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Source Flow in Heterogeneous Aquifers with Application to Hydraulic Tomography

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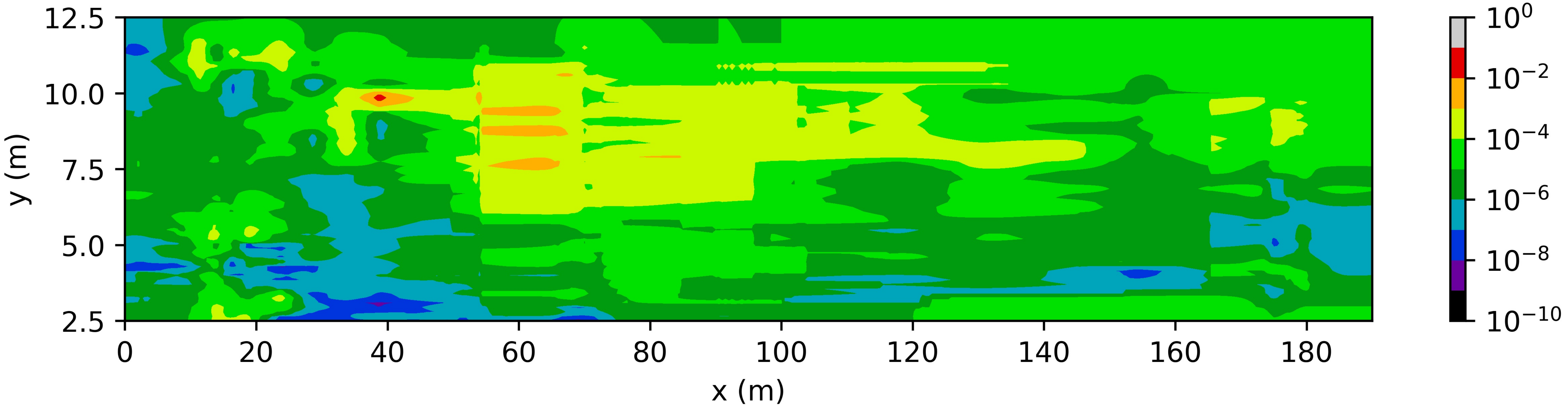
Heterogeneous formations

Map of the hydraulic conductivity at a section of the MADE site (DPIL data)



Fiori e al. (2019), Groundwater Contaminant Transport: Prediction Under Uncertainty, With Application to the MADE Transport Experiment, Front. Environ. Sci., 06 June 2019 doi:10.3389/fenvs.2019.00079

$$\mathbf{v} = -\frac{\mathbf{K}}{n} \cdot \nabla h$$

K spanning several orders of magnitude

Difficulties in characterizing hydraulic property variations at scales relevant for transport are still the main hurdle to modeling solute transport

