



# Discrete Cascade Disaggregation of Climate Models for High Resolution Rainfall Estimation in urban environment

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# Analyzing climate models outputs using Multifractal formalism

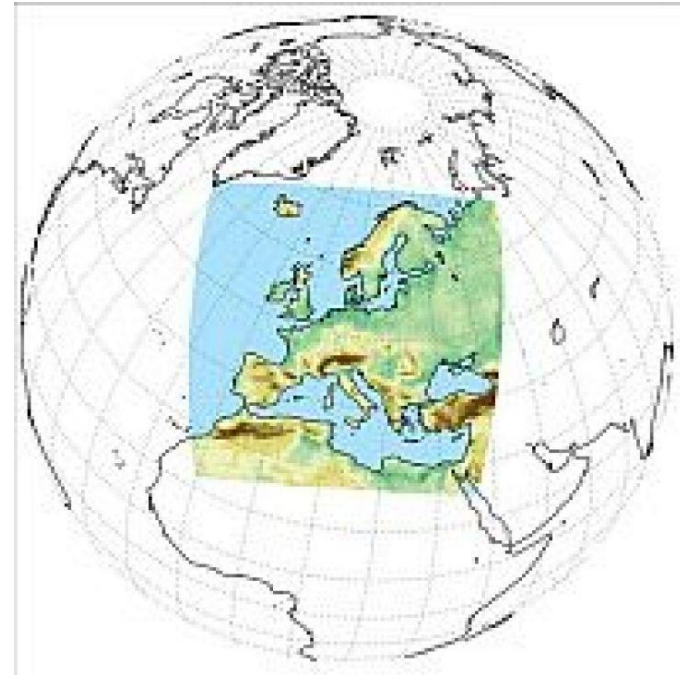
Outputs downloaded from Euro-Cordex data  
(<https://www.euro-cordex.net/> )

Models investigated :

-- 0.11°/12.5 km, hourly resolution --

*GCM*: CNRM-CERFACS-CM5 (1)

*RCM*: RCA4 (2)



We analyze precipitation outputs in  
a multifractal framework

# Analyzing climate models outputs using **Multifractal formalism**

The multifractal formalism (3) is used to statistically describe and simulate a field by means of the scale-invariant properties.

$l_0$ : outer scale ;  $l$  = **observation** scale;

$\lambda = \frac{l_0}{l}$  : resolution ;  $\varepsilon_\lambda$ : field considered at resolution  $\lambda$

$$\Pr(\varepsilon_\lambda \geq \lambda^\gamma) \sim \lambda^{-c(\gamma)}$$

$\gamma$ : Singularity of the field,  $c(\gamma)$ : codimension function of  $\gamma$  for this field

If  $D$  is the dimension of space:  $\#\{\text{points where } \varepsilon_\lambda \geq \lambda^\gamma\} \propto \lambda^{D-c(\gamma)}$

# Analyzing climate models outputs using **Multifractal formalism**

$$\Pr(\varepsilon_\lambda \geq \lambda^\gamma) = \lambda^{-c(\gamma)}$$

Equivalent to :

$$\langle \varepsilon_\lambda^q \rangle \sim \lambda^{K(q)}$$

with  $K(q) =$  Legendre transform of  $[c(\gamma)]$

## Generalized Central Limit Theorem:

$K(q)$  and  $c(\gamma)$  governed by  
 3 parameters with physical meaning :

$$K(q) = \frac{C_1}{\alpha-1} (q^\alpha - q) - Hq, \alpha \neq 1$$

$$K(q) = C_1 q \log(q) - Hq, \alpha = 1$$

$\alpha$  : Multifractality index  
 $\alpha \in [0,2]$   
 $C_1 = \left. \frac{dK(q)}{dq} \right|_{q=1}$   
 Mean intermittency of the field

$H$  characterizes the non-conservativity of the field:  
 $H \neq 0$  if  $\langle \varepsilon_\lambda \rangle \neq \text{cst.}(\lambda)$

# Analyzing climate models outputs using **Multifractal formalism**

## Evaluation of $H$ :

$\beta$  := spectral slope of energy density of the field

$$\beta = 1 + 2H - K|_{H=0}(2) \quad (\text{see } (3))$$

If  $\beta > 1$ , one deals with the fluctuations field to extract parameters:

$$\phi_\lambda = \frac{|\delta\varepsilon_\lambda|}{\langle |\delta\varepsilon_\lambda| \rangle}$$

Where :  $\delta\varepsilon = \varepsilon(i + 1) - \varepsilon(i)$

So that all parameters are taken from conservative fields with approx. scale independent mean

