Existence and Stability of Morphodynamic Equilibria in Double Inlet Systems

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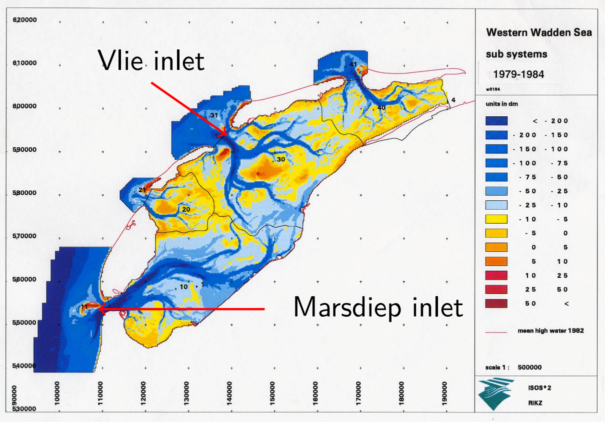
Delft University of Technology

GHENT UNIVERSITY

Xiao Deng
Thomas Boelens
Tom De Mulder
Henk Schuttelaars

Introduction



In the inlets and back-barrier basins, complex channel-shoal patterns are often observed, and these morphodynamic features are highly dynamic.

Research Question:

- Under what external forcing conditions can a double inlet system in morphodynamic equilibrium exist?
- Is this morphodynamic equilibirum unique?

Idealized Model Approach

- Simplified geometry (see figures on the right).
- Only essential dynamics taken into account.
- In experiments, forcing at inlet I is fixed and at inlet II is varied, i.e., $\hat{\zeta}(0, y, t) = 0.74\ [m]$ and $\hat{\zeta}(L, y, t) = A^{II}_{M_2} \cos(t - \phi^{II}_{M_2})$.

Typical Parameter Values
L=59 km
$H^I = H^{II} = 12m$

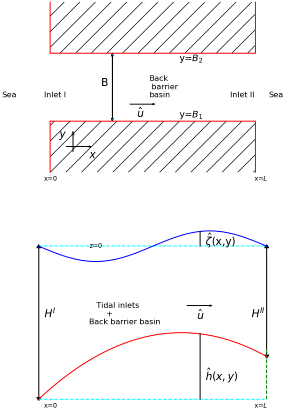


Figure: Top View and Side View

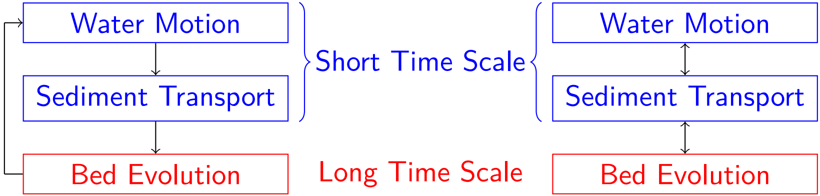
Model: The system of equations

Time integration method

Use time stepping
to find an equilibrium

Root–finding method

Find an equilibrium directly
using a nonlinear root finder

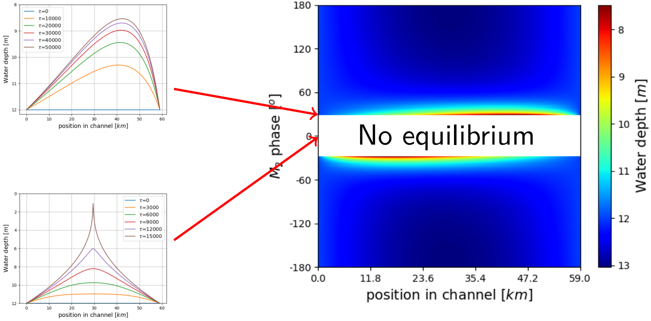


- Morphodynamic equilibria are obtained when the bed does not change anymore on the long (morphodynamic) timescale.

Solution method

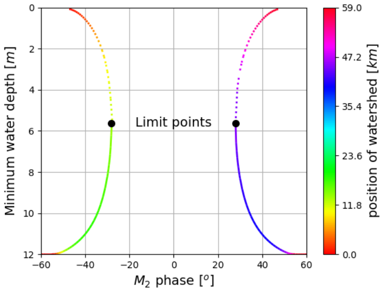
- Expand the physical variables and the equations in small parameter $\epsilon = A^I_{M_2}/H^I$.
- Expand the solution in terms of tidal constituents.
- Discretize in space with finite element method.
- To obtain an equilibrium, the Newton-Raphson method is used, combined with the Arclength method.

