



Iterative Schwarz method to couple ocean and atmosphere in a Global Climate Model

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Abstract

For historical and practical reasons, present-day coupling algorithms implemented in ocean-atmosphere models are primarily driven by the necessity to conserve energy and water at the air-sea interface. However the asynchronous coupling algorithms currently used in ocean-atmosphere do not allow for a correct phasing between the ocean and the atmosphere.

In an asynchronous coupling algorithm, the total simulation time is split into smaller time intervals (a.k.a. coupling periods) over which averaged-in-time boundary data are exchanged. For a particular coupling period, the average atmospheric fluxes are computed in the atmospheric model using the oceanic surface properties computed and averaged by the oceanic model over the previous coupling period. Therefore, for a given coupling period, the fluxes used by the oceanic model are not coherent with the oceanic surface properties considered by the atmospheric model. The mathematical consistency of the solution at the interface is not guaranteed.

The use of an iterative coupling algorithm, such as Schwarz methods, is a way to correct this inconsistency and to properly reproduce the diurnal cycle when the coupling period is less than one day. In Lemarie et al. (2014), preliminary numerical experiments using the Schwarz coupling method for the simulation of a tropical cyclone with a regional coupled model were carried out. In ensemble simulations, the Schwarz iterative coupling method leads to a significantly reduced spread in the ensemble results (in terms of cyclone trajectory and intensity), thus suggesting that a source of error is removed with respect to the asynchronous coupling case.

In the present work, the Schwarz iterative method is implemented in IPSLCM6, a state-of-the-art global ocean-atmosphere coupled model used to study past, present and future climates. We analyse the convergence speed and the quality of the convergence. A partial iterative method is also tested: in a first phase, only the atmosphere physics and the vertical diffusion terms are computed, until the convergence. This provide a first guess for the full model which is then iterated until convergence of the whole system. The impact on the diurnal cycle will also be presented.

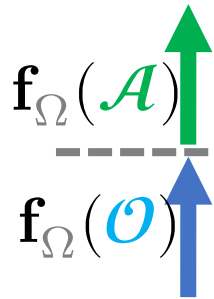
Air-sea coupling in theory

\mathcal{A} : the atmospheric state vector
(Temperature θ , Humidity q , Velocity \mathbf{U} , sea ice fraction, albedo α ...)

$$\frac{d\mathcal{A}}{dt} = \mathcal{F}_{\mathcal{A}}(\mathcal{A})$$

Fluxes across the boundary Ω

Heat
Water
Momentum
...



Ω

\mathcal{O} : the oceanic state vector
(Temperature θ , Salinity S , Velocity \mathbf{U} , ...)

$$\frac{d\mathcal{O}}{dt} = \mathcal{F}_{\mathcal{O}}(\mathcal{O})$$

Coupling = continuity at interface Ω

$$\left\{ \begin{array}{l} \mathcal{A}_{\Omega} = \mathcal{O}_{\Omega} \\ \mathbf{f}(\mathcal{A}_{\Omega}) = \mathbf{f}(\mathcal{O}_{\Omega}) \\ \left. \frac{\partial \mathcal{A}}{\partial z} \right|_{\Omega} = \left. \frac{\partial \mathcal{O}}{\partial z} \right|_{\Omega} = \mathbf{f}_{\Omega} \end{array} \right.$$

Air-sea coupling in models

Atmosphere \mathcal{A} is integrated with a Dirichlet lower boundary condition :

$$\mathcal{A}_\Omega = \mathcal{O}_\Omega.$$

i.e. surface values computed by the ocean are used as a lower boundary condition.

Time evolution of atmosphere can be written $\frac{d\mathcal{A}}{dt} = \mathcal{F}_\mathcal{A}(\mathcal{A}, \mathcal{O}_\Omega)$

Surface flux are computed by atmosphere from atmosphere state \mathcal{A} and surface boundary conditions given by ocean \mathcal{O}_Ω

$$\text{Surface flux : } \mathbf{f}_\Omega = \mathbf{f}_\Omega(\mathcal{A}, \mathcal{O}_\Omega)$$

Ocean is integrated with a Neumann surface boundary condition, using fluxes \mathbf{f}_Ω computed by the atmosphere