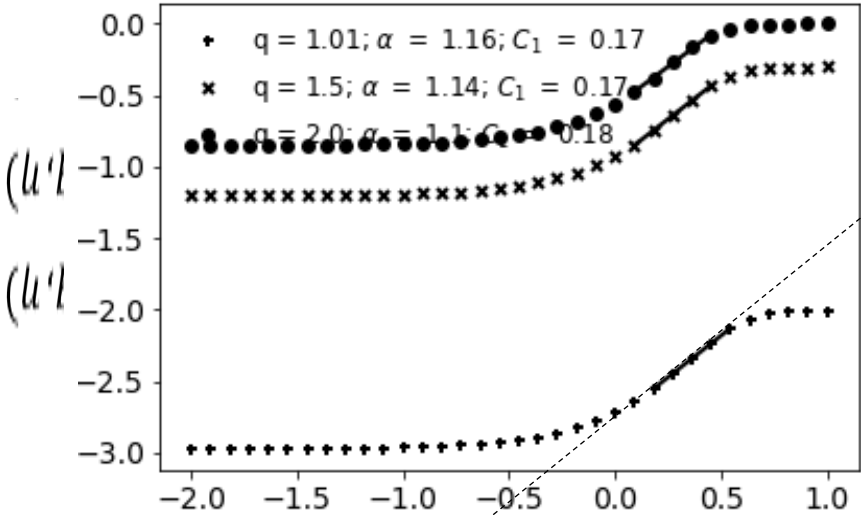
# Analyzing climate models outputs using **Multifractal formalism**

$$K(q) = \frac{C_1}{\alpha-1} (q^\alpha - q) , \alpha \neq 1 \quad \text{most frequent case}$$

$$K(q) = C_1 q \log(q), \alpha = 1$$

**Evaluation of  $\alpha$  and  $C_1$**  (*Double Trace Moment method*):

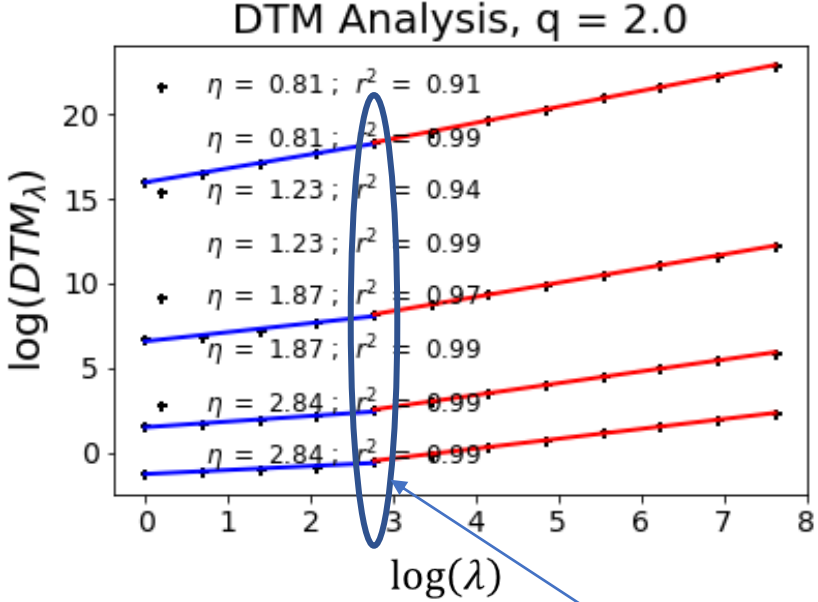
$$\langle (\varepsilon_\lambda^\eta)^q \rangle = \lambda^{K(q,\eta)} \quad K(q,\eta) = \eta^\alpha K(q)$$



$$\log K(q, \eta) = a \log \eta + b$$

$$\alpha(q) := a \quad \alpha = \langle \alpha(q) \rangle_q$$

$$C_1(q) := \frac{e^b (\alpha - 1)}{(q^\alpha - q)} \quad C_1 = \langle C_1(q) \rangle_q$$



Scale break: each regime has its own parameters

# Analyzing climate models outputs using **Multifractal formalism**

$$K(q) = \frac{C_1}{\alpha-1} (q^\alpha - q) , \alpha \neq 1$$

Finite-size sampling effects:

$$N_{\text{samples}} = \lambda^{D_s}$$

$\exists \gamma_s > 0$ , largest observable singularity:

$$\gamma_s = \frac{\alpha}{\alpha - 1} C_1 \left( \frac{D + D_s}{C_1} \right)^{\frac{\alpha-1}{\alpha}}$$