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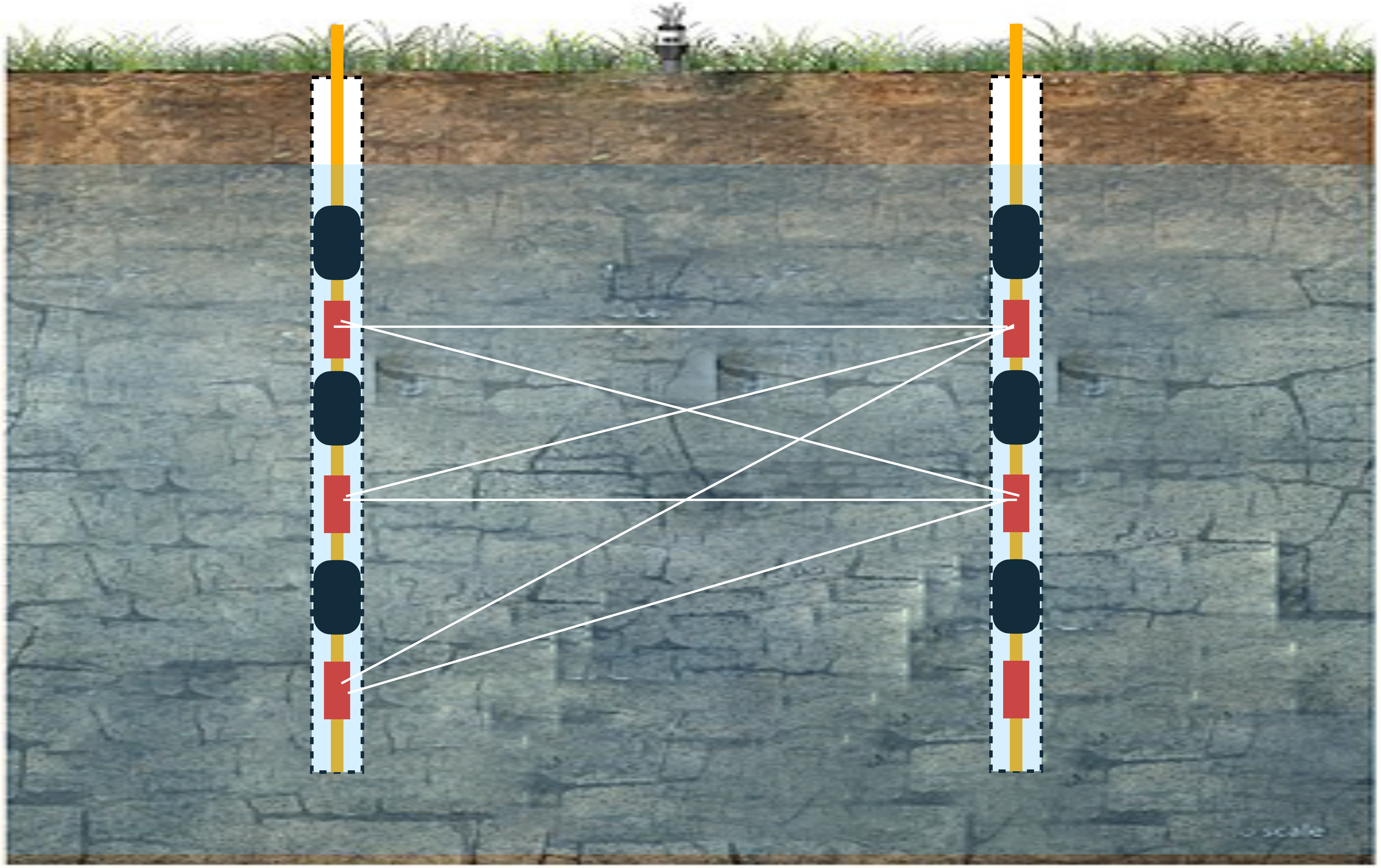
Cross-hole hydraulic tomography



Technology is available

The method is “direct” and appealing

Inversion methods are available

Cheap and accurate characterization methodology

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DICAM

Objective of the work

- Relate the structural parameters of the conductivity field to expressions of the equivalent hydraulic conductivity in view of inversion of hydraulic tomography data;

- Simplified experimental setup: point sink at the pumping port, head measurements at several receiving ports at given horizontal and vertical distances (and thereby at different r) from the sink;

- Analysis of the equivalent hydraulic conductivity under radial flow (in the mean) conditions;

