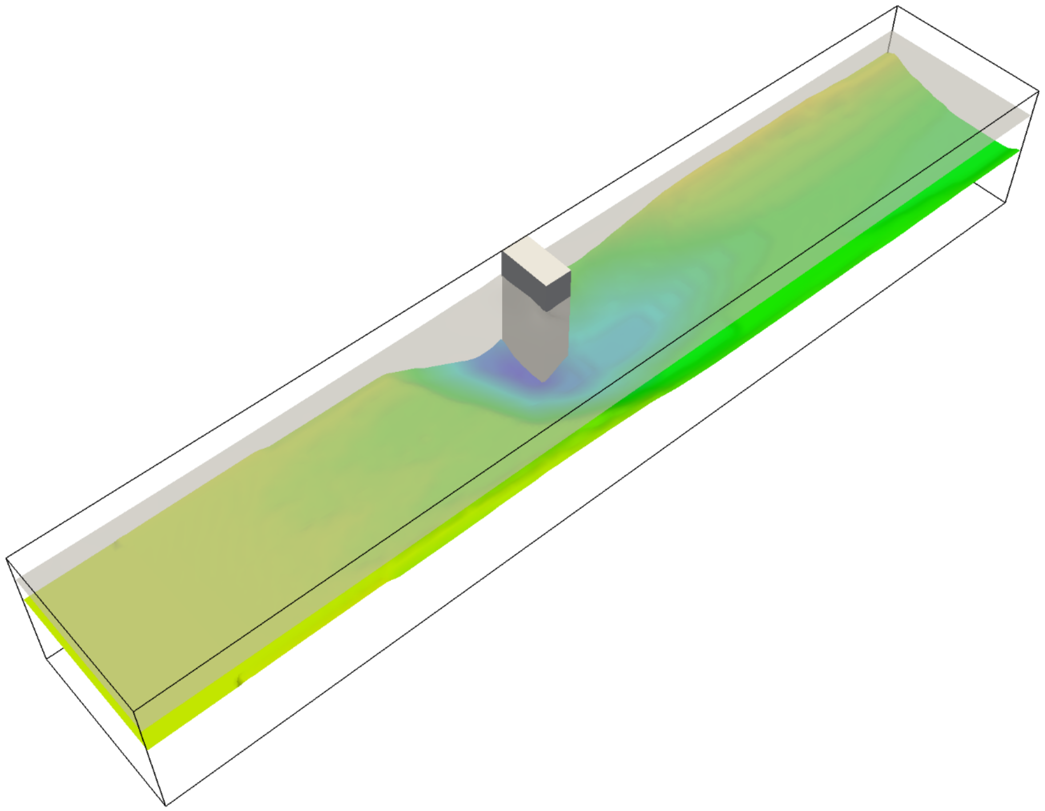
with

$$V = max\left(\nu + \nu_t\right) \cdot \left(\frac{2}{(\delta x)^2} + \frac{2}{(\delta y)^2} + \frac{2}{(\delta z)^2}\right)$$

Removing viscous time step constraint:
- implicit diffusion treatment

Sedimenttransport

- **Algorithmus**
  - bed shear stress from flow
  - bedload, e.g. van Rijn
  - sandslide algorithm
  - decoupling of time scales
  - Exner-Equation: bed changes

