

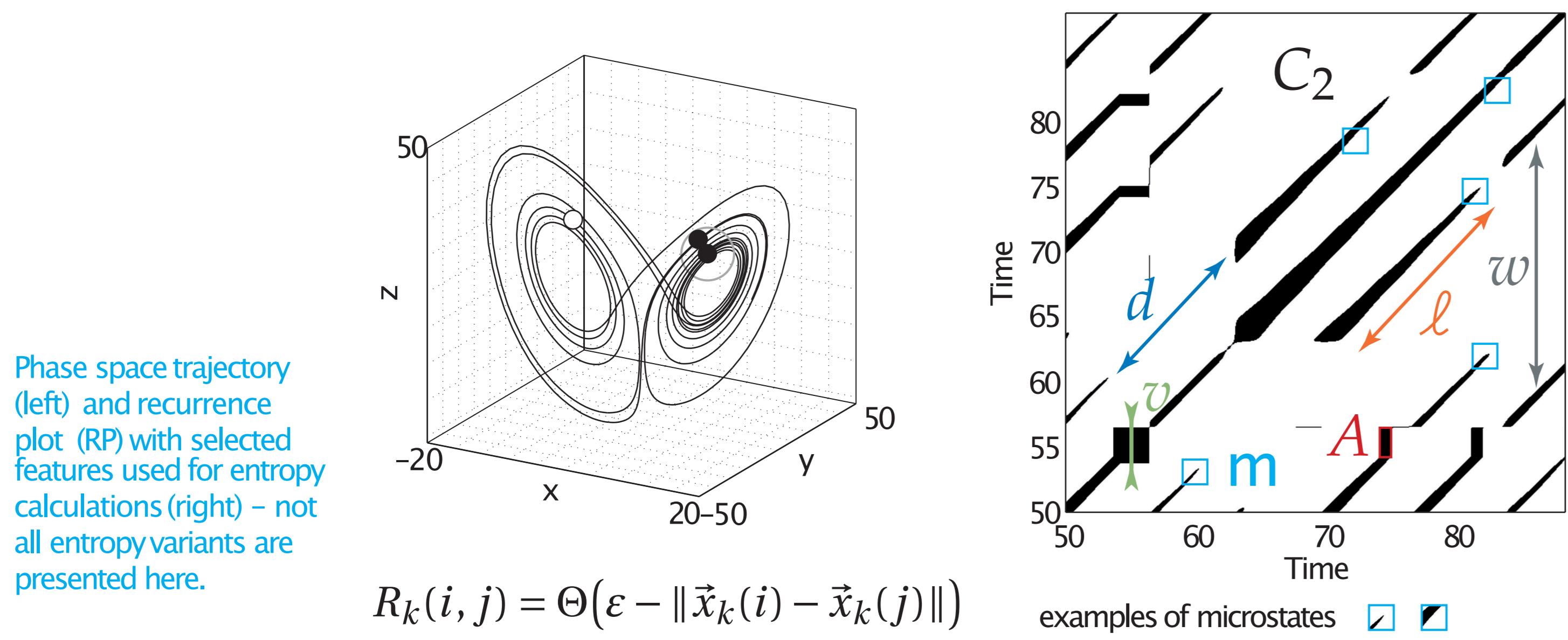
Recurrence Based Entropies^[1]

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Recurrence plots...

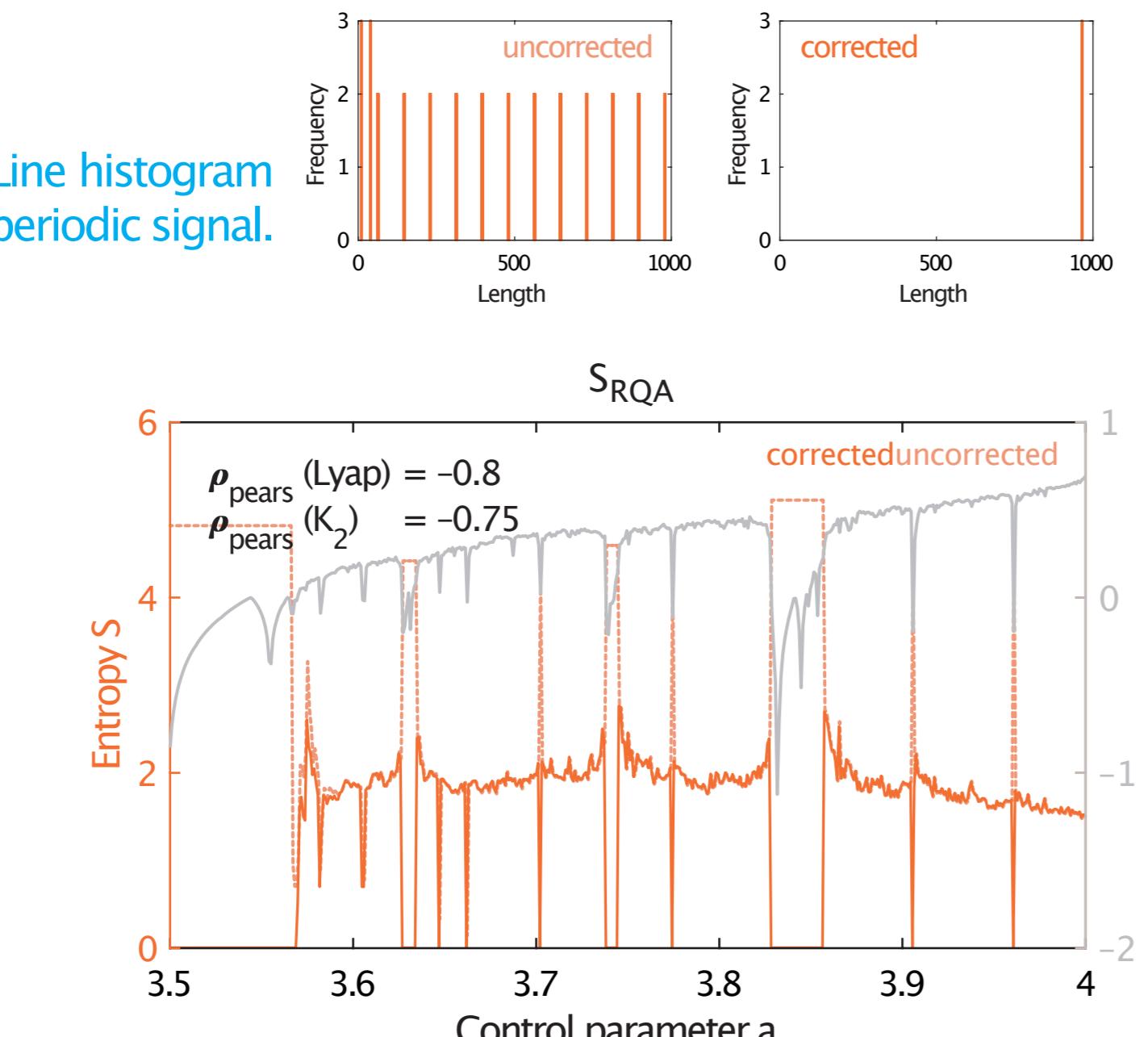
- ... store information from dynamical systems in their structures/patterns
- quantification of certain structures can give access to this information
- entropies based on different RP-patterns, in turn, require different interpretation, which remains unclear in some cases
- ability to detect different transition types



