Stochastic modeling of extreme El-Niño and La Niña events by nonlinearly coupled oscillators

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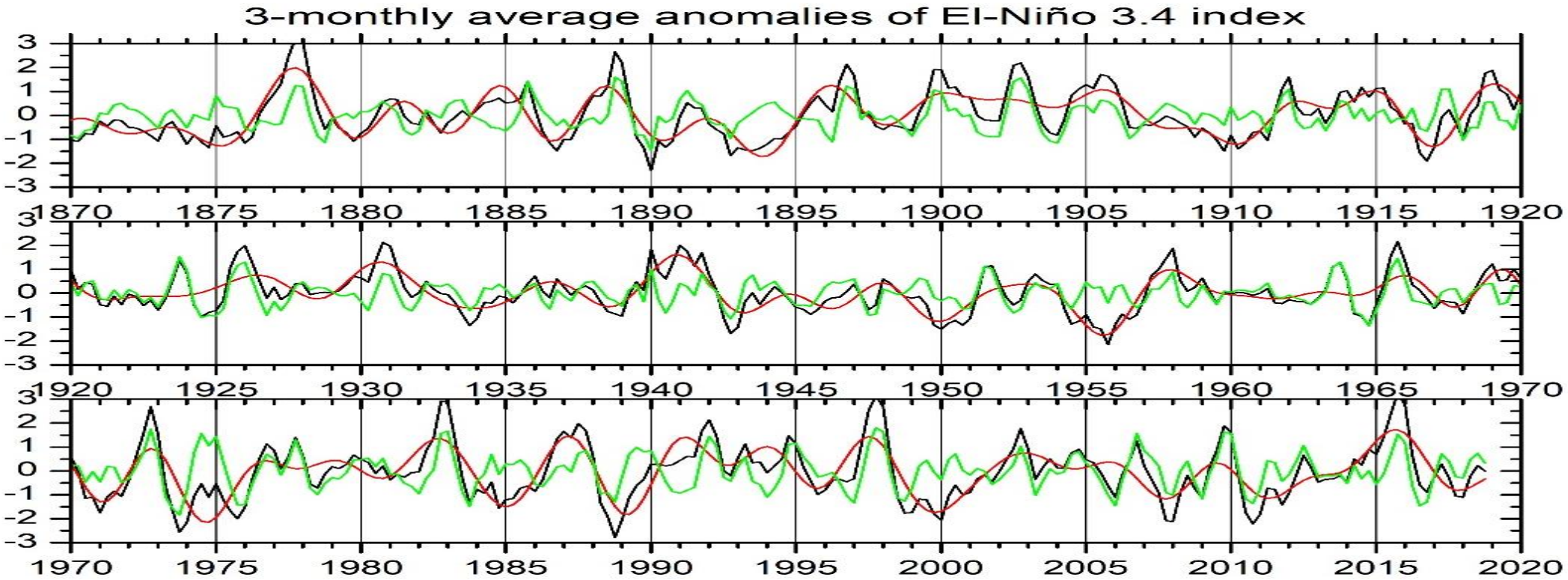
The timeseries 1870-2019 of the 3.4 El-Niño index three-monthly averages x(t) exhibits a skewness=0.46. We show x(t) (black) and corresponding inter-triannual slow component s(t) (red) and intra-triannual fast component f(t) (green). The skewness of x(t) can be decomposed into self skewness terms (*SSS* and *FFF*) and cross skewness terms: *SSF, SFF*.