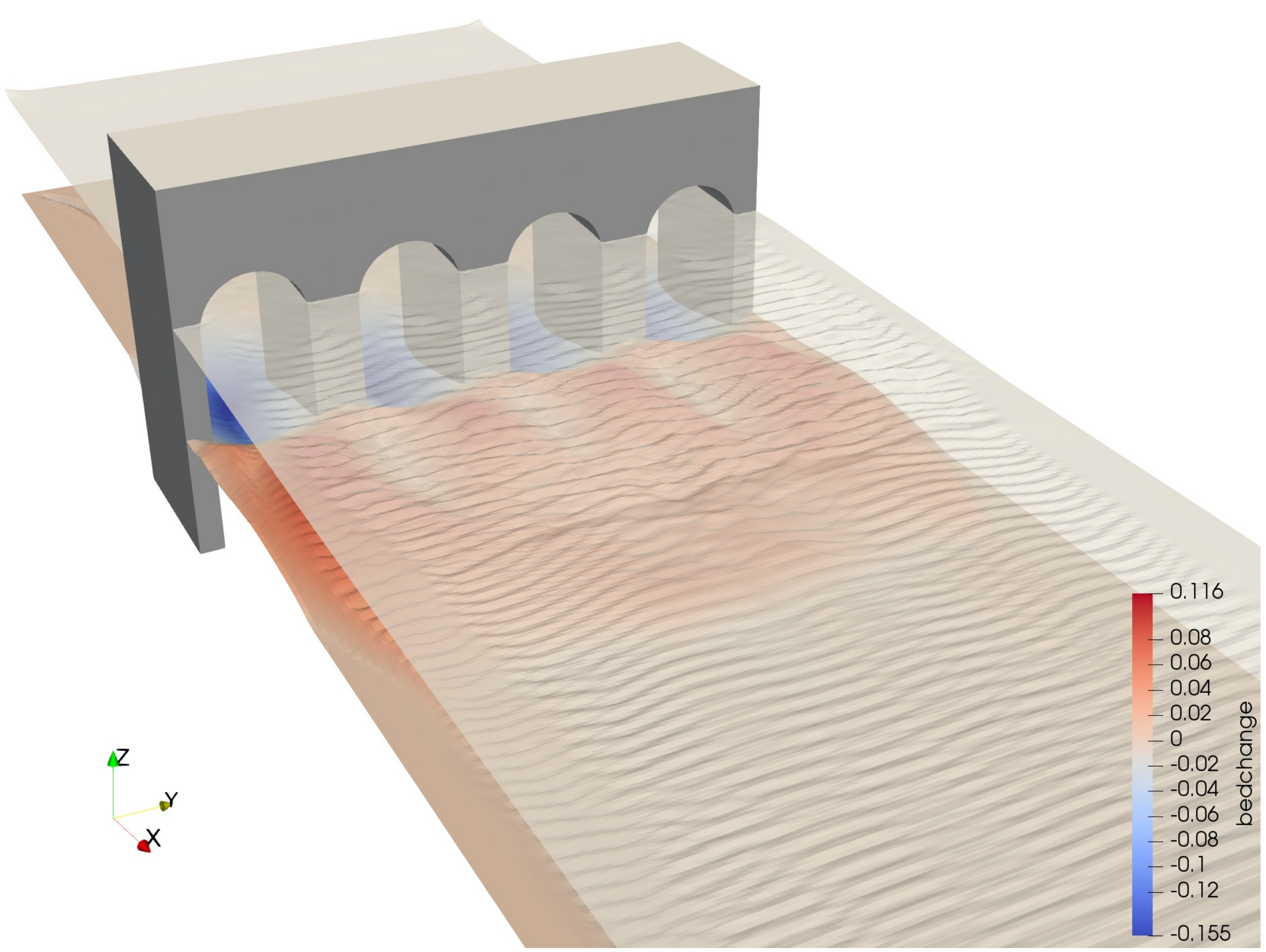


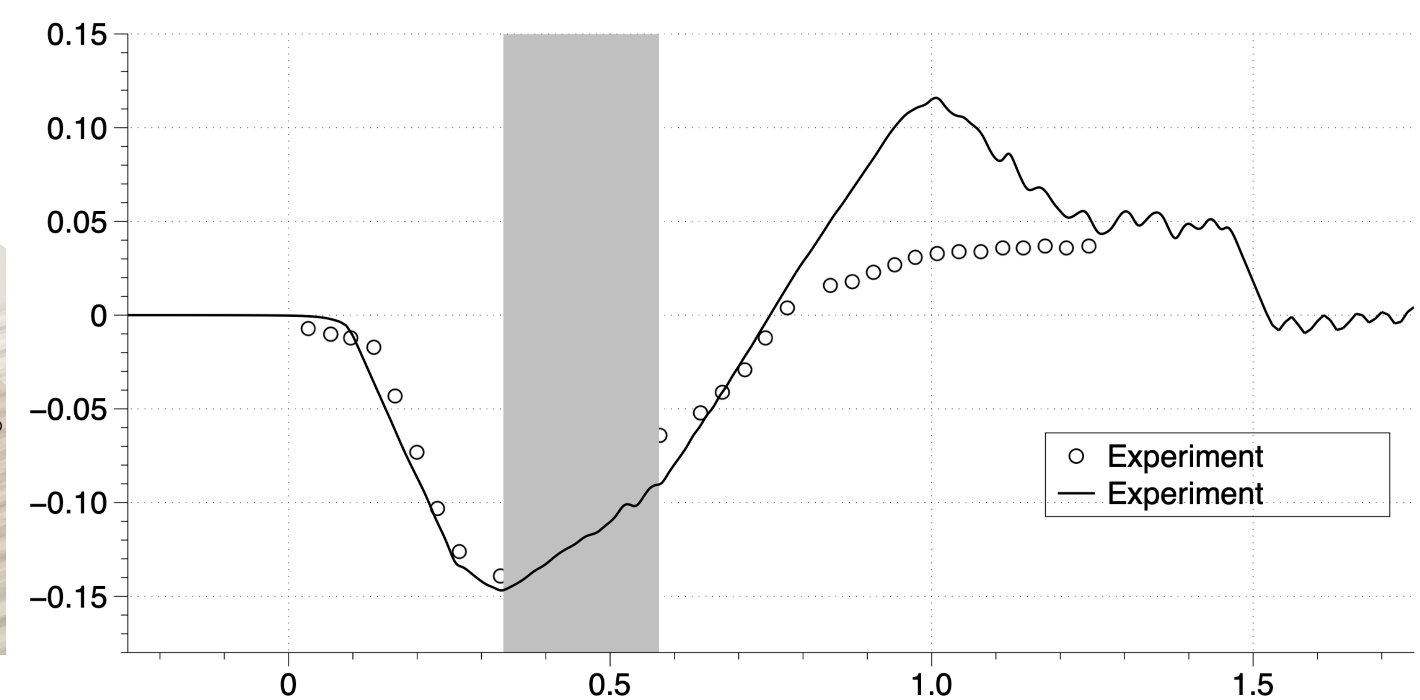
$$(1-n)\frac{\partial z_b}{\partial t} = -\frac{\partial q_{b,x}}{\partial x} - \frac{\partial q_{b,y}}{\partial z}$$

