D443: Numerical benchmark study for flow in highly heterogeneous aquifers

C. D. Alecsa[1], I. Boros[1,2], F. Frank[3], P. Knabner[3], M. Nechita[1,4], A. Prechtel[3], A. Rupp[3,5], **N. Suciu**[1,3]

[1] *Tiberiu Popoviciu Institute of Numerical Analysis, Romanian Academy, Cluj-Napoca, Romania,* [2] *Department of Mathematics, Babes-Bolyai University, Cluj-Napoca, Romania,* [3] *Department of Mathematics, Friedrich-Alexander University of Erlangen-Nuremberg, Erlangen, Germany,* [4] *Department of Mathematics, University College London, London, United Kingdom,* [5] *Interdisciplinary Center for Scientific Computing, Ruprecht-Karls-University, Heidelberg, Germany*

EGU: Sharing Geoscience Online / HS8.1.1, May 4 2020

Highlights

- Heterogeneous aquifers are modeled with log-normal random conductivity fields.
- Numerical schemes are tested on two-dimensional benchmark flow problems.
- Code verification performed by comparison with analytical manufactured solutions.
- Validation done through comparisons between MC statistics and first-order results.
- Limited computational feasibility for exponentially correlated log-conductivity fields.

Context

- While the Gaussian correlation of the log-hydraulic conductivity ensures the sample-smoothness, in case of the exponential correlation the realizations of the K field are rather noisy, approaching non-differentiable functions (e.g. Yaglom, 1987; Trefry et al., 2003; Suciu, 2010).
- Numerical simulations are often conducted with exponentially correlated fields with large variance of the $\ln(K)$ field up to $\sigma^2 = 9$ (de Dreuzy et al, 2007) or $\sigma^2 = 16$ (Kurbanmuradov and Sabelfeld, 2010).
- Some numerical investigations indicate that accurate flow solutions in case of exponential correlation with $\sigma^2 \geq 2$ require exceedingly large computing resources (Gotovac et al., 2009; Cainelli et al., 2012).

This study (https://doi.org/10.1016/j.advwatres.2020.103558)

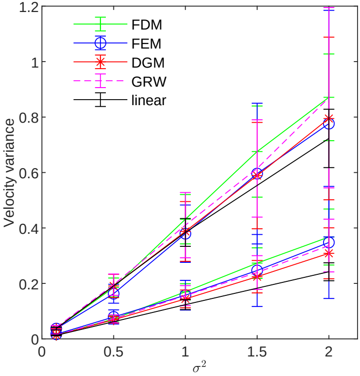
ln(K)-fields generated as sums of N cosine modes with random wavenumbers and phases $\Rightarrow$ analytical realizations of the hydraulic conductivity K,
$\Rightarrow$ manufactured exact solutions of the incompressible flow in groundwater,
$\Rightarrow$ allow direct code verification and evaluation of the numerical schemes.

Relevant parameters:
N - proportional to the number of correlation lengths λ traveled by the solute,
σ^2 - associated with the heterogeneity of the aquifer system.

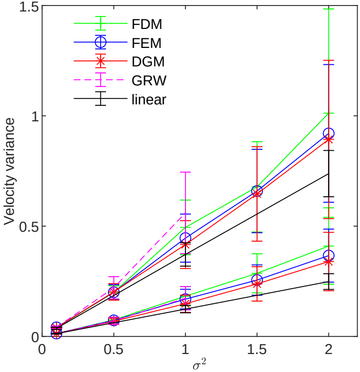
Tested in the benchmark (https://github.com/PMFlow/FlowBenchmark):
finite difference (FDM), finite element (FEM), discontinuous Galerkin (DGM), Chebyshev spectral (CSM), and global random walk (GRW) schemes.

Spatial resolution:
$\lambda/\Delta x = 50$ (FDM, FEM, DGM, GRW) / optimal nr. of Chebyshev points (CSM).