**The larger  $\gamma_s$ , the stronger the extremes are in the observed field (4)**

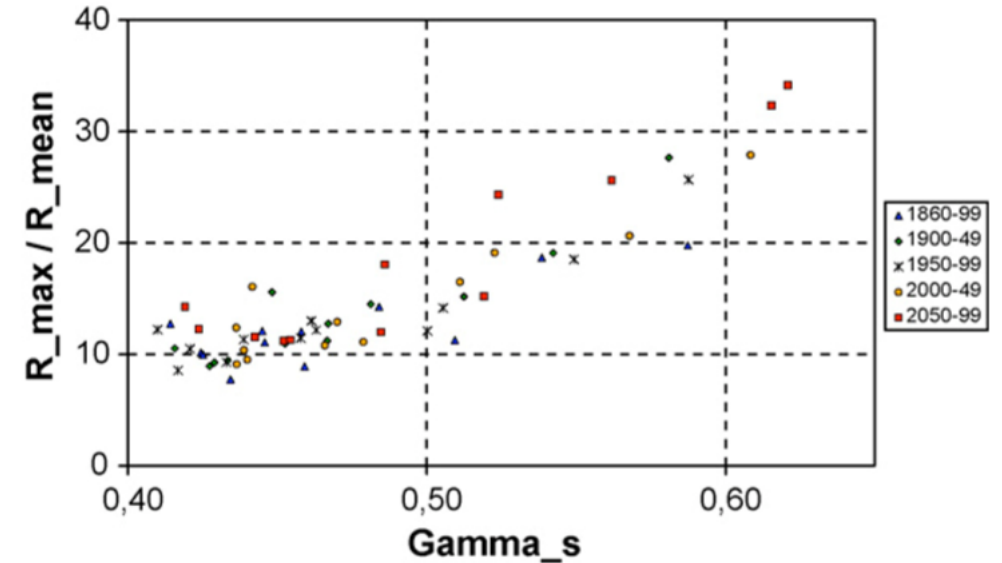


Fig. 8. Dispersion diagram showing the ratio of maximum to mean precipitation ( $R_{\max}/R_{\text{mean}}$ ) as a function of the maximal probable singularity  $\gamma_s$  for the different grid points and time intervals.

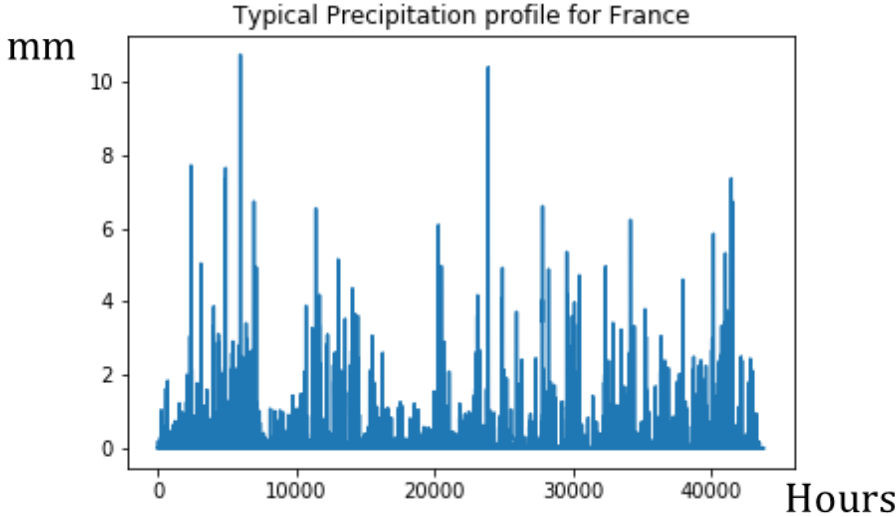
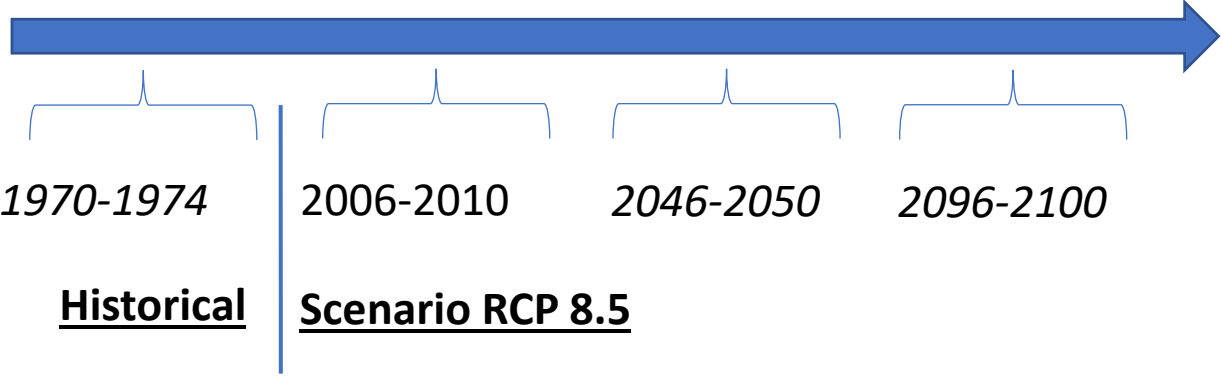
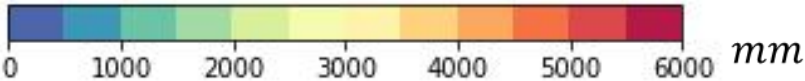
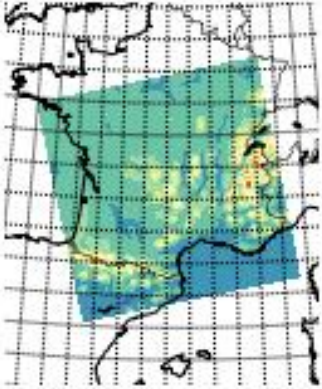
Figure taken from (4)