Scour Around Arch Bridge

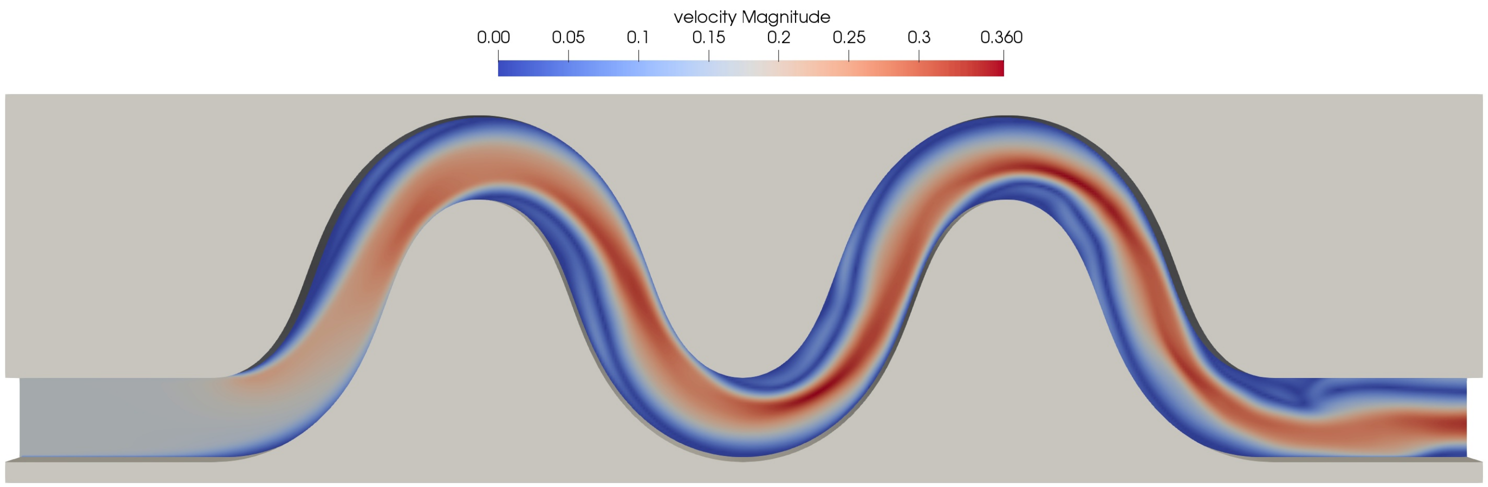
Experiment
Martin-Vide and Prio
JHR, 2005, "Backwater of arch bridges under free and submerged conditions"