"Standard" RQA entropy

$$S_{RQA} = - \sum_{\ell=1}^N p(\ell) \ln p(\ell).$$

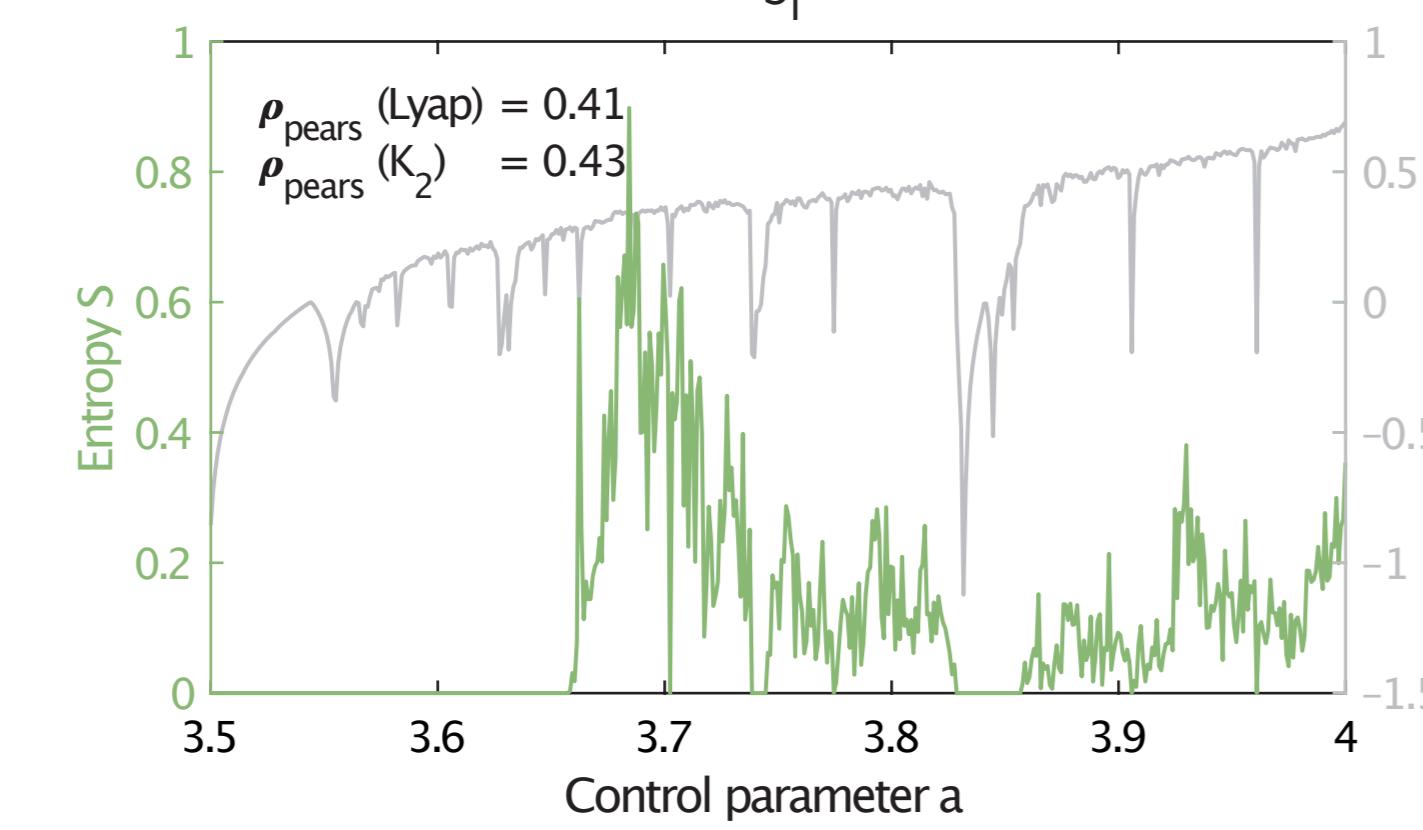
- variability of diagonal lines
- mostly used RP based entropy
- △ border effects (cut diagonal lines), noise, tangential motion, fixed by [2]



