Geocenter Motion from a Combination of GRACE Mascon and SLR Data

Claudio Abbondanza\textsuperscript{1}, Toshio M Chin\textsuperscript{1}, Richard S Gross\textsuperscript{1},
Michael B Heflin\textsuperscript{1}, Jay W Parker\textsuperscript{1}, Benedikt S Soja\textsuperscript{1},
David N Wiese\textsuperscript{1}, Xiaoping Wu\textsuperscript{1}

May 6, 2020

\textsuperscript{1} Jet Propulsion Laboratory - California Institute of Technology
Surficial mass variability observed through space gravimetry can be converted into load-induced deformations at Space-geodetic (SG) observing sites by adopting a spectral formalism [9].

GRACE-derived elastic displacements would represent, if accurate, band-limited load-induced deformations that can be removed from SG-derived station displacements in order to recover degree-1 surface deformation signature, and therefore geocenter motion [3].

In so doing, the residual SG station displacements, insofar as expressed in a geocentric frame, would reflect a “pure” degree-1 deformation signature that can be recovered via spectral inversion.

We will recover such degree-1 deformation from residual displacements (SG-GRACE) and will compare it to standard geocenter motion solution determined via network shift approach.
1. Data Sets

- We adopt **GRACE JPL Mascons (RL06)** [12] solution available at https://grace.jpl.nasa.gov/data

- in conjunction with **load Love numbers** inferred from **Preliminary Earth Reference Model** [see e.g. 11] to derive load-induced elastic displacements. The assumption we are making here is that (i) Earth’s response to loading and mass transport is purely elastic and (i) Earth is isotropic (no lateral variation of its mechanical properties).

- State-of-art **SLR** solution (weekly station positions and daily EOPs in SINEX format) produced by the **Italian Space Agency (ASI)** in preparation for the ITRF2020.
2(a). SLR-ASI Data Set

- ASI Preliminary Solution compliant with the ITRF2020 Call for Participation standards.
- In order to improve SLR scale and geocenter, ILRS implements new treatment of station-dependent range biases, mean pole and time variable gravity.
- To match GRACE time span, we restrict ASI-SLR data to the interval 2002-2016, during which 49 stations, not all simultaneously co-observing, are available.
- SLR SINEX files are monthly binned and interpolated at the midpoint epoch of JPL-RL06M GRACE Mascons solutions.
Per-Session Weighted RMS of the Residuals computed after removal of a 7-parameter transformations. Green Curve relates to ASIv290. Orange is ILRS2014.
3. JPL Mascon RL06 (JPL-RL06M) Data Set

- State-of-art JPL Mascon solution (RL06M.MSCNv01) obtained by reducing Level-1 GRACE observations [12] and available on http://grace.jpl.nasa.gov
- JPL-RL06M are gridded data types reporting surface mass changes in Equivalent Water Thickness with a spatial sampling of 0.5° in both latitude and longitude.
- **Why Mascons instead of Spherical Harmonic Solutions?**
  - The JPL-RL06M Kalman filter formulation allows direct use of its gravity field products without signal attenuation from smoothing and de-striping.
4. Spectral Properties of JPL-RL06M vs CSR-RL06

Median amplitude (square root of Equation 1) spectra for JPL-RL06M and CSR-RL06. In both solutions, de-aliasing products were restored and linear trends removed from the Stokes coefficients. CSR solution is unfiltered, i.e. neither smoothing nor destriping was applied. JPL solution is spatially and temporally constrained.

- Power Spectrum as a function of the Harmonic Degree:

\[ P_\ell = \sum_{m=-\ell}^{+\ell} |c_{\ell m}|^2 \]  

(1)

- CSR-RL06 solution is bandlimited up to \( \ell = 96 \)
- JPL-RL06M has been expanded in SH up to \( \ell = 179 \)
- The geophysical signal is approximately concentrated within the spectral band \( 2 \leq \ell \leq 70 \).
- The higher degrees in JPL-RL06M are used to reconstruct the “geometry” of the mascon caps.
- The unfiltered CSR-RL06 solution is dominated by high wavenumber noise (\( \ell > 40 \)), hence the necessity of smoothing/destriping on conventional SH solutions.
If the surficial load admits a representation in real spherical harmonics (rSH) $Y_{\ell m}$, then

$$u(x) = \left( \sum_{lm} U_{lm} Y_{lm} \right) e_r + \left( \sum_{lm} V_{lm} Y_{lm} \right) e_t, \quad l \geq 2$$

where the rSH coefficients $(U_{lm}, V_{lm})$ encode Earth’s elastic response and are functions of the load Love numbers $(h'_l, l'_l)$.

JPL-RL06M was expanded in bandlimited ($0 \leq \ell \leq 179$) rSH coefficients that have then been converted into Stokes, i.e. geopotential, coefficients [10].

Non-tidal de-aliasing products (ocean and atmosphere) have been restored [4] and the Stokes coefficients were finally detrended.

