Critical fluid volumes and the start of 'self-sustaining' fracture ascent

Tim Davis and Eleonora Rivalta
EPA fracking executive summary 2016:

“...fracture growth during hydraulic fracturing can be controlled by limiting the rate and volume of hydraulic fracturing fluid injected...”

“...thousands of feet of rock between hydraulically fractured rock formations and underground drinking water resources can reduce the frequency of impacts on drinking water resources...”
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Do volumetric limits/rates exist in the literature?

“...thousands of feet of rock between hydraulically fractured rock formations and underground drinking water resources can reduce the frequency of impacts on drinking water resources...”

Can fractures propagate vertically thousands of metres?
Theory

‘$K_{IC}$’ = ribbon strength

$\rho_{\text{air}} - \rho_{\text{fluid}} \gg \Delta \gamma \cdot V > K_{IC}$
Theory

For a given volume \( (\mathcal{V}_c) \) the balloon will begin to rise indefinitely.
New solution: Boundary conditions

‘Critical’ fracture length (3D) can be described by:

\[ K_I = K_{IC} \]

\[ c = \left( \frac{3\sqrt{\pi} K_{IC}}{8\Delta \gamma} \right)^{2/3} \]
‘Critical’ fracture length (3D) can be described by:

Volume of a crack due to internal pressure \( p \) 3D.

Substituting ‘\( c \)’ and ‘\( p \)’

\[
V_{\text{min}} = \frac{(1 - \nu)}{16\mu} \left( \frac{9\pi^4 K_{IC}^8}{\Delta \gamma^5} \right)^{1/3}
\]

Independent of shape
Is that all?

Differences:

V too low?
• Our boundary conditions are such that the fracture is trapped

V too high?
• Circle has greater area towards upper tip
How to test?

Numerically:
- Typical buoyant ascent methods 2D (BEM).
- 3D propagation code that can mesh on the fly as front changes.
- Efficiently compute closing of multiple tail elements.

\[ \text{Davis et al 2019. 3D } K \text{ calculation} \]

\[ \text{CutAndDisplace + CGAL} \]

\[ \text{Complementarity conditions} \]

\[ 2D \text{ Ritz et al 2012} \rightarrow 3D \text{ Davis et al 2019.} \]
Numerical scheme

Volume inserted

$\Delta \gamma$ acts as it grows

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Numerical scheme

Stable propagation
Numerical results

Numerical critical volume is $0.75^*$ analytical formula.

$$V_{\text{min}} = \frac{(1 - \nu)}{16\mu} \left( \frac{9\pi^4K_{IC}^8}{\Delta\gamma^5} \right)^{1/3}$$

$$V_c = \frac{(1 - \nu)}{10.6\mu} \left( \frac{9\pi^4K_{IC}^8}{\Delta\gamma^5} \right)^{1/3}$$
Reasons why this estimate is ‘conservative’
Comparing to experimental data

Formula matches critical volume data from gelatine experiments of Heimpel and Olson (1994)

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Comparing to hydrofracturing data

- $K_{IC}^{lab} = 1-3$
- $K_{IC}^{field\ veins} = 8-25$
- Around scale of mine injection experiments\textsuperscript{a}  
  \(1-15\text{m}^3\)
- Typical ‘hydrofrac’ job per well\textsuperscript{b}  
  \(5600-21500\ \text{m}^3\)

Using $K_{IC}$ from field our equation provides critical volumes of:
- 2.5-55m$^3$

\textsuperscript{a} Max: 2m$^3$ Warpinski et al. [1982].  
15.6m$^3$ Jeffrey et al. [2009]  
9.24m$^3$ Zimmermann et al. [2019]

\textsuperscript{b} https://www.americangeosciences.org/critical-issues/faq/how-much-water-does-typical-hydraulically-fractured-well-require
Conclusions

• Introduced scale independent volumetric limits on stability of fluids inside fractures.

• The limit matches well with critical volumes observed in gelatine experiments.

• More research to constrain this tipping point is required, the problem remains poorly quantified.