$$\frac{\partial \mathcal{O}}{\partial z} \Big|_\Omega = \mathbf{f}_\Omega$$

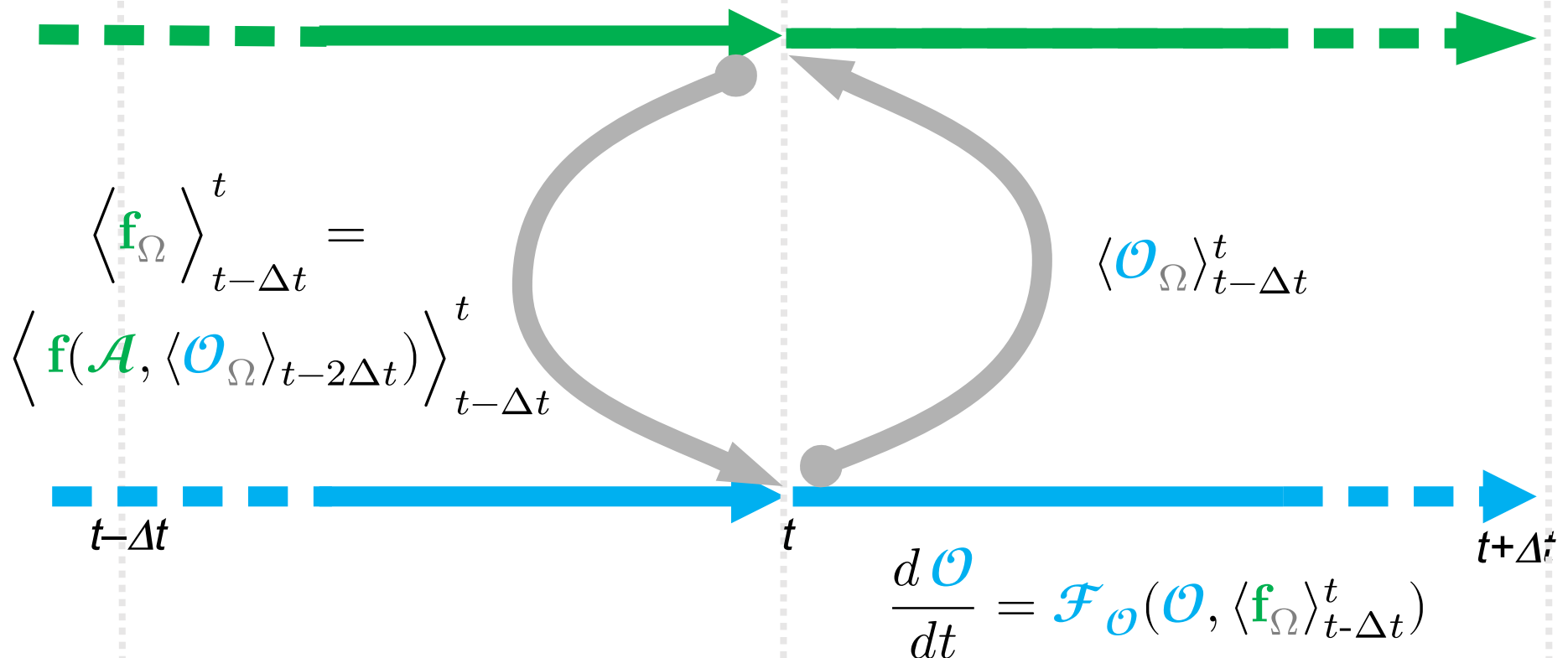
Time evolution of the ocean can be written : $\frac{d\mathcal{O}}{dt} = \mathcal{F}_\mathcal{O}(\mathcal{O}, \mathbf{f}_\Omega)$

Time stencil of the exchanges between ocean and atmosphere (without Schwarz)

Redrawn from Fig. 5.4, Lemarié et al.

$[t, t+\Delta t]$ is the coupling time step. $\langle \bullet \rangle$ is the time average between to times

$$\frac{d\mathcal{A}}{dt} = \mathcal{F}_{\mathcal{A}}(\mathcal{A}, \langle \mathcal{O}_{\Omega} \rangle_{t-\Delta t}^t)$$