Intermittency entropy

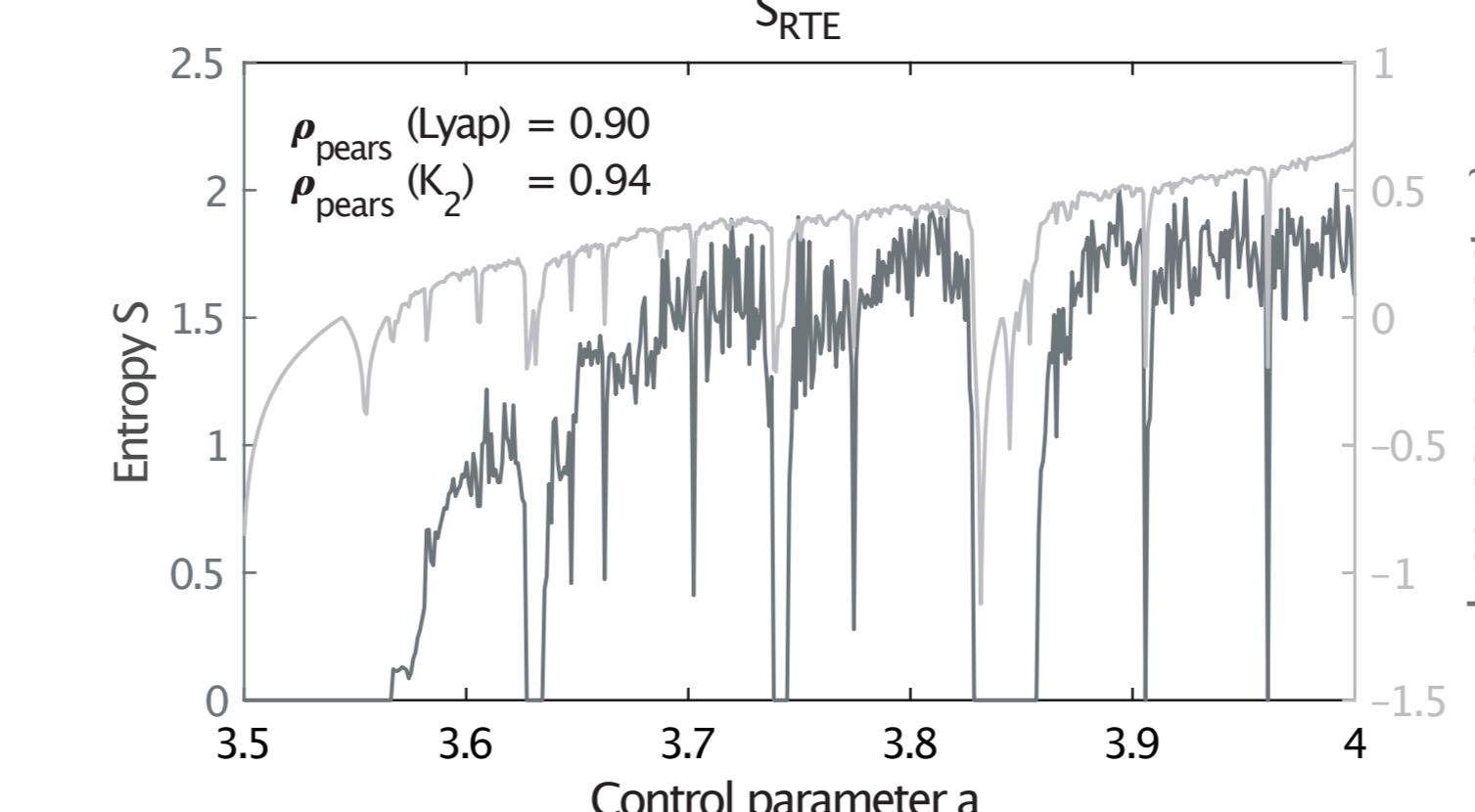
$$S_I = - \sum_{v=1}^N p(v) \ln p(v).$$

- variability of vertical lines, i.e., laminar phases
- △ only valid for dynamics with intermittent/ laminar regimes; meaning still unclear