Results: Diffusively dominated transport



The stable equilibrium bed profiles with $A^{II}_{M_2} = 0.74$ $[m]$ and varying $\phi^{II}_{M_2}$ are shown in the right figure. Left top figure shows bed evolution starting from an initially flat bottom using time–integration method with $\phi^{II}_{M_2} = 30^o$. Left bottom figure shows the same experiment as in left top figure but with $\phi^{II}_{M_2} = 0^o$, in this case the depth vanishes in the middle.

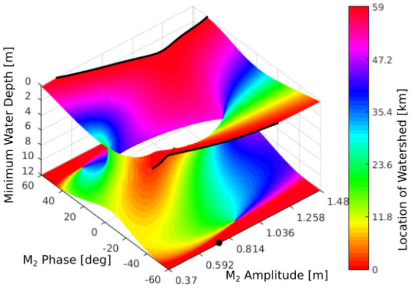
Results: Diffusively dominated transport



The minimum water depth (MWD) of equilibrium bed profiles with $A^{II}_{M_2} = 0.74$ [m] and a varying $\phi^{II}_{M_2}$.

- The solid lines are MWD of stable equilibrium bed profiles.
- The dotted lines are MWD of unstable equilibrium bed profiles.
- No equilibrium is found between the two limit points.
- Arclength method is an effective way to continue across limit points from stable equilibrium to unstable equilibrium.

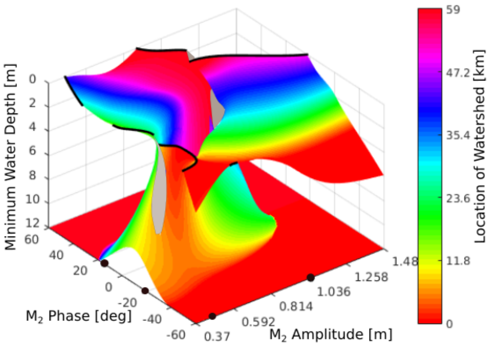
Results: Diffusively dominated transport

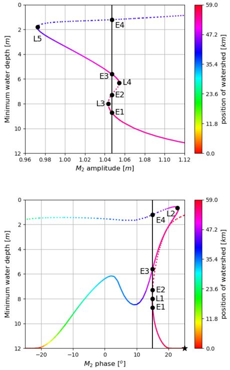


The MWD of equilibrium bed profiles as function of $A^{II}_{M_2}$ and $\phi^{II}_{M_2}$ is shown.

- The number of equilibria depends on both $A^{II}_{M_2}$ and $\phi^{II}_{M_2}$.
- For conditions with two equilibria, the stable one has a larger MWD.
- Two black contour lines indicate where the MWD becomes zero.

Results: All transport contributions included

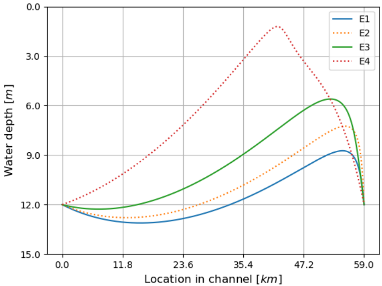




Left figure shows the MWD as a function of $A^{II}_{M_2}$ and $\phi^{II}_{M_2}$, locations where a complex bifurcation structure exists are indicated by grey. Right figures show the bifurcation sturcture for fixed $\phi^{II}_{M_2} = 15^o$ and varying $A^{II}_{M_2}$ (top right) and for fixed $A^{II}_{M_2} = 1.0471$ $[m]$ and varying $\phi^{II}_{M_2}$ (bottom right).

Results: All transport contributions included

- For $\phi^{II}_{M_2} = 15^o$ and $A^{II}_{M_2} = 1.0471$ [m] four equilibria exist. Their bed profiles are shown in the figure on the right.
- Two equilibria ($E1$ and $E3$) are stable, the other two ($E2$ and $E4$) are unstable.
- Comparing the water depth, $E1 > E2 > E3 > E4$ at most locations.



The bed profiles of equilibria $E1$ to $E4$.

Conclusion

- The number of morphodynamic equilibria depends strongly on the M_2 tide: either no, one or multiple equilibria are found.
- If no equilibria are found, the two inlets do not communicate with each other anymore: two single inlet systems have formed.