Mathematical inconsistency of state-of-the-art ocean atmosphere models

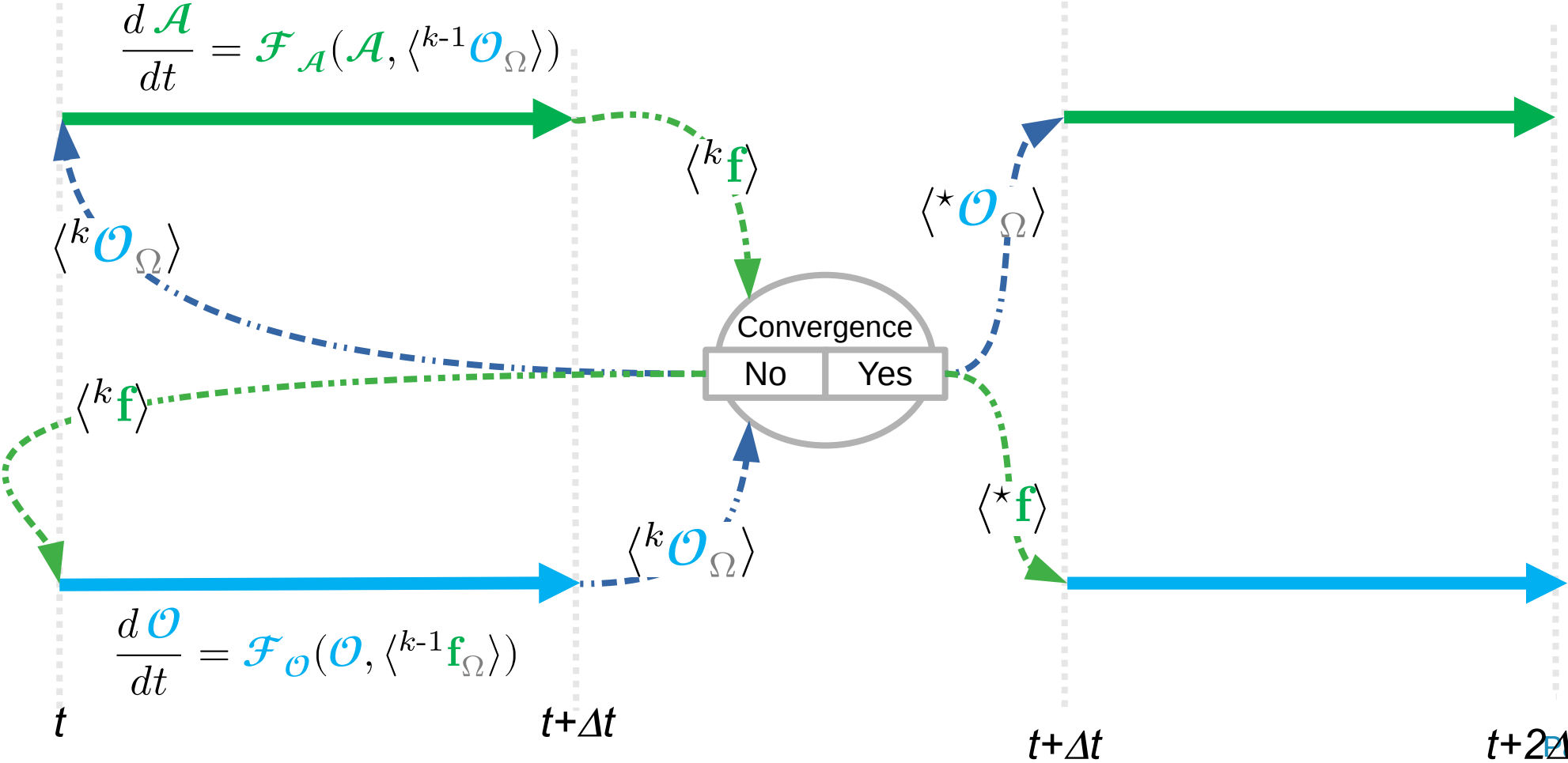
- During a coupling period, each component uses a boundary condition from the previous coupling period.
- i.e. the evolution of ocean \mathcal{O} from t to $t+\Delta t$ can be written as :

$$\left. \frac{d\mathcal{O}}{dt} \right|_t^{t+\Delta t} = \mathcal{F}_{\mathcal{O}}(\mathcal{O}, \left\langle \mathbf{f}(\mathcal{A}, \langle \mathcal{O}_{\Omega} \rangle_{t-2\Delta t}^{t-\Delta t}) \right\rangle_{t-\Delta t}^t)$$

- **The different intervals of time demonstrates a mathematical inconsistency.**
- Time stencils may differ between model, but no model uses a synchronous time stepping, no model is mathematically consistent
- Some models may use an implicit time stepping, but only for vertical diffusion. Not for the full model.

Schwarz iterative procedure during one coupling time step $[t, t+\Delta t]$

- Iterations from $k=1$ to convergence
- For each iteration, initial state of \mathcal{A} and \mathcal{O} are the solutions at the end of the previous time step $[t-\Delta t, t]$, when interface values of \mathcal{O}_Ω and \mathbf{f}_Ω have converged.
- * denotes the converged solution



The Schwarz iterative method

- If we call the converged solutions $^* \mathcal{A}$ and $^* \mathcal{O}$

$$\frac{d^* \mathcal{O}}{dt} \Big|_t^{t+\Delta t} = \mathcal{F}_{\mathcal{O}} \left(^* \mathcal{O}, \left\langle \mathbf{f}(^* \mathcal{A}, \langle ^* \mathcal{O}_{\Omega} \rangle_t^{t+\Delta t}) \right\rangle_t^{t+\Delta t} \right)$$

- **Re-synchronize components**
- **Mathematically consistent**

Schwarz in IPSL-CM

Model

Earth System Model IPSL-CM at low resolution (ocean 2°, atmosphere 96x95x39).