Recurrence time entropy

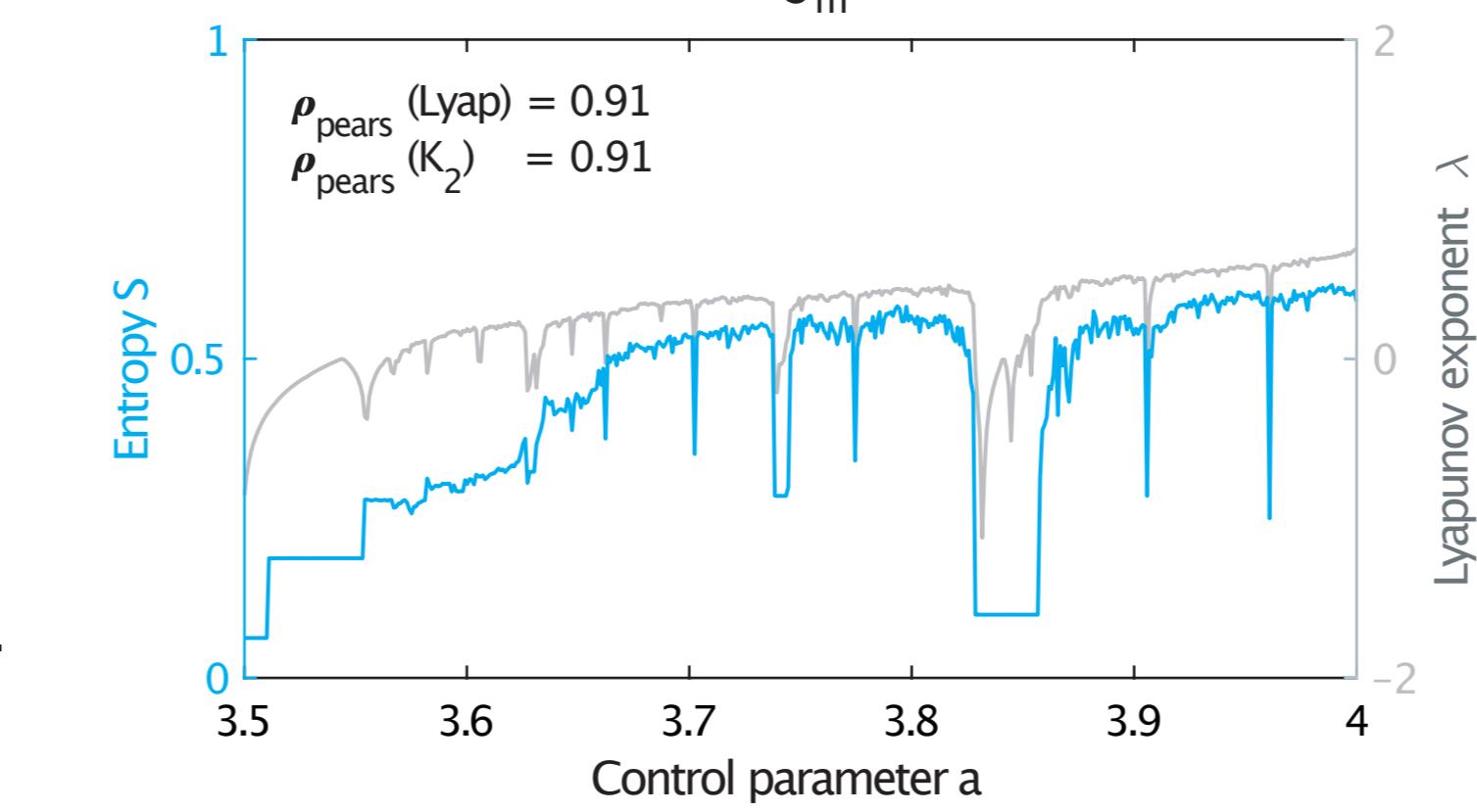
- $S_{RTE} = - \sum_{w=1}^N p(w) \ln p(w)$.
- variability of recurrence times (periods)
- related to KS entropy
- △ border effects (cut vertical white lines, possibly fixed by [2])



