2-D spherical simulations of mantle convection are popular, either in spherical axisymmetric or spherical annulus geometry. A problem is that the geometrical restriction forces a downwelling to deform as it sinks, whereas in 3D it can sink with no deformation. Basically, it is “squeezed” in the plane-perpendicular direction, forcing it to expand in the in-plane directions. A rigid/high viscosity downwelling resists this deformation, sinking with a greatly reduced and unrealistic velocity. This can be solved by subtracting the geometrically-forced deformation (“squeezing”) from the strain-rate tensor when calculating the stress tensor. Specifically, components of in-plane and plane-normal strain rate that are proportional to radial velocity are subtracted, a procedure that is here termed “anti-squeeze”. It is here demonstrated that this leads to realistic sinking velocities whereas without it, abnormal and unrealistic results can be obtained for high viscosity contrasts. This correction has been used since 2010 in the code StagYY for spherical annulus calculations (Tackley, PEPI 2008; Hernlund and Tackley, PEPI 2008).

In 3-D geometry a rigid/high viscosity block can sink without deforming:

In 2-D spherical it must deform as it sinks, which causes it to sink very slowly (blue curves below) or get stuck (model car, right) unless the anti-squeeze correction given here is applied (red curves):

Incompressible spherical annulus geometry

The stress terms are:

\[
\tau_{rr} = 2\eta \frac{\partial \psi}{\partial r} \quad \tau_{\theta \theta} = 2\eta \left( \frac{\partial \psi}{\partial r} + \frac{\psi}{r} \right) \quad \tau_{rr} = 2\eta \frac{\psi}{r} \quad \tau_{\theta \theta} = 0
\]

The radial component of the momentum equation is,

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \tau_{rr} \right) + \tau_{\theta \theta} = 0
\]

while the angular component of momentum is,

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \tau_{\theta \theta} \right) + \tau_{rr} = 0
\]

The deviatoric stresses \( \tau_{ij} \) are given by,

\[
\tau_{ij} = 2\eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

Regarding spherical axisymmetric geometry

Stresses are:

\[
\tau_{rr} = 2\eta \frac{\partial \psi}{\partial r} \quad \tau_{\theta \theta} = 2\eta \left( \frac{\partial \psi}{\partial r} + \frac{\psi}{r} \right) \quad \tau_{rr} = 2\eta \frac{\psi}{r} \quad \tau_{\theta \theta} = 0
\]

The cut of plane stress (pgn phg) is not ok!

High viscosity material doesn’t want to deform \( \Rightarrow \) gets ‘stuck’.

Anti-squeeze could be used in both directions, or use spherical annulus instead…

Regarding compressibility

Now, normal stresses have an addition term subtracting the strain-rate due to velocity divergence (due to compression/decompression associated with increasing/decreasing pressure).

\[ \epsilon_{rr, \text{forced}} = \epsilon_{rr} + \epsilon_{\theta \theta} \]

Anti-squeeze would solve the problem! Subtract forced strain-rates in the normal stress terms. Assume equal deformation in the \( \phi \) and theta-theta directions.

\[ \epsilon_{rr} \Rightarrow \text{Anti-squeezed normal stress} \]

\[
\tau_{rr} = 2\eta \frac{\partial \psi}{\partial r} + \frac{\psi}{r} \quad \tau_{\theta \theta} = 2\eta \left( \frac{\partial \psi}{\partial r} + \frac{3\psi}{2r} \right) \quad \tau_{rr} = 0
\]

It works! Sinking velocity now \( \Rightarrow \) independent of viscosity (see tests).

In 2-D spherical, use anti-squeeze!