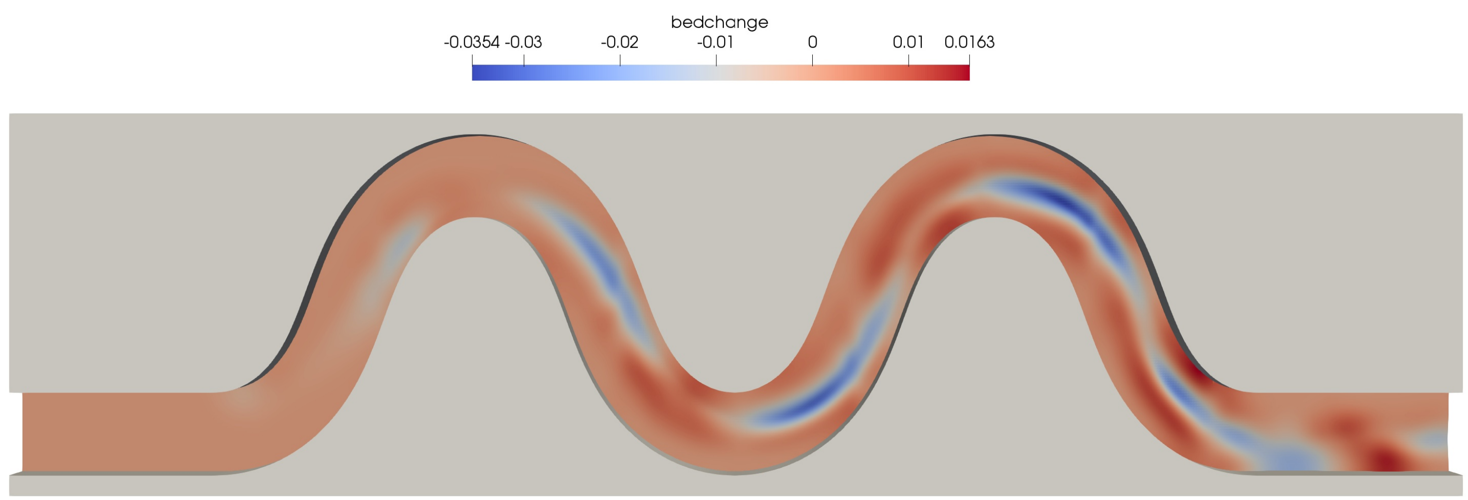
Setup
$Q = 0.1243\ m^3/s$
$h_{out} = 0.252\ m$
$b = 1.48\ m$
$D_{50} = 0.86\ mm$
$T_{sed} = 14400\ s$

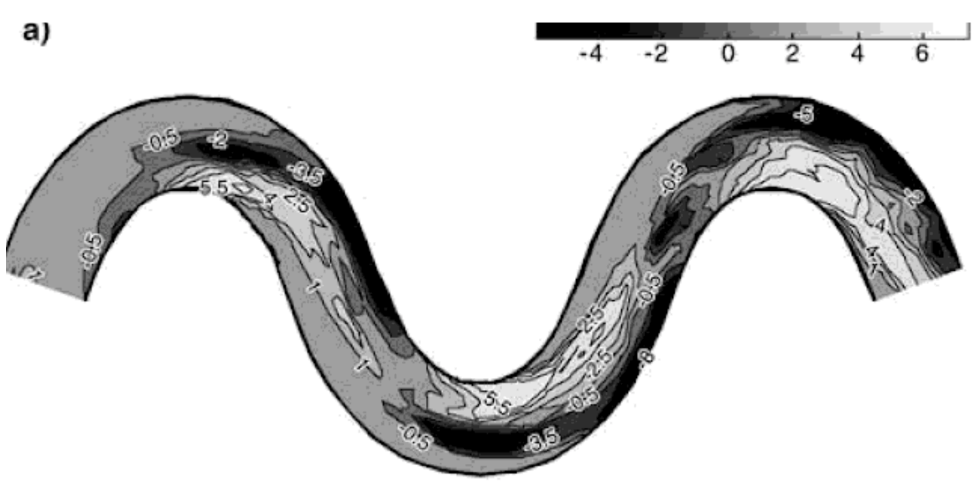




Sediment Transport in a Meandering Channel







Experiment
Ferreira da Silva and El-Tahawy Riverflow, 2006, "Location of hills and deeps in meandering streams: an experimental study "

Setup
$\Theta = 70°$
$Q = 0.011\ m^3/s$
$h_{av} = 0.075$ m
$b = 0.8$ m
$D_{50} = 0.65$ mm
$T_{sed} = 3600$ s

[experimental data from Ferreira da Silva & El-Tahawy, Riverflow 2006]

Conclusions

- new non-hydrostatic SWE-model
- high-performance computing
- high-order finite differences

- sediment transport with free surface
- more testing to be undertaken
- continuous code development

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