# Analyzing climate models outputs using Multifractal formalism

Analysis on 5-years periods

Hourly resolutions

Map of cumulative precipitations on the region of interest



Regions with  $64 \times 64$  pixels with 12.5 km resolution are analyzed pixel-wise, as was done in (5)



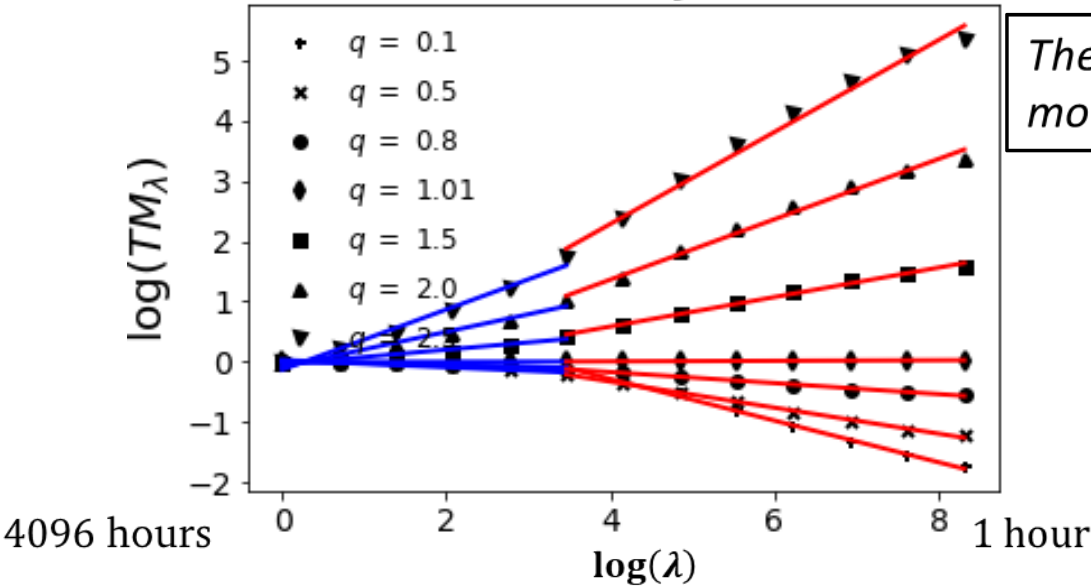
# Analyzing climate models outputs using Multifractal formalism

Quality of scaling estimated using Trace Moment method :  $\log \langle \varepsilon_\lambda^q \rangle = K(q) \log \lambda$