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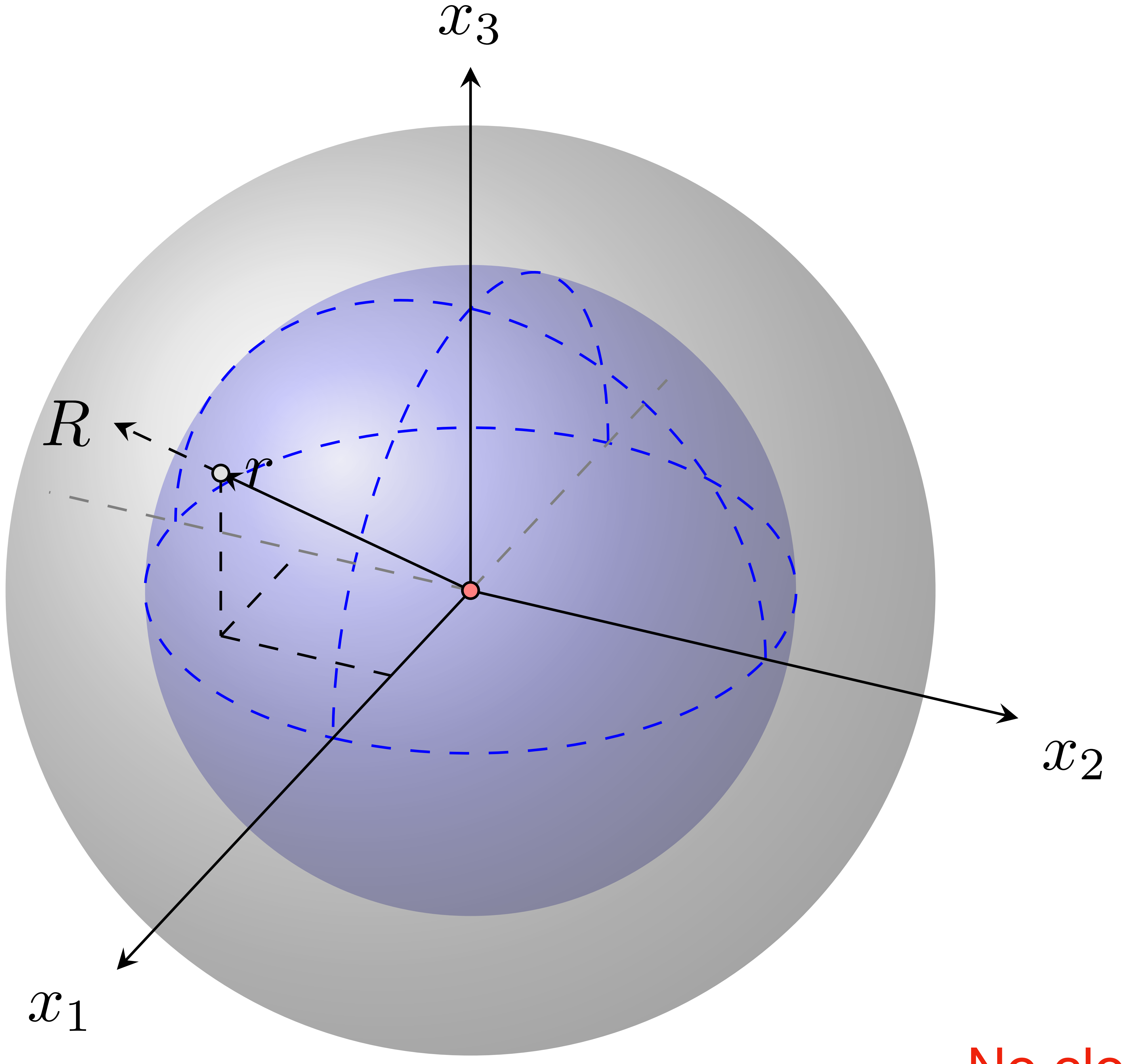
Mathematica statement

Spherical domain



Flow equation

$$\nabla \cdot \mathbf{q} + Q\,\delta(\mathbf{x}) = 0; \quad \mathbf{q} = -K\,\nabla H$$

Definitions of equivalent K

$$K_{eq}^{\langle H \rangle}(r) = \frac{Q}{4\pi\langle H(r)\rangle}\left(\frac{1}{r} - \frac{1}{R}\right) \qquad K_{eq}^{H}(\mathbf{x}) = \frac{Q}{4\pi H(\mathbf{x})}\left(\frac{1}{r} - \frac{1}{R}\right),\ r = |\mathbf{x}|$$

First Order Approximation (FOA)

See Indelman, (2001); Dagan and Lessoff (2007)

$$K_{eq}^{\langle H \rangle}(r) = K_G\left[1 - \sigma_Y^2\left(\psi - \frac{1}{2}\right)\right]$$

$$\psi = \frac{1}{3} + \left(\frac{2}{3} + \frac{r'}{6} - \frac{r'^2}{6}\right)\exp(-r') + r'\left(1 - \frac{r'^2}{6}\right)\mathrm{Ei}(-r'),\ r' = r/I$$

$$\langle K_{eq}^{H}(r)\rangle = K_G\left[1 - \sigma_Y^2\left(\psi - \frac{1}{2}\right) + \frac{\sigma_h^2}{H_0^2}\right]\ ,\ \sigma_{K_{eq}}^2(r) = K_G^2\frac{\sigma_h^2}{H_0^2}$$

No closed form solutions for σ_h^2; Severino (2011a, 2011b) provided an expression which needs quadratures and a simplification for a fully penetrating well