Microstates entropy

$$S_m = - \sum_{\mu}^{N^*} p(\mu) \log p(\mu)$$

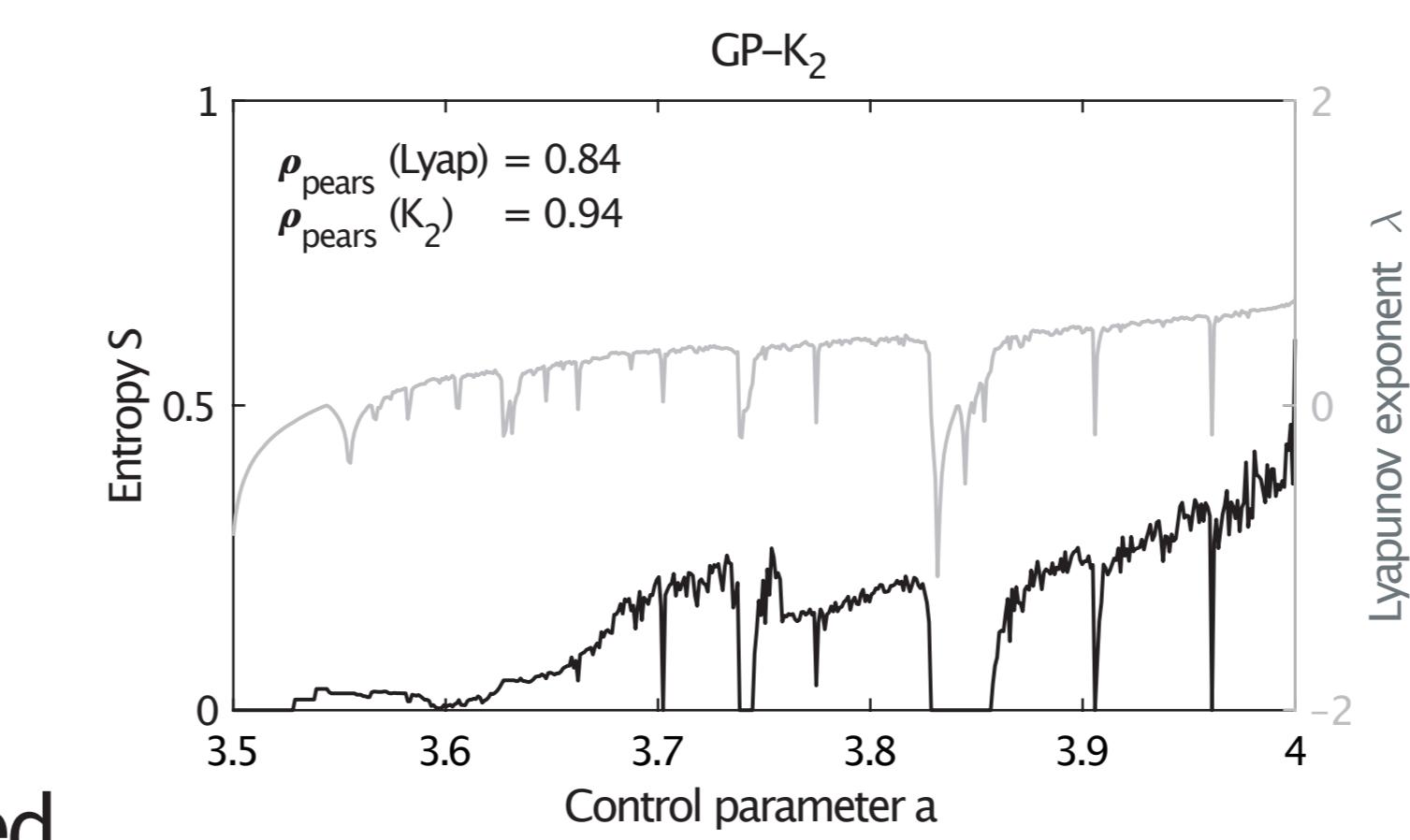
- variability of short time recurrence patterns
- fast (when using random subset)
- △ mixing of structures (no clear physical meaning); sensitive to embedding



K2 entropy (GP algorithm)

$$S_{K_2} = \frac{1}{k\Delta t} \ln \frac{\frac{1}{N_l(N_l-1)} \sum_{i,j} R_{i,j}^{(l)}}{\frac{1}{N_{l+k}(N_{l+k}-1)} \sum_{i,j} R_{i,j}^{(l+k)}}$$

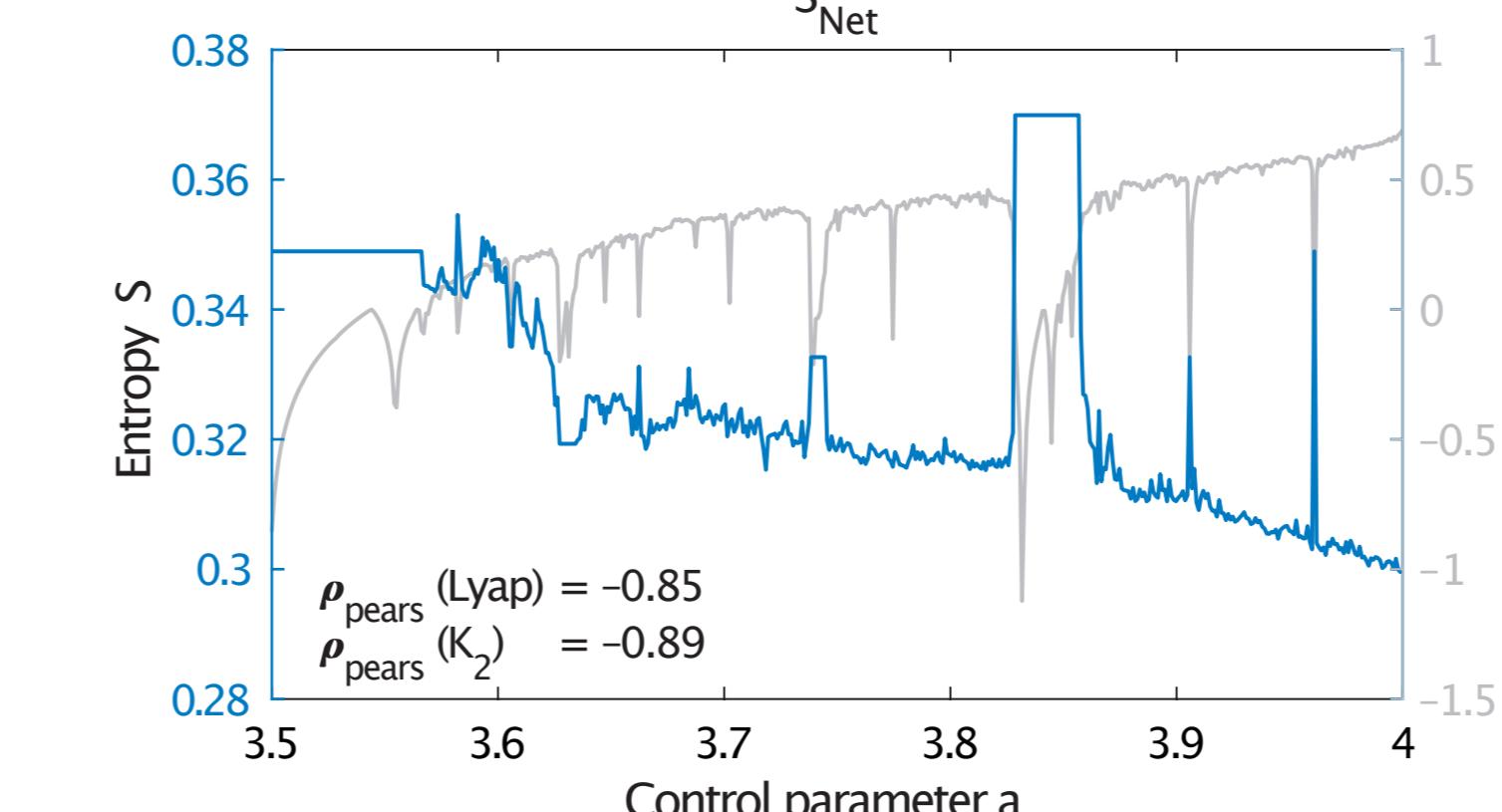
- correlation entropy, mathematically straightforward, using correlation sum C_2
- △ high embedding dim. and scaling region required