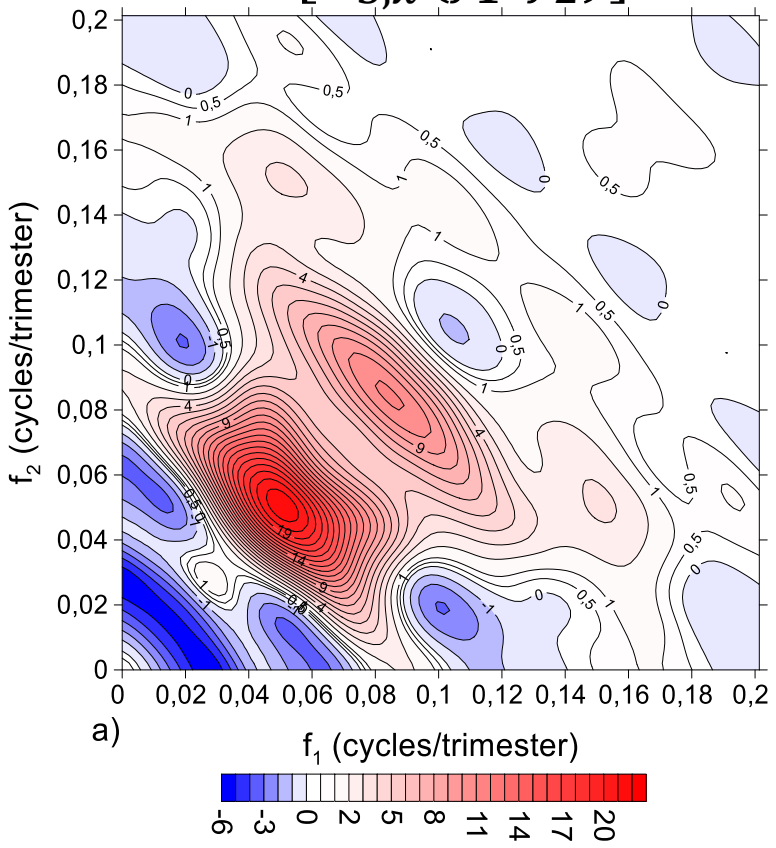
$$Skew(x) = E(x^3) = E(s^3) + 3E(s^2 f) + 3E(s f^2) + E(f^3)$$ ***= SSS + SSF + SFF + FFF***



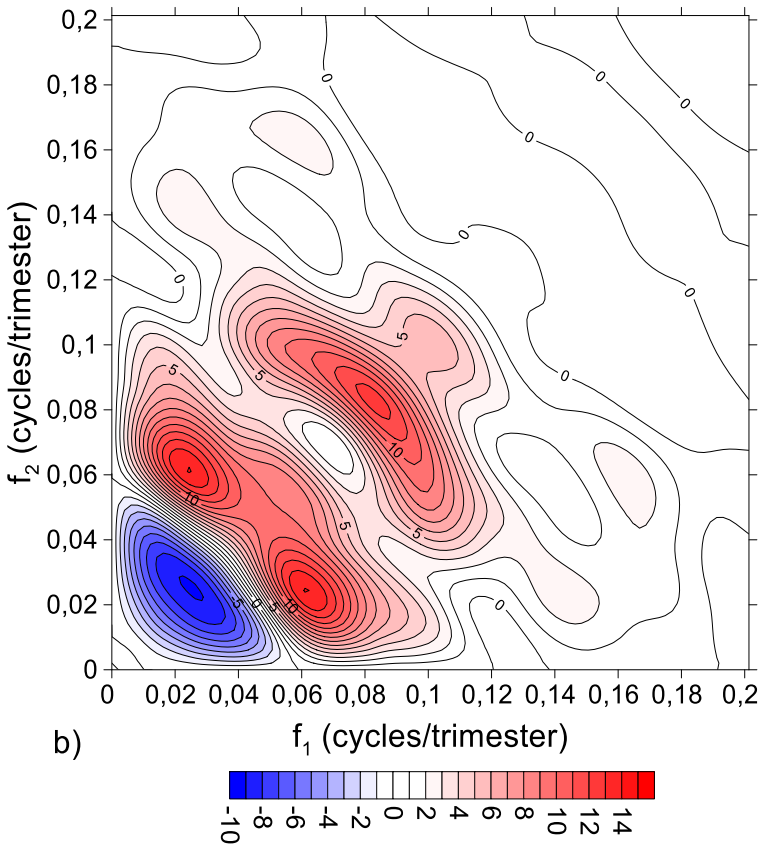
The skewness $E(x^3)$ decomposes as contributions from the bispectrum $\Gamma_{3,x}(f_1, f_2)$ = two-dimensional Fourier transform of the bicovariance $\gamma_x(\tau_1, \tau_2) = E[x(t)x(t+\tau_1)x(t+\tau_2)]$