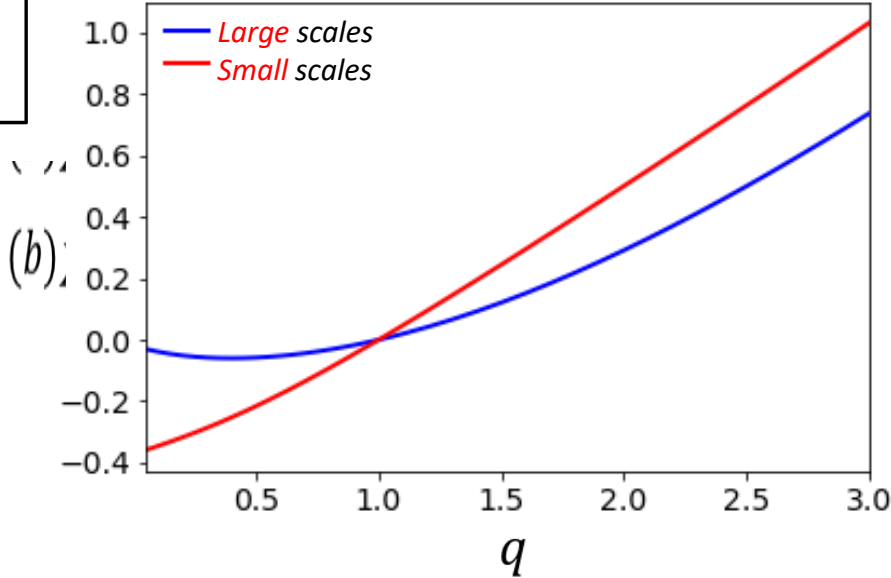
Graphs are fit using sample-averaged data from 8x8 pixels

France, 1970-1974, Winter

TM Analysis



$K(q)$  estimation from TM analysis



Scale break is at 128 hrs  $\approx$  5 days

Scaling is poorer for very high resolution (i.e smallest scales) & high moments, consistently with RCA4 underestimation of extremes events for timescales <12h (6)

# Analyzing climate models outputs using Multifractal formalism

## Seasonal analysis –

Year split into:

2048 hrs « **Summer** » (mid-June –September)

4096 hrs « **Winter** » (January –June)

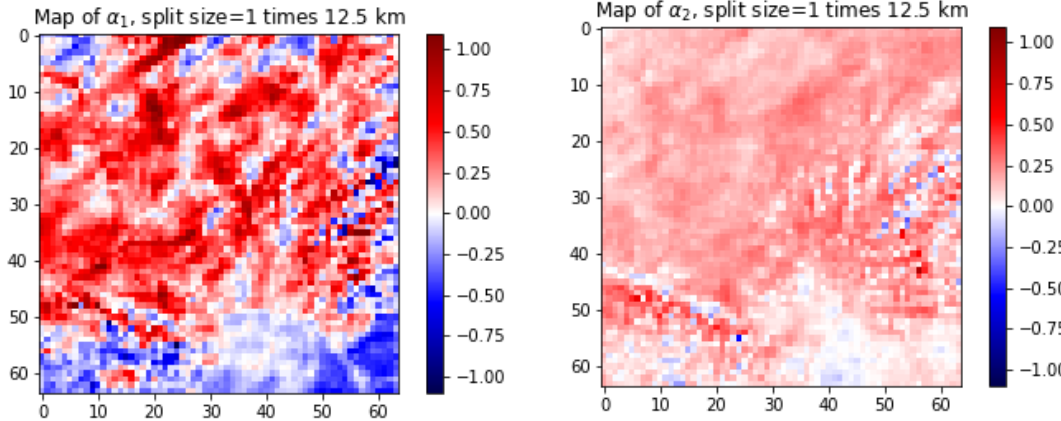
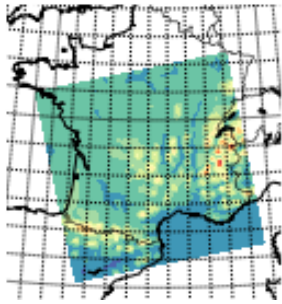
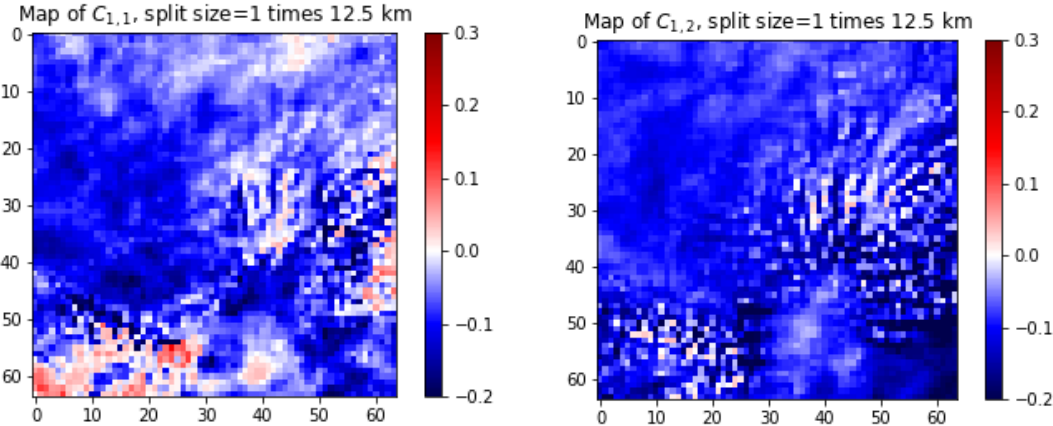
Separate MF analysis for each season, on each pixel