Simplified land surface model (bucket)

Sea-ice model : LIM3 monocategory

4 experiments :

Sw4h1i	Sw1h1i	Sw5h50i	Sw1h50i
Coupling time step			
$\Delta t=4h$	$\Delta t=1h$	$\Delta t=4h$	$\Delta t=1h$
Schwarz			
No	No	50 iterations	50 iterations

5 days runs

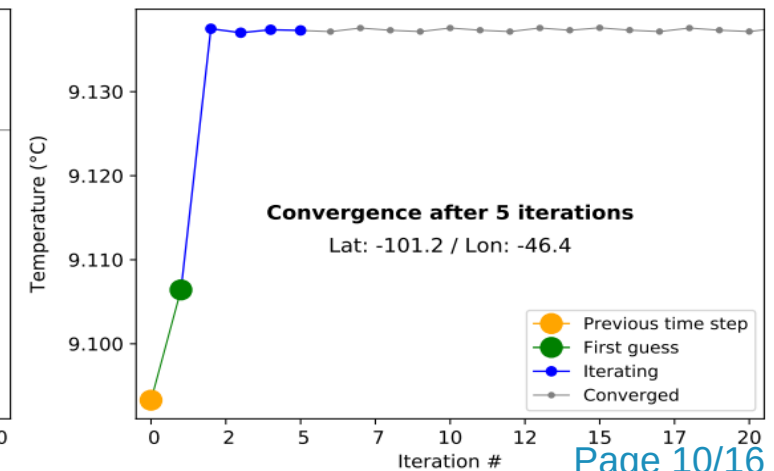
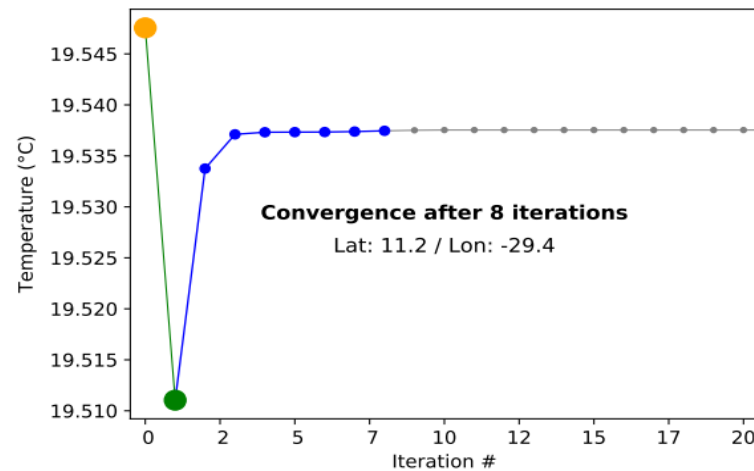
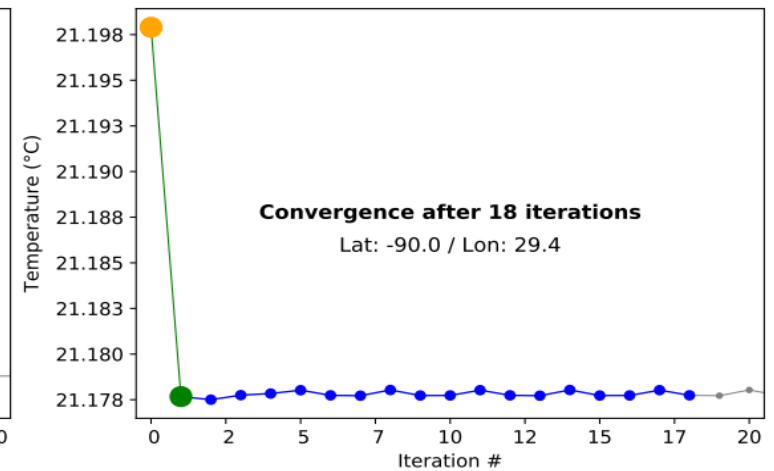
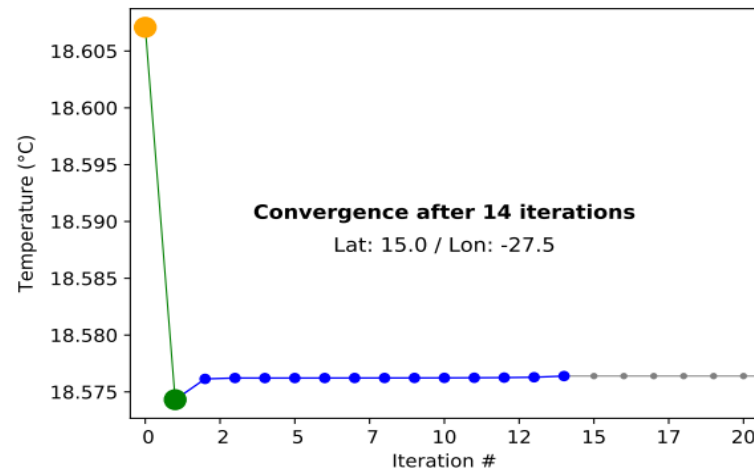
Fixed number of iterations

At each coupling time step, each grid point reaches convergence at it's own speed.