$E(x^3) = \gamma_x(0,0) = 12 \iint_{PD} Re[\Gamma_{3,x}(f_1, f_2)] df_1 df_2$ = sum over the bi-frequency domain

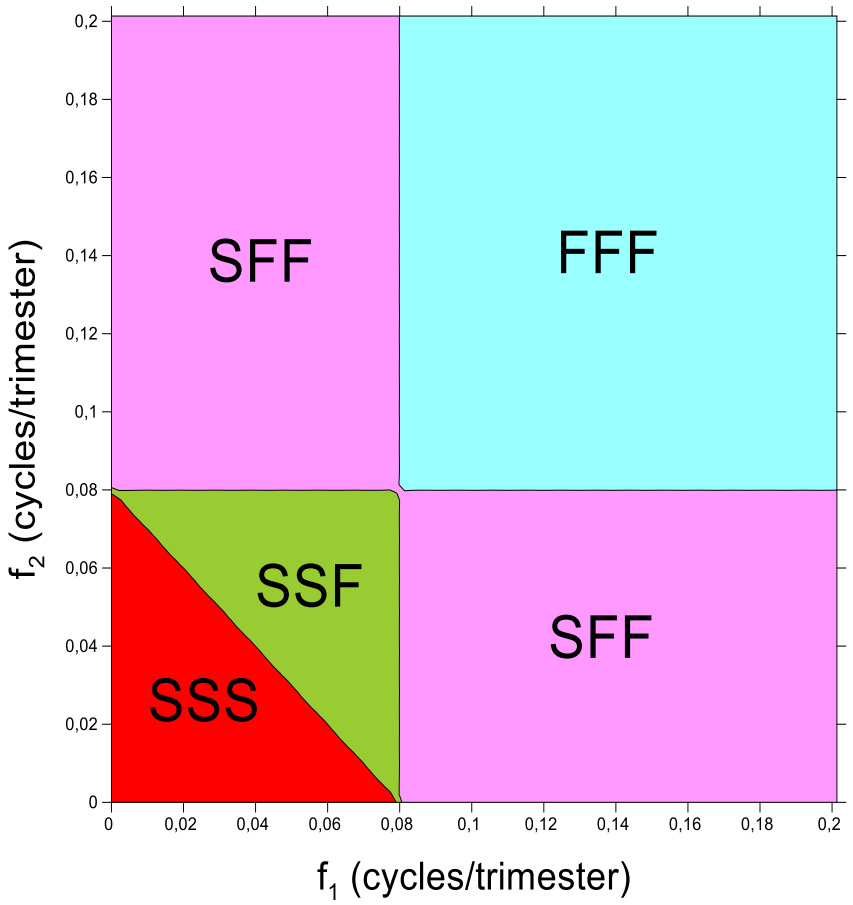
$Re[\Gamma_{3,x}(f_1, f_2)]$