Elastic displacements $u(x)$ were computed by using load Love numbers derived from PREM [11]. Note that the $u(x)$ was reconstructed from $Y_{\ell m}$ within the band $2 \leq \ell \leq 179$, in such a way that they do not reflect the degree-1 surface deformation.
Spatial variability of the **annual oscillation** of the elastic displacements from JPL Mascon-RL06 determined through the load Love numbers outlined in [11] and based upon PREM [6]. Non-tidal variability of Ocean and Atmosphere has been restored into the Mascons. The elastic displacements deliberately exclude degree-1 and hence do not reflect annual geocenter motion. Yellow dots represent the location of SLR stations.
7. Estimation Model

- At time $t_k$, with $n_s$ SLR stations simultaneously observing, we can construct the model $Ax - L = v$, solved via least-squares, where
- $L$ is the vector containing differences of SLR-observed crustal deformation and JPL-RL06 elastic displacements in the local tangent (ENU) space.
- $x = [c_{10}, c_{11}, s_{11}]$ is the vector of degree-1 surface deformation at $t_k$
- $A \in \mathcal{M}_{3n_s \cdot 3}(\mathbb{R})$ where for the $i$-th Station:

$$A_i = \frac{a}{1 + k'_1} \cdot \begin{bmatrix} h'_1 P_{10}(\mu) & h'_1 P_{11}(\mu) \cos(\lambda_i) & h'_1 P_{11}(\mu) \sin(\lambda_i) \\ 0 & -\frac{l'_1}{\sin(\theta_i)} P_{11}(\mu) \sin(\lambda_i) & -\frac{l'_1}{\sin(\theta_i)} P_{11}(\mu) \cos(\lambda_i) \\ -l'_1 \partial_\theta P_{10}(\mu) & -l'_1 \partial_\theta P_{11}(\mu) \cos(\lambda_i) & -l'_1 \partial_\theta P_{11}(\mu) \sin(\lambda_i) \end{bmatrix}$$

where $\mu = \cos(\theta_i), (\theta_i, \lambda_i)$ are polar coordinates, $a$ is Earth’s radius, $(h'_1, l'_1, k'_1)$ degree-1 load Love numbers, $P_{lm}$ the associated Legendre functions.
8. Network Shift Approach for SLR-based CM-CN

- First sketched out in the 90’s in the context of “fiducial-free” GPS data analysis by Heflin et al. [7], the *Network Shift Approach* entails
  - applying and estimating the parameters of a linearized *similarity transformation* (cf Eqn 2),
  - whose translations (T) are related to (CN-CM) offsets, where CN is the Center-Of-Network.
- The estimation model adopted in this exercise relies upon the following transformation [1]:

\[
X_s(t) = X + (t - t_0) \cdot \dot{X} + T(t) + \lambda(t)X + R(t)X
\]  

(2)

Since SLR input SINEX files are rotationally aligned to ITRF prior to the application of Equation 2, R (rotation) parameters are not estimated.

- For a methodological discussion on the *Network Shift* and other approaches to Geocenter Motion determination, the interested reader is invited to consult [see e.g. 5, 8].
Time series of (CM-CN) as determined through the Network Shift approach (gray thick line) and the Degree-1 Spectral Inversion (green line). Both the time series result from equally weighted observations, i.e. the measurement covariance matrix is assumed to be the identity $I$. 
## 10. Comparison of Seasonal Geocenter Motion (CM-CN)

<table>
<thead>
<tr>
<th>Approach</th>
<th>Annual</th>
<th>Semi-Annual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$ [mm]</td>
<td>$\varphi$ [deg]</td>
</tr>
<tr>
<td>Degree1</td>
<td>$T_x$ 1.2 (0.2)</td>
<td>342.7 (8.7)</td>
</tr>
<tr>
<td></td>
<td>$T_y$ 0.8 (0.2)</td>
<td>296.9 (7.9)</td>
</tr>
<tr>
<td></td>
<td>$T_z$ 2.8 (0.2)</td>
<td>307.9 (4.4)</td>
</tr>
<tr>
<td>Net Shift</td>
<td>$T_x$ 2.4 (0.4)</td>
<td>319.1 (9.0)</td>
</tr>
<tr>
<td></td>
<td>$T_y$ 2.9 (0.3)</td>
<td>244.9 (5.2)</td>
</tr>
<tr>
<td></td>
<td>$T_z$ 4.2 (0.5)</td>
<td>297.4 (6.0)</td>
</tr>
</tbody>
</table>

The model adopted for the least squares fit of the seasonal terms is $A \cdot \sin [\omega (t - t_0) - \varphi]$, where $\omega = 2\pi/\tau$, with $\tau = 1, 0.5$ yr, and $t_0$ January 1 2005. Amplitudes $A$ are given in mm. Phases $\varphi$ are in deg. Parenththesized are $1 \cdot \sigma$ formal errors.
Acknowledgments

- This work was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration (NASA).

- We gratefully acknowledge funding support from NASA’s Space Geodesy Program and from NASA ROSES (GRACE) Program NNH15ZDA001N-GRACE.

- Cinzia Luceri (e-geos) and Antonio Basoni (e-geos) are gratefully acknowledged for providing the ASI-SLR preliminary solution.
References