The number of iteration is ridiculously large to ensure convergence for all point, on to allow to study the behaviour of the algorithm

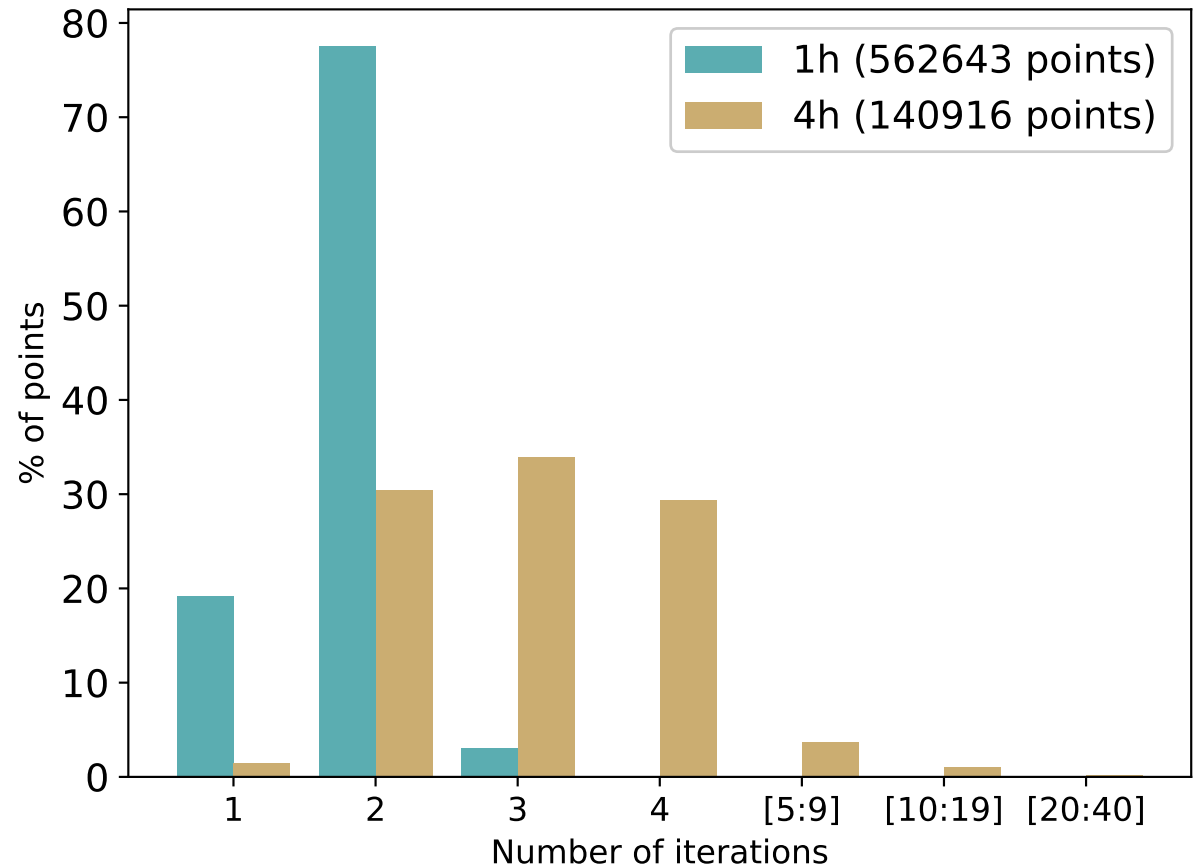
Iterations and convergence on a few selected points

Examples of behaviour of the sea surface temperature θ_{Ω} along the iterative process for four selected points in time and space. The yellow dots show the value at the end of the previous time step. The green dots show the value after the first iteration. It is the value that the models will use without Schwarz. The blue dots shows the iterative process. Dots become grey when θ_{Ω} is considered to be converged.