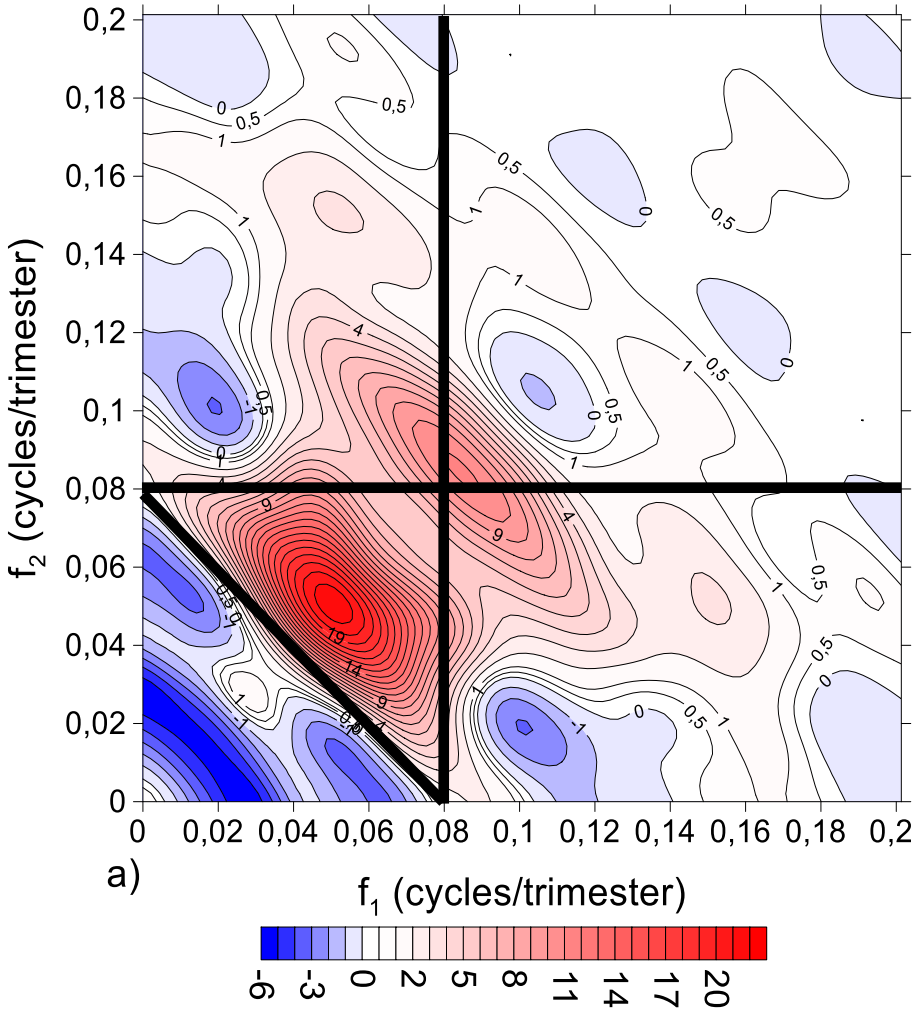


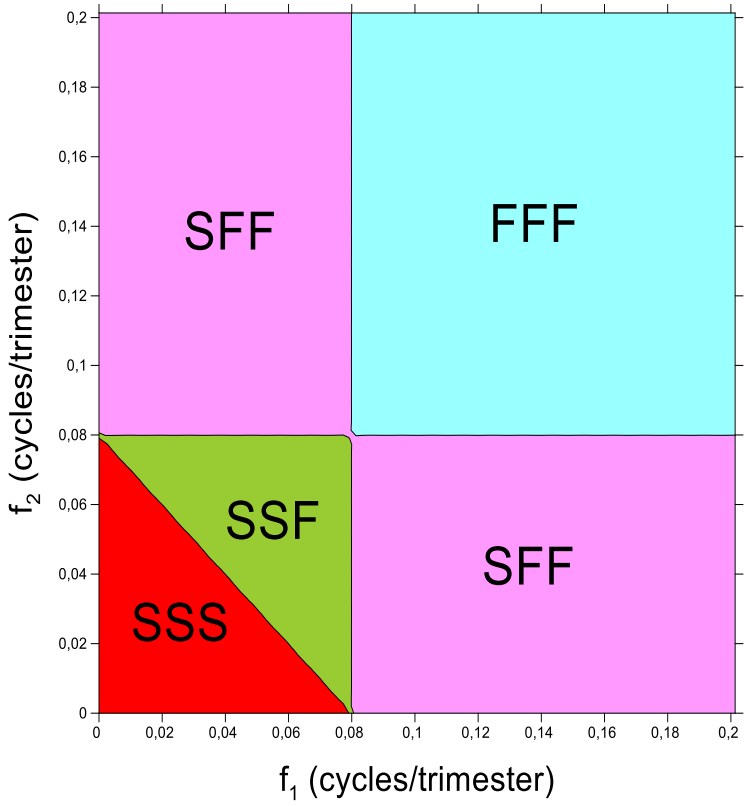
$Im[\Gamma_{3,x}(f_1, f_2)]$

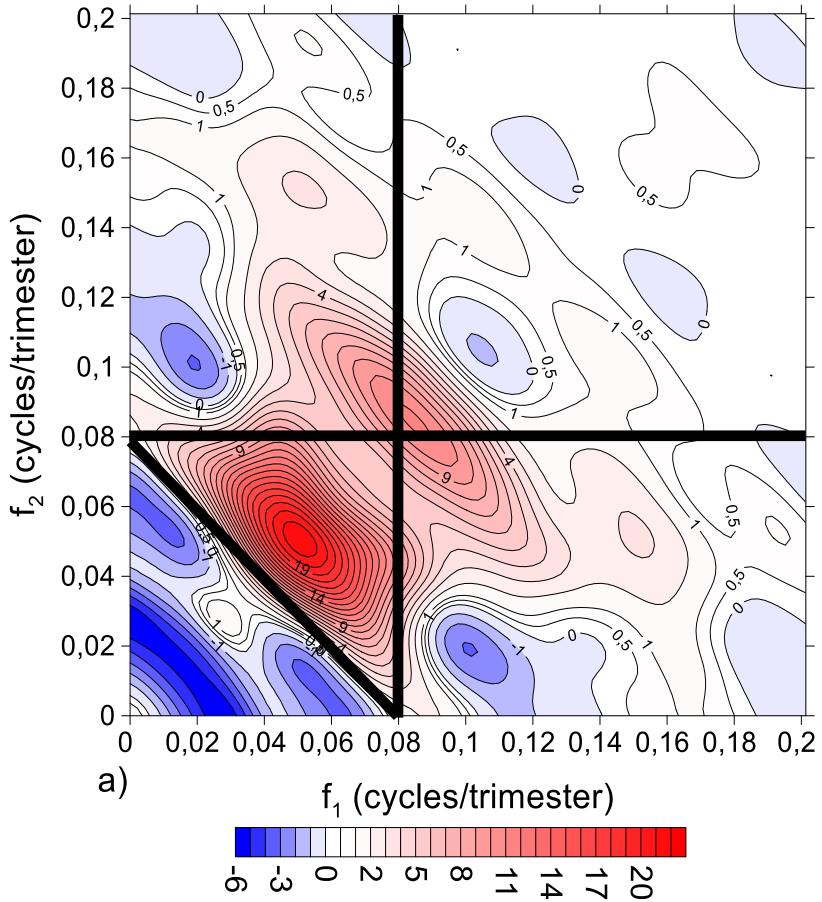


The SSS, SSF, SFF and FFF terms of skewness $E(x^3)$ are associated to prescribed regions of the bi-frequency domain with a cutoff frequency of 0.08 cycles/trimester (corresponding to a period of 3 years).