Powers of 2 used to numerically extract slopes

France, 1970-1974, *Winter-Summer*

$C_1(\text{Winter}) - C_1(\text{Summer})$

$\alpha(\text{Winter}) - \alpha(\text{Summer})$



Large Scales

VS

Small Scales

Large Scales

VS

Small Scales

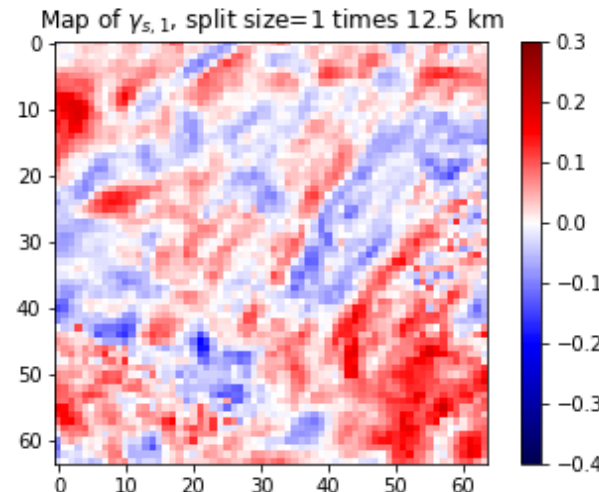
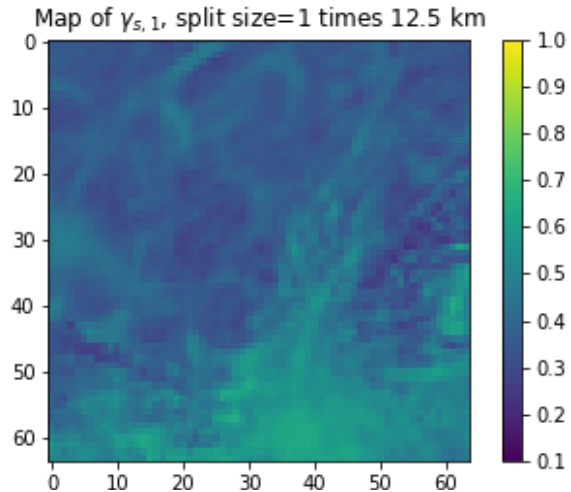
# Effects of changing climate on the MF parameters: influence on extremes