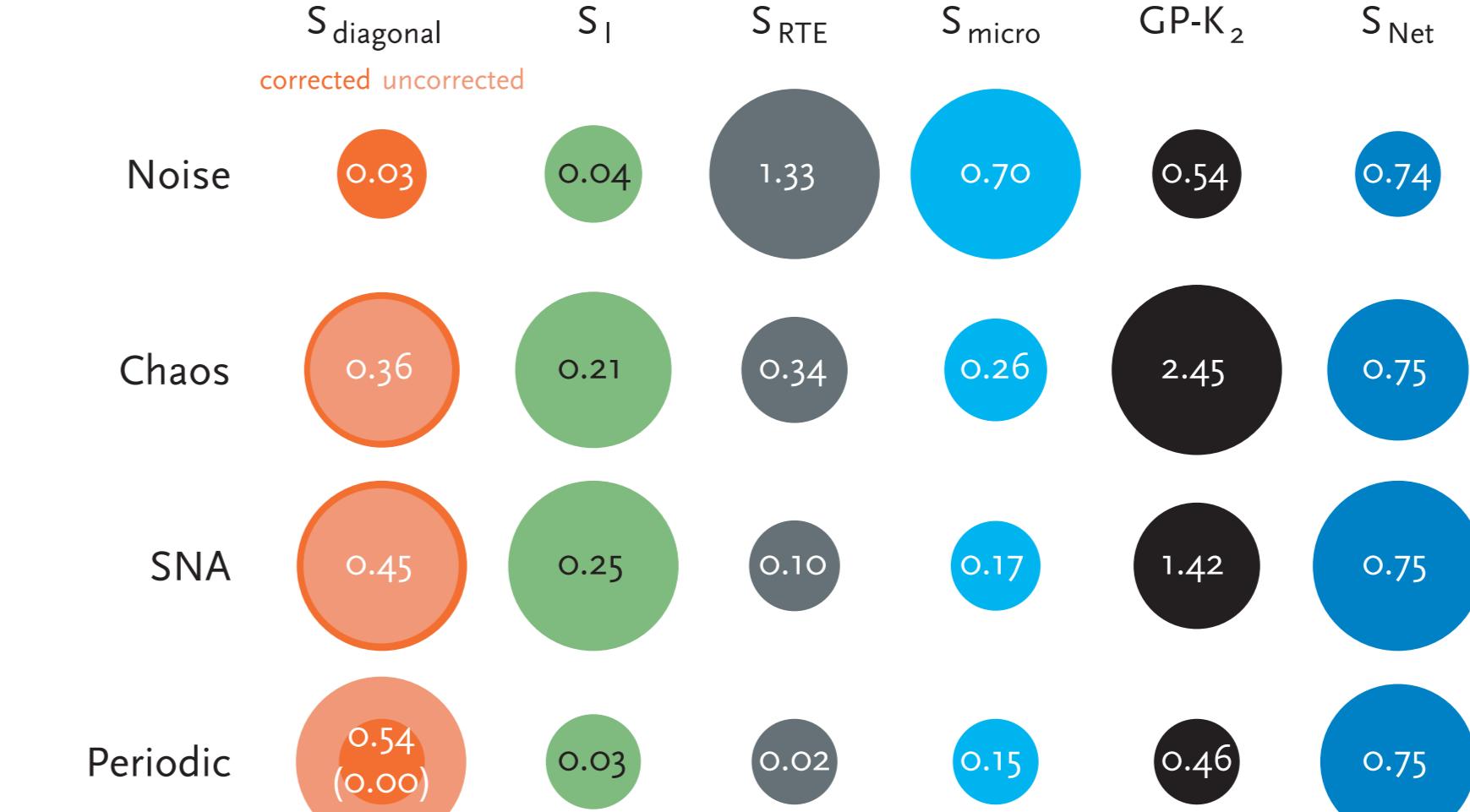
Network entropy

$$S_{Net} = \frac{1}{N \log(N-1)} \sum_i \log \sum_j (R_{i,j} - \delta_{i,j})$$

- heterogeneity of phase space density