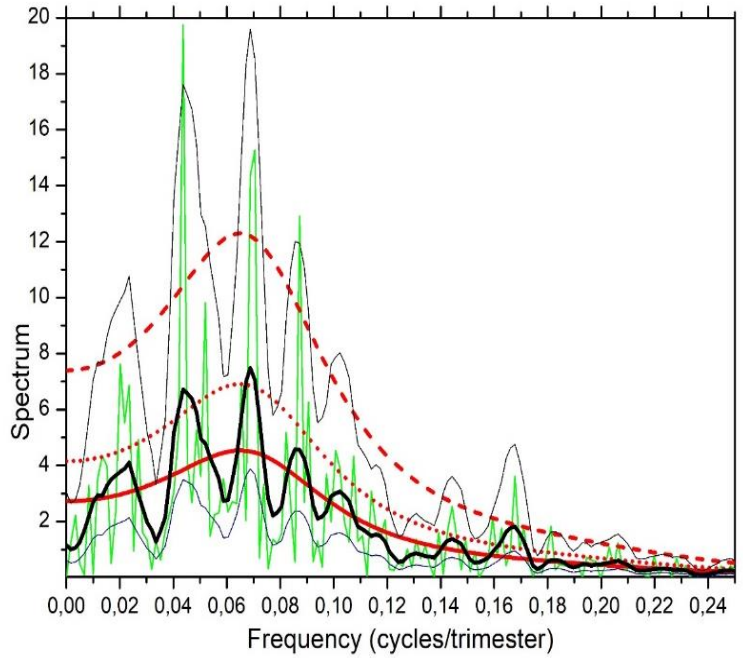




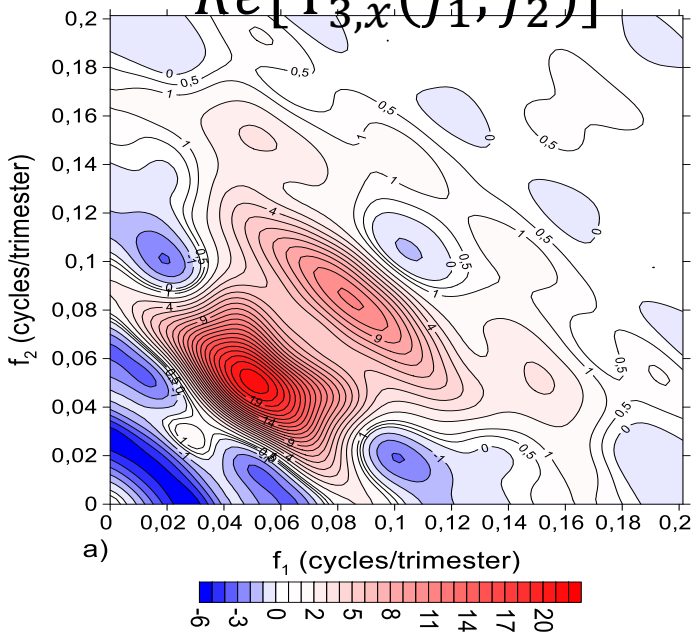
	$E(s^3)$ **SSS**	$3E(s^2f)$ **SSF**	$3E(sf^2)$ **SFF**	$E(f^3)$ **FFF**
Time series	-0.066	0.263	0.185	0.071
$Re(\hat{\hat{S}}_{3,x})$	-0.023	**0.216**	**0.212**	0.052
$Re\left(\hat{\hat{S}}_{3,x}\right) < 0$	**-0.037**	0.000	-0.031	-0.012
$Re\left(\hat{\hat{S}}_{3,x}\right) > 0$	0.014	0.216	0.243	0.063

CONCLUSION: Positive terms leading to El-Niños are mostly due to cross-scale interactions SSF and SFF whereas negative terms leading to La-Niñas are mostly due to slow self interactions SSS and cross-scale interactions SFF (seen in the time-series graphs)

Some experiments have been performed to built stochastic processes (e.g. bilinear and nonlinear AR) with the same coarse-grained spectrum and bispectrum of the El-Niño 3.4 index (see below the spectrum and bispectrum)



$Re[\Gamma_{3,x}(f_1, f_2)]$



$Im[\Gamma_{3,x}(f_1, f_2)]$