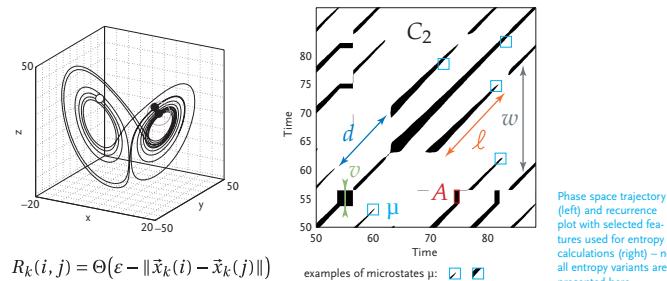


Recurrence Based Entropies

Norbert Marwan^{1,5}, Hauke Krämer^{1,2}, Karoline Wiesner³, Sebastian F. M. Breitenbach⁴, Jens Leonhardt⁵

¹Potsdam Institute for Climate Impact Research, ²Institute for Geosciences, University of Potsdam, ³School of Mathematics, University of Bristol, ⁴Institute of Geology, Ruhr University Bochum, ⁵Thüringer Höhlenverein, Erfurt

Recurrence plot



"Standard" RQA entropy

$$S = - \sum_{\ell=1}^N p(\ell) \ln p(\ell).$$

- variability of diagonal lines
- mostly used RP based entropy
- △ border effects (cut diagonal lines), noise, tangential motion

Intermittency entropy

$$S = - \sum_{v=1}^N p(v) \ln p(v).$$

- variability of vertical lines, i.e., laminar phases
- △ only valid for dynamics with intermittent/laminar regimes; meaning still unclear

References

- Breitenbach et al.: Holocene interaction of maritime and continental climate in Central Europe: New speleothem evidence from Central Germany, *Global and Planetary Change*, 176, 144–161 (2019).
 Krämer et al.: Recurrence Plot Based Entropies, in prep.
 Marwan et al.: Recurrence Plots for the Analysis of Complex Systems, *Physics Reports*, 438(5–6), 237–329 (2007).

Recurrence time entropy

$$S = - \sum_{w=1}^N p(w) \ln p(w).$$

- variability of recurrence times (periods)
- related to KS entropy
- △ border effects (cut vertical white lines)

Microstates entropy

$$S_\mu = - \sum_{\mu} p(\mu) \log p(\mu)$$

- variability of short time recurrence patterns
- fast (when using random subset)
- △ mixing of structures (no clear physical meaning); sensitive to embedding

K2 entropy (GP algorithm)

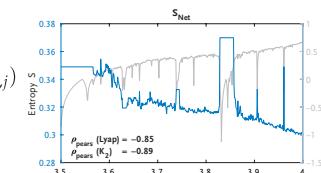
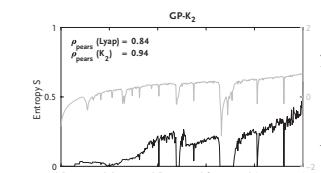
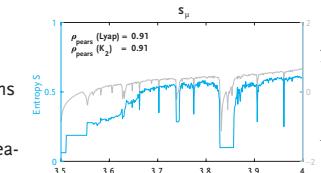
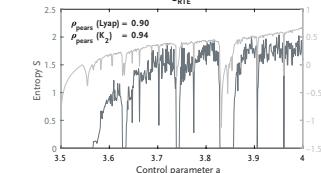
$$S_{K_2} = \frac{1}{k\Delta t} \ln \frac{\frac{1}{N_l(N_l-1)} \sum_{i,j} R_{i,j}^{(l)}}{\frac{1}{N_{l+k}(N_{l+k}-1)} \sum_{i,j} R_{i,j}^{(l+k)}}$$

- correlation entropy, mathematically straightforward, using correlation sum C_2
- △ high embedding dim. and scaling region required

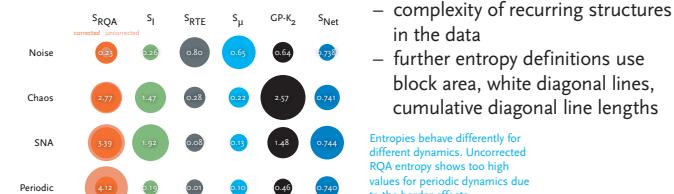
Network entropy

$$S_{Net} = \frac{1}{N \log(N-1)} \sum_i \log \sum_j (R_{i,j} - \delta_{i,j})$$

- heterogeneity of phase space density
- △ geometrical, not a dynamical measure



Abilities regarding different dynamics



Regime transition in palaeoclimate