Number of iterations

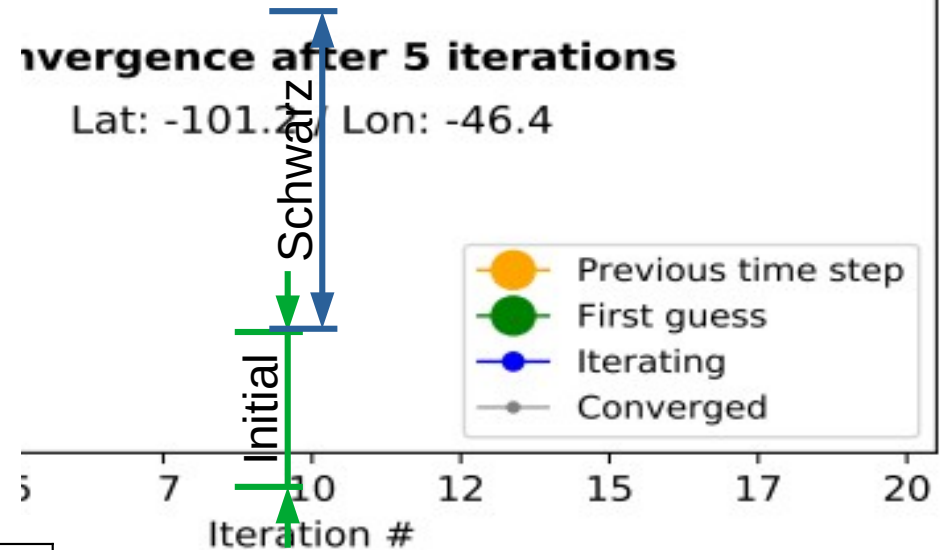
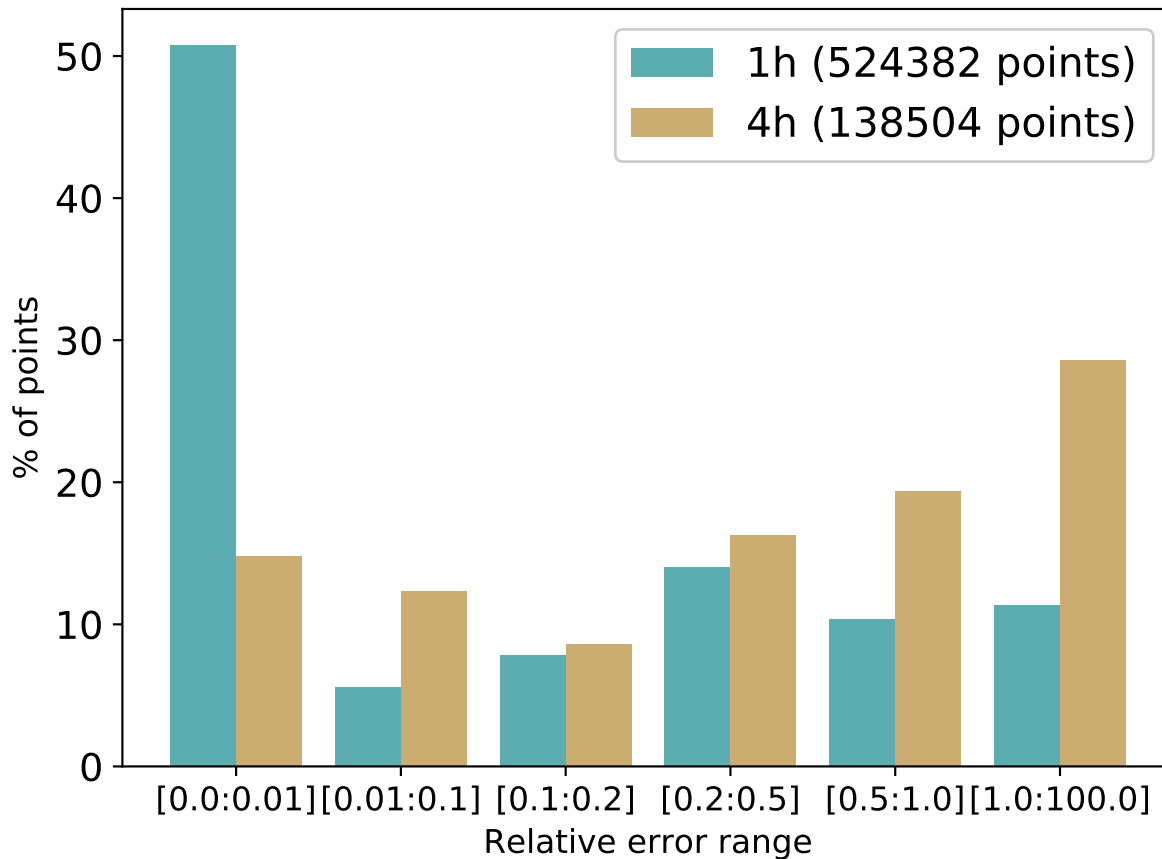
Number of iterations for the $\Delta t = 1$ hour and $\Delta t = 4$ hours (coupling time step) simulations. We have 140 916 cases in space x time in the $\Delta t = 4$ hour simulation if we consider only the points with no sea ice. And 562 643 points in space x time for the $\Delta t=1$ hour simulation. The ordinate shows the number of points that have converged in percentage of the total number of points in space x time points



Relative error

Relative error. We consider the ratio between i) the solution change between t to $t+\Delta t$ with no Schwarz iteration (**Initial** on the figure on right), and ii) the correction due to the iterative process (**Schwarz**).