Gaussian correlation



Exponential correlation



- FDM, FEM, DGM, GRW: results similar to MC estimates published in the past.
- CSM: the number of collocation points needed to represent the solution of the flow problem with $\sigma^2 > 0.1$ exceed the maximum Matlab array size.

Code verification: $\varepsilon(\sigma^2, N) = \|h - \widetilde{h}\|$, $\widetilde{h}(x, y) = 1 + sin(2x + y)$

Gaussian correlation: $\varepsilon(\sigma^2, N = 10^2)$

	σ^2 =0.1	σ^2 =1	σ^2 =2	σ^2 =4	σ^2 =6
FDM	1.03e-03	2.00e-03	7.95e-03	4.34e-02	1.45e-01
FEM	1.61e-03	2.97e-03	3.93e-03	5.60e-03	7.06e-03
DGM	1.11e-03	1.15e-03	1.41e-03	2.10e-03	2.76e-03
CSM	3.58e-13	6.75e-13	9.00e-12	1.06e-10	6.54e-10
GRW	3.16e-02	4.80e-02	1.35e-01	–	–

Gaussian correlation: $\varepsilon(\sigma^2, N = 10^3)$

	σ^2 =0.1	σ^2 =1	σ^2 =2	σ^2 =4	σ^2 =6
FDM	1.09e-03	8.91e-03	4.23e-02	3.65e-01	1.88e+00
FEM	2.17e-03	5.60e-03	7.60e-03	1.01e-02	1.16e-02
DGM	1.11e-03	1.19e-03	1.41e-03	1.84e-03	2.29e-03
CSM	3.71e-13	4.50e-12	1.25e-11	2.50e-10	2.23e-09
GRW	6.81e-02	5.59e-01	1.78e+00	–	–

- All the five schemes preform well for Gaussian correlation of the $\ln(K)$ field.

Exponential correlation: $\varepsilon(\sigma^2, N = 10^2)$

	σ^2 =0.1	σ^2 =1	σ^2 =2	σ^2 =4	σ^2 =6
FDM	1.17e-01	2.60e+00	3.61e+00	9.41e+01	7.15e+02
FEM	9.08e-03	5.07e-01	3.57e+00	4.28e+01	2.41e+02
DGM	4.58e-02	2.59e-01	5.31e-01	2.91e+00	1.53e+01
CSM	8.11e-12	2.15e-11	7.24e-11	3.58e-10	1.00e-09
GRW	9.45e-02	7.57e-01	1.64e+00	–	–

Exponential correlation: $\varepsilon(\sigma^2, N = 10^3)$

	σ^2 =0.1	σ^2 =1	σ^2 =2	σ^2 =4	σ^2 =6
FDM	3.58e-02	3.09e+00	1.42e+01	9.79e+01	1.45e+03
FEM	1.29e-02	8.46e-01	3.66e+00	4.87e+01	6.06e+02
DGM	1.37e-02	1.72e-01	1.20e+00	1.32e+01	6.87e+01
CSM	1.63e-11	3.42e-11	3.38e-11	2.47e-10	1.67e-09
GRW	1.13e-01	1.28e+00	4.12e+00	–	–

- FDM, FEM, DGM, GRW solutions - practically not feasible for large σ^2 and N.
- CSM - uses analytical derivatives of K and produces ε values close to the roundoff plateau. Otherwise, the spectral schemes (e.g., Galerkin SM) are constraint by the allowable array size and fail to solve the benchmark tests.

This study was partially supported by the Deutsche Forschungsgemeinschaft (DFG) – Germany under Grants RU 2179 - 251268514 "MAD Soil - Microaggregates: Formation and turnover of the structural building blocks of soils" and EXC 2181 - 390900948 "STRUCTURES: A unifying approach to emergent phenomena in the physical world, mathematics, and complex data", DFG Grant SU 415/4-1 - 405338726 "Integrated global random walk model for reactive transport in groundwater adapted to measurement spatio-temporal scales".