- △ geometrical, not a dynamical measure



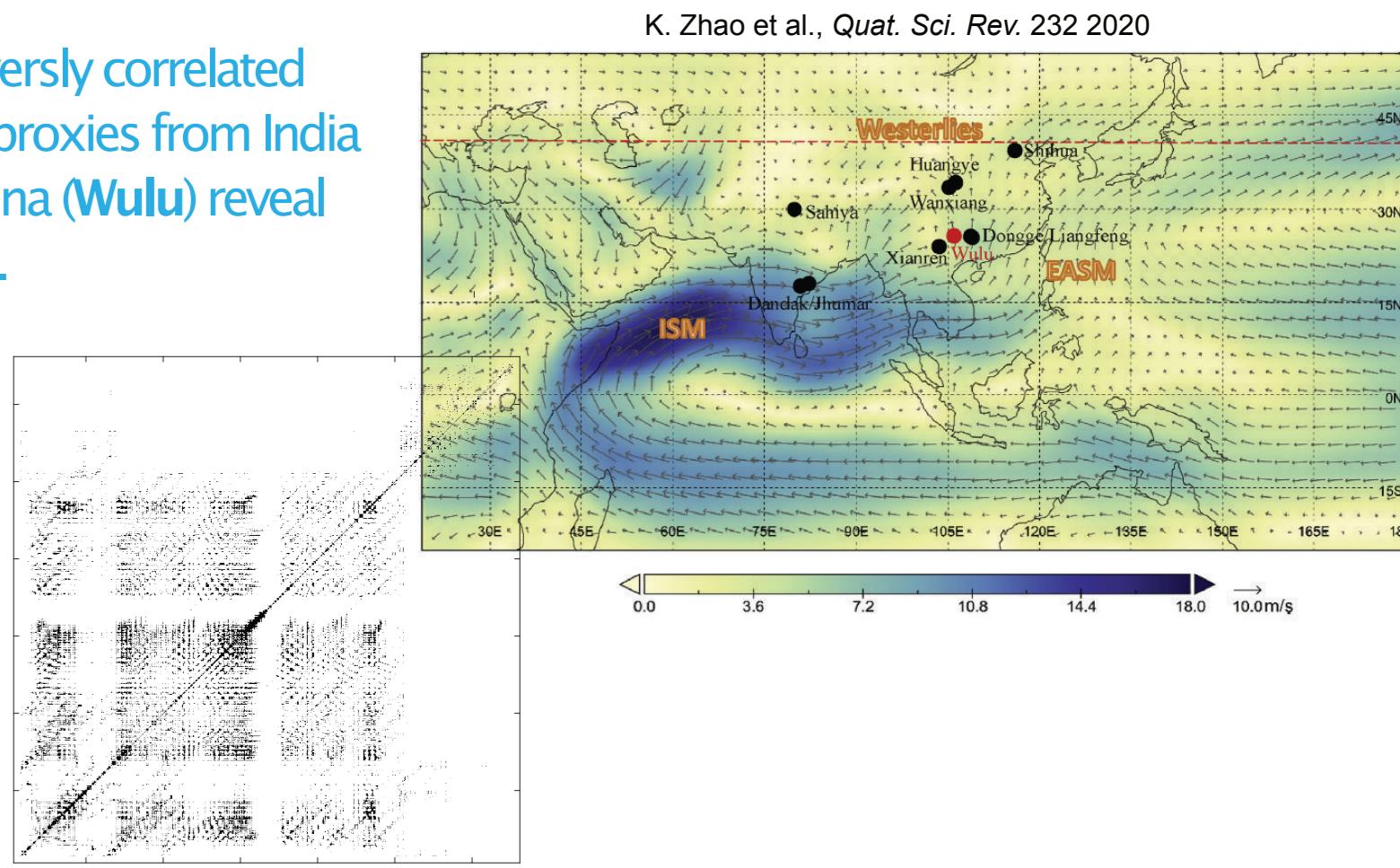
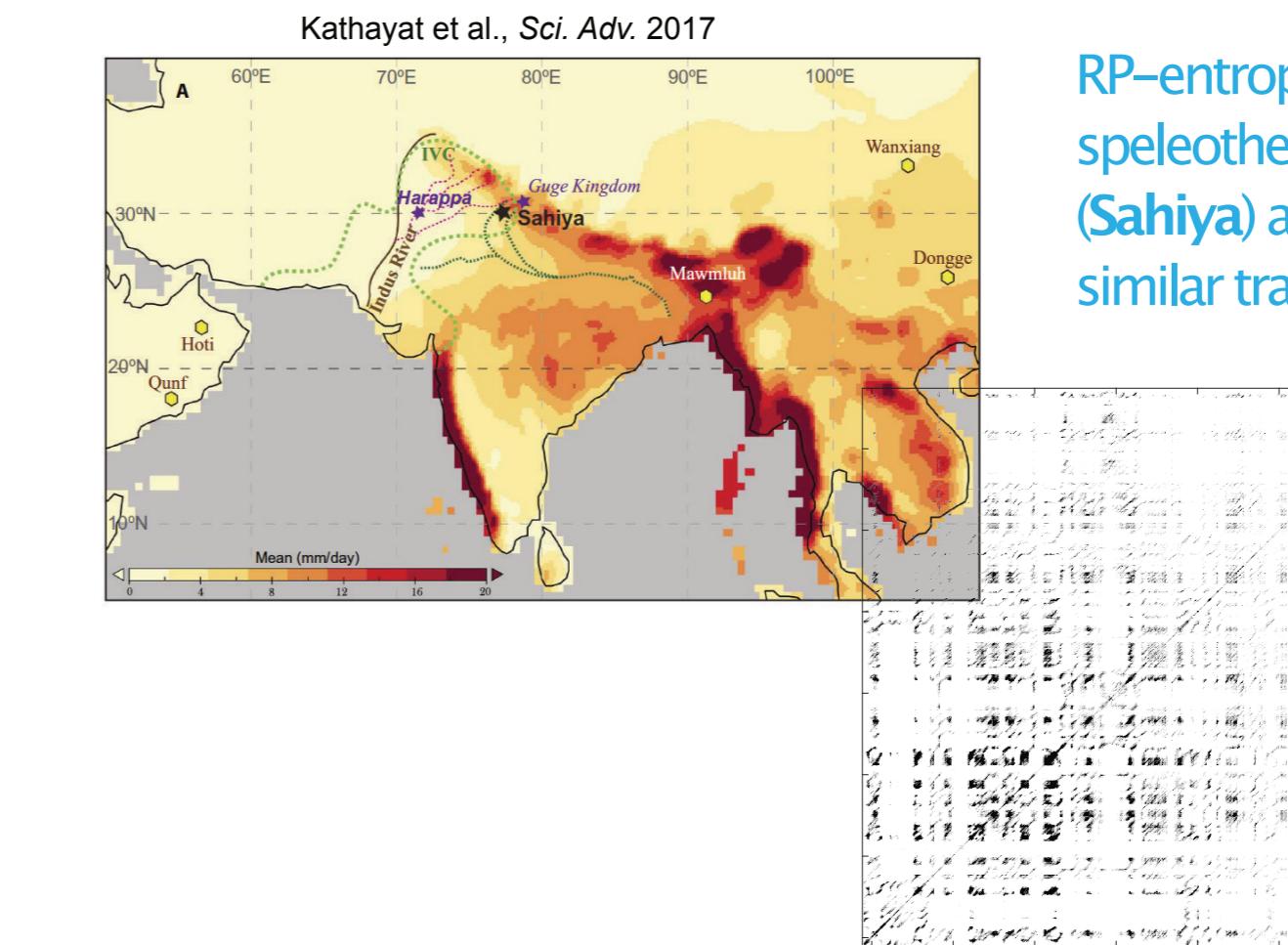
Abilities regarding different dynamics