RCP 8.5 SCENARIO

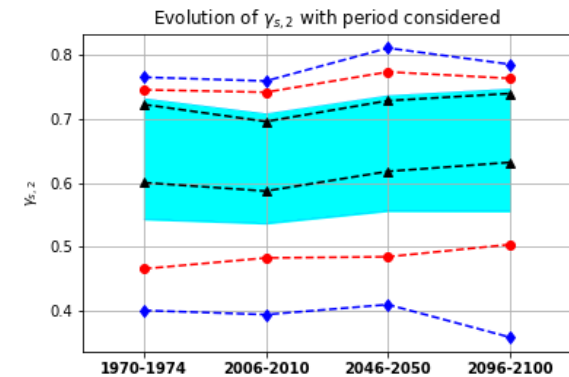
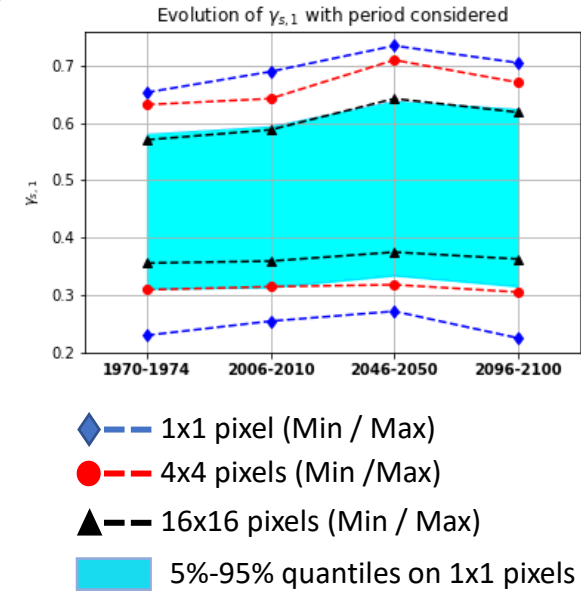
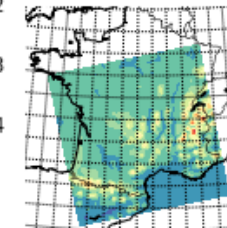
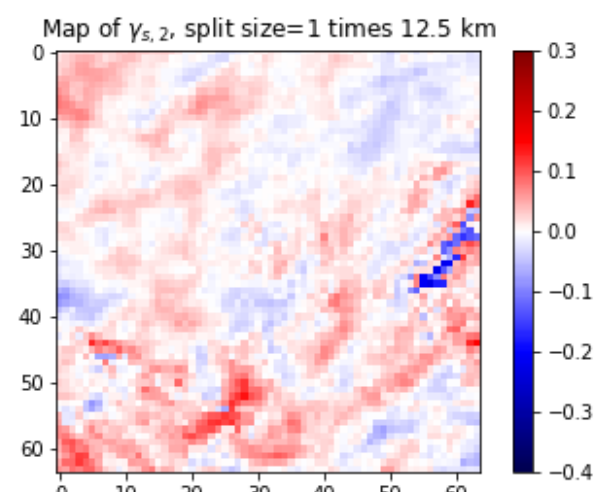
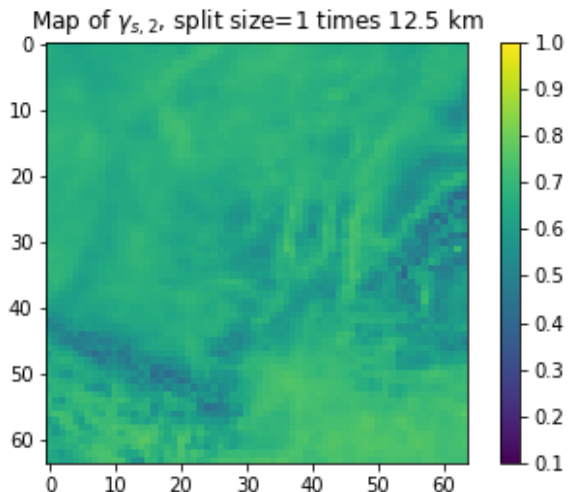
Winter 1970

Winter  $\delta(2100-1974)$

Large Scales  $\gamma_s$



Small Scales  $\gamma_s$



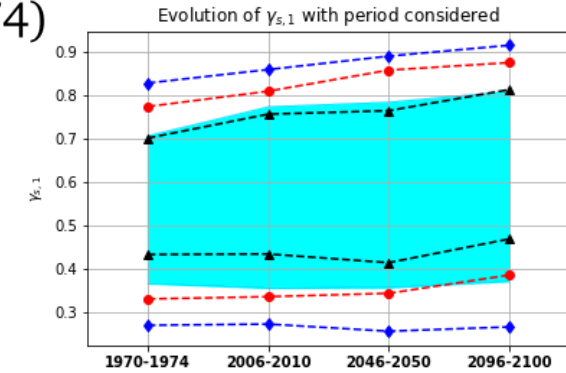
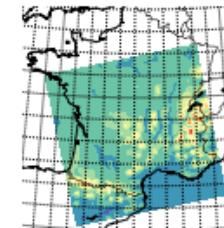
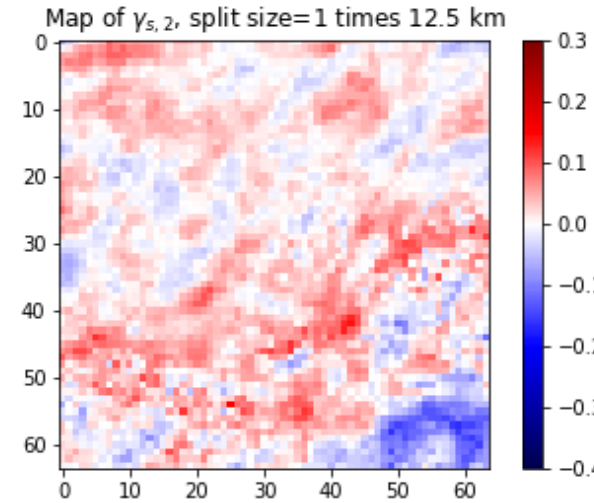
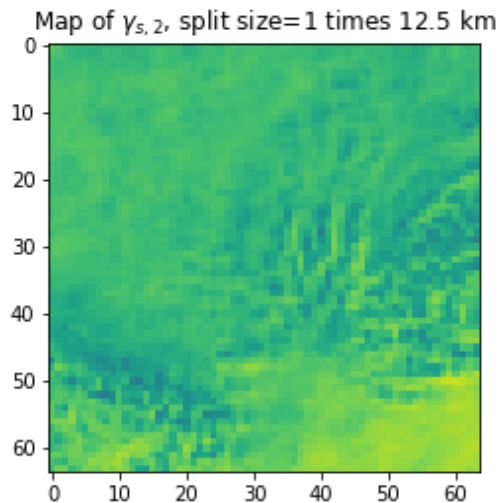
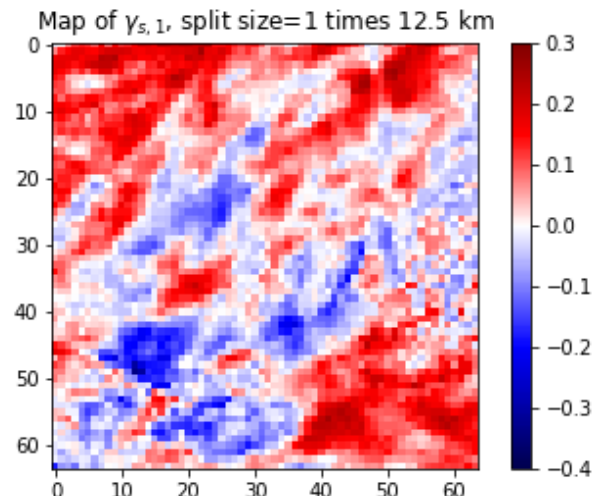
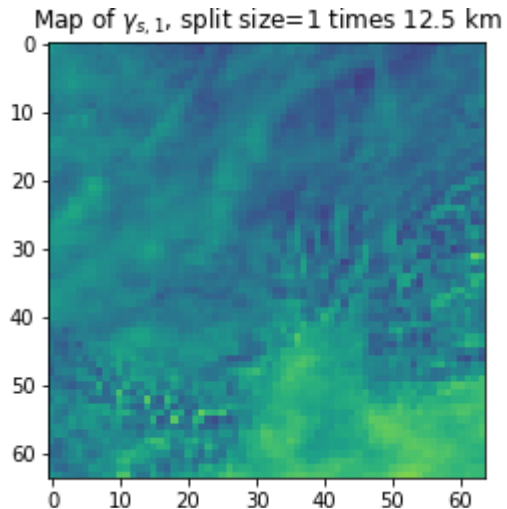
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RCP 8.5 SCENARIO