The histogram plots the number of points (in space x time) for different range of relative error **Schwarz/Initial**.



With $\Delta t=1h$, the first guess (with no Schwarz iteration) yields an «negligible» relative error (less than 0.01) in 51 % of the cases, an «acceptable» relative error (less than 0.1) in 56 % of the cases

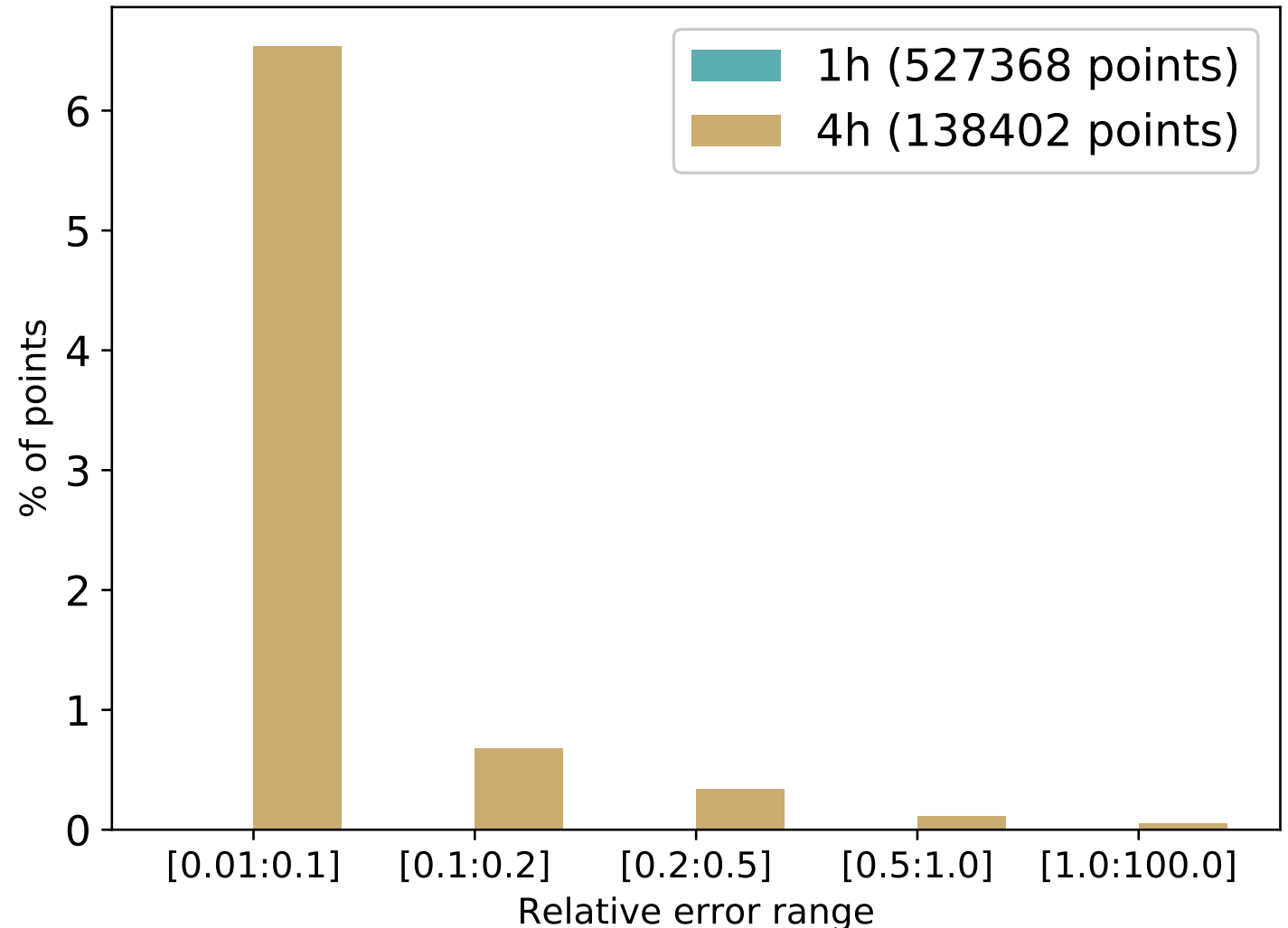
For $\Delta t=4h$, the error is « acceptable » for only 28 % of the cases.

Relative error

Same as previous, but we compare the final iterated solution to the solution obtained after 2 iterations.

For $\Delta t=1h$, the error is always negligible, showing that iterating 2 times gives of solution very closed to the converged one.

For $\Delta t=4h$. A few cases still show large error after 2 iterations.