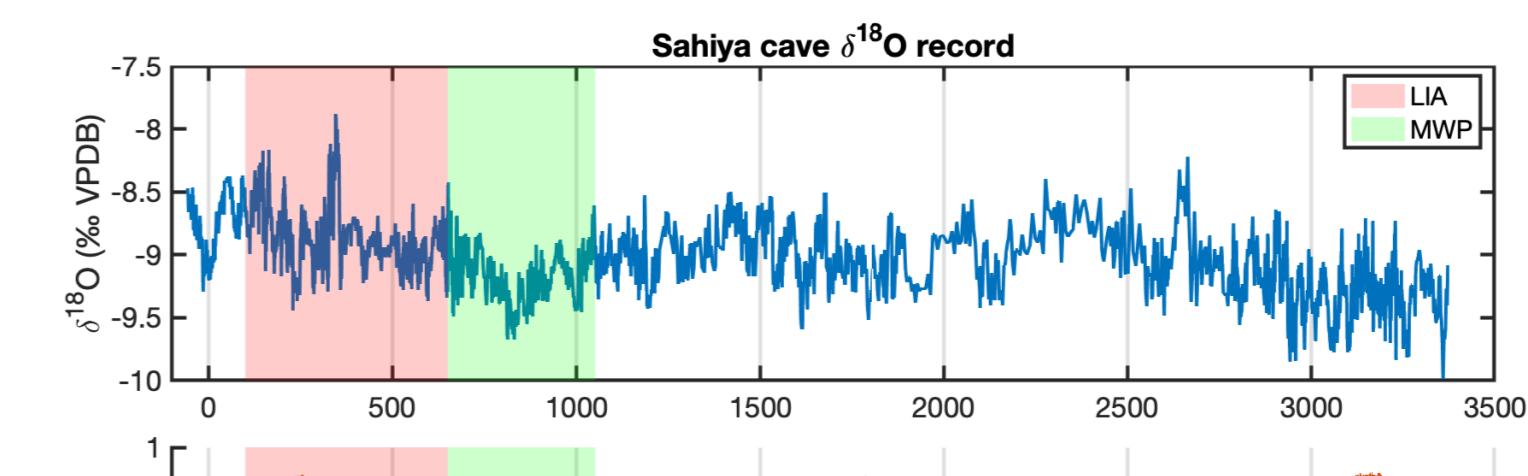
- complexity of recurring structures in the data
- further entropy definitions use block area, white diagonal lines, cumulative diagonal line lengths

Entropies behave differently for different dynamics. Uncorrected RQA entropy shows too high values for periodic dynamics due to the border effects.

Regime transitions in palaeoclimate



- preliminary results indicate that entropies of the RPs of precipitation proxies show a certain pattern of (correlating) transitions



- transitions not apparent in the time series
- especially into/off the Medieval Warm Period (MWP) and the Little Ice Age (LIA)