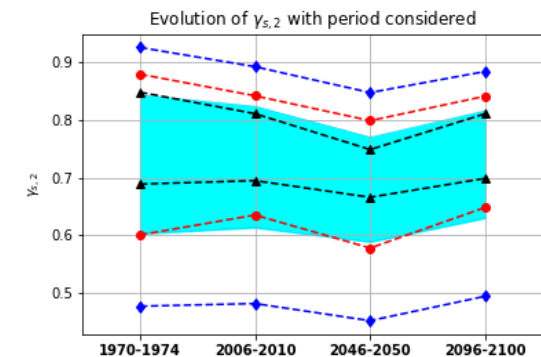
Summer 1970

Summer  $\delta(2100-1974)$

Large Scales  $\gamma_s$



- ◆ — 1x1 pixel (Min / Max)
- — 4x4 pixels (Min / Max)
- ▲ — 16x16 pixels (Min / Max)
- 5%-95% quantiles on 1x1 pixels



Small Scales  $\gamma_s$

# Conclusions and perspectives

- Significant seasonal variations for MF parameters:  
**Robust wrt numerical choices ? To be confirmed with further analysis**
- CNRM-CM5/RCA4 Model exhibits a scale break at small time scales (1h-8h) :  
**Physical feature or model artifact related to precipitation parametrization ?**
- Effect of climate change on extremes :
  - MF framework useful tool to extract **scale invariant quantification**
  - **Inhomogeneous** and Region-dependent (Mediterranea VS Continental)
  - Showing **contrasted evolution**, either augmenting or diminishing
  - **Stronger** in summer, but the trend is **weak on average**
- Further and refined investigations needed to confirm results
- Is it possible to design discrete **multiplicative cascade processes** including seasonal variations?

# Bibliography

*[1] Voldoire et al., Climate Dynamics, vol. 40 (9), 2011*

*[2] Strandberg et al., Report Meteorology and Climatology No. 116, 2014*

*[3] Schertzer & Lovejoy, Journal of Geophysical Research, 1987*

*[4] Royer et al., C.R. Geoscience 340, 2008*

*[5] Wolfensberger et al., Atmos. Chem. Phys., vol. 17, 2017*

*[6] Berg et al., Nat. Hazards Earth Syst. Sci., vol. 19, 2019*