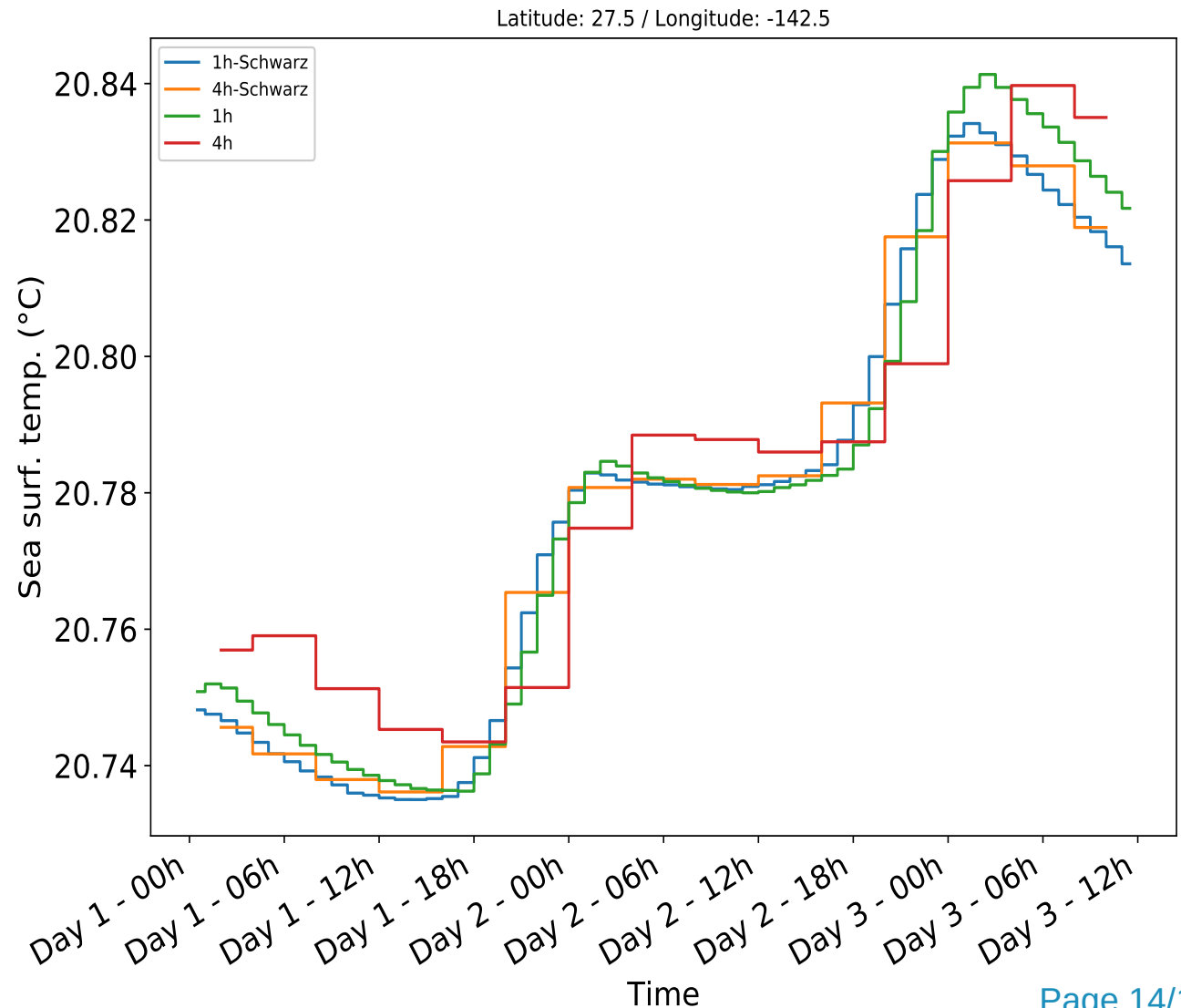


Phasing the diurnal cycle

Evolution of the interface temperature θ_{Ω} .

Without Schwarz, the diurnal cycle is out of phase with regards to the reference solution (1h-Schwarz).

Even with a large time step of $\Delta t=4h$, the Schwarz method yields a correct pace of the diurnal cycle.



Conclusions and perspectives

Current time schemes in state-of-the-art Earth System Model are mathematically inconsistent.

A Schwarz iterative has been implemented in the IPSL coupled model. It is mathematically consistent.

The comparison of simulations with and without Schwarz allows us to quantify the error done when Schwarz is not used.

This error is quite large.

With a coupling time step $\Delta t =$ of 1 hour, 2 Schwarz iterations can almost cancel the error.

2 Schwarz iterations doubles the cost of the model, which is clearly unacceptable. We will investigate two methods :

- Performs Schwarz iterations on a sub part of the model : probably vertical diffusion only.
- Improve the first guess to increase the convergence speed.

The present study focus on the ocean-atmosphere interface, with no sea-ice. The case with three domains (ocean / sea-ice / atmosphere) remains to be investigated.

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