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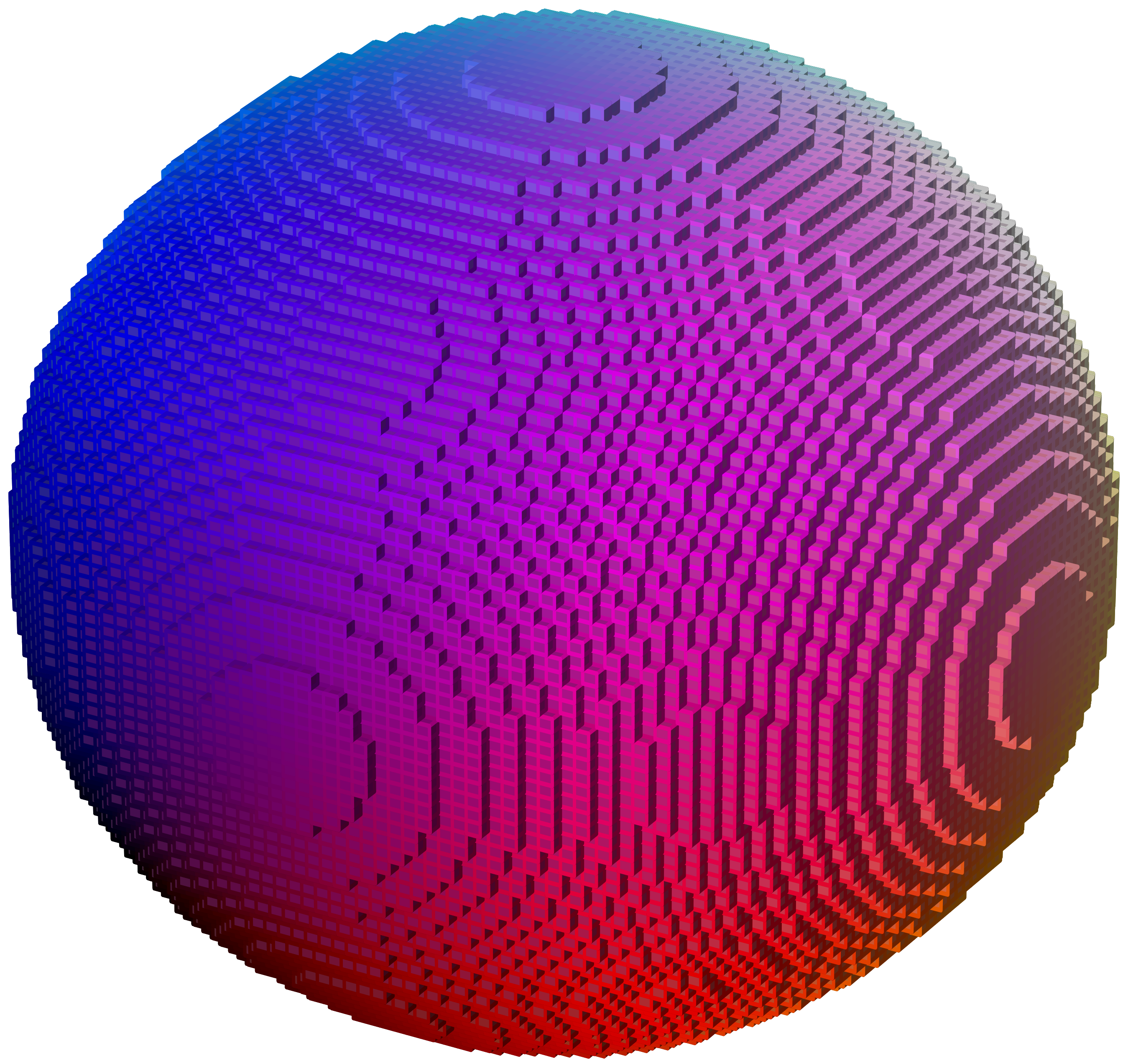
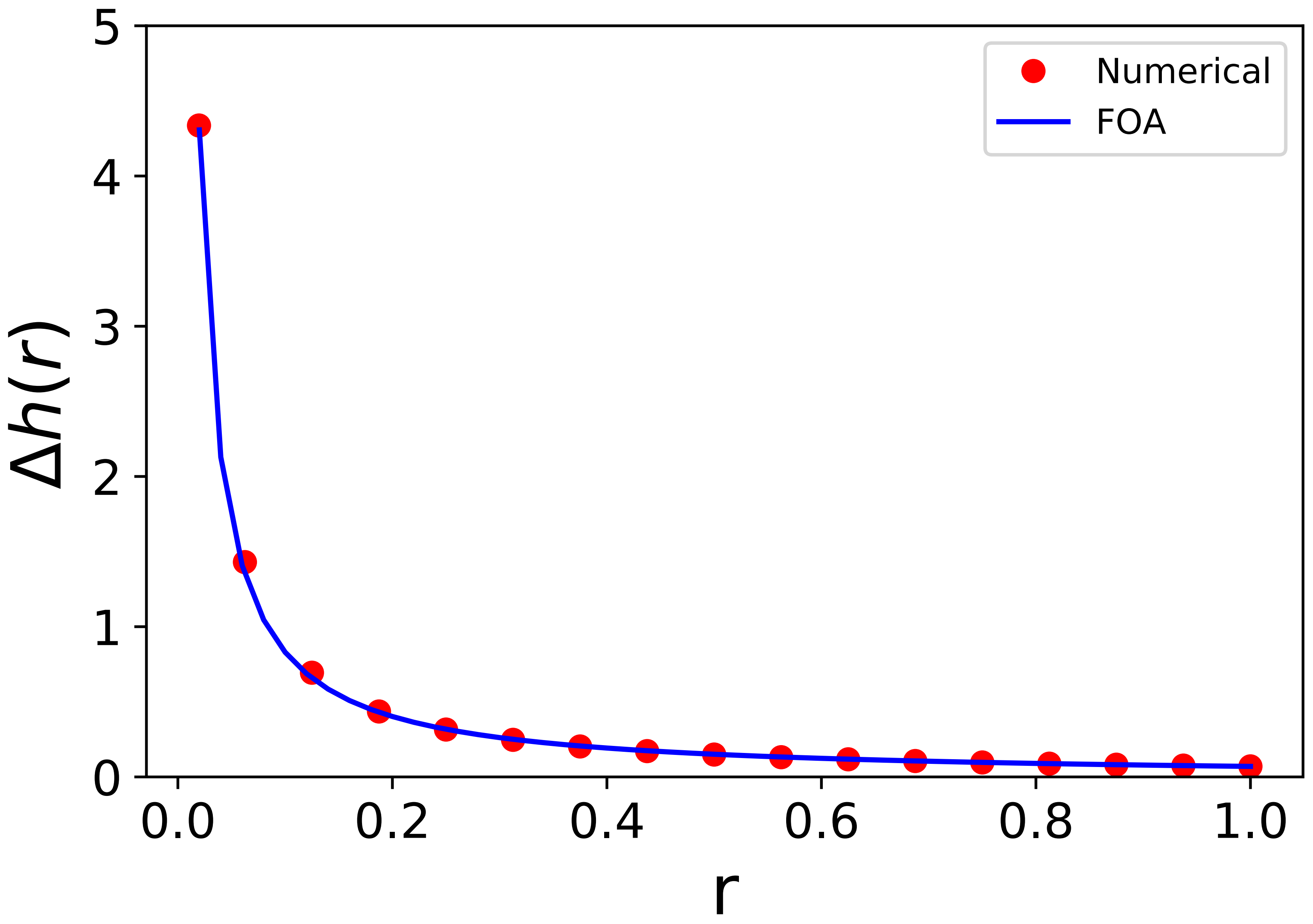
Numerical Simulations (NS)



- Flow solved numerically by Modflow 2005 (Harbaugh, 2005) + Flowpy (Bakker et al., 2016);
- domain: sphere with R = 20.0625 I;
- 16 points per integral scale ($dx_i = 0.0625\, I$) resulting in N = 17,155,325 nodes;
- MultiGaussian Y=ln K fields generated by Hydrogen (Bellin and Rubin, 1996), $\sigma_Y^2 = 0.2, 0.5, 1, 2, 4$;
- The numerical scheme approximates very well FOA at low heterogeneity, i.e. $\sigma_Y^2 = 0.2$
- 1000 Monte Carlo realizations

Note: for illustration purposes the depicted grid is much coarser than the one used in the simulations ($187\ 10^3$ against $17\ 10^6$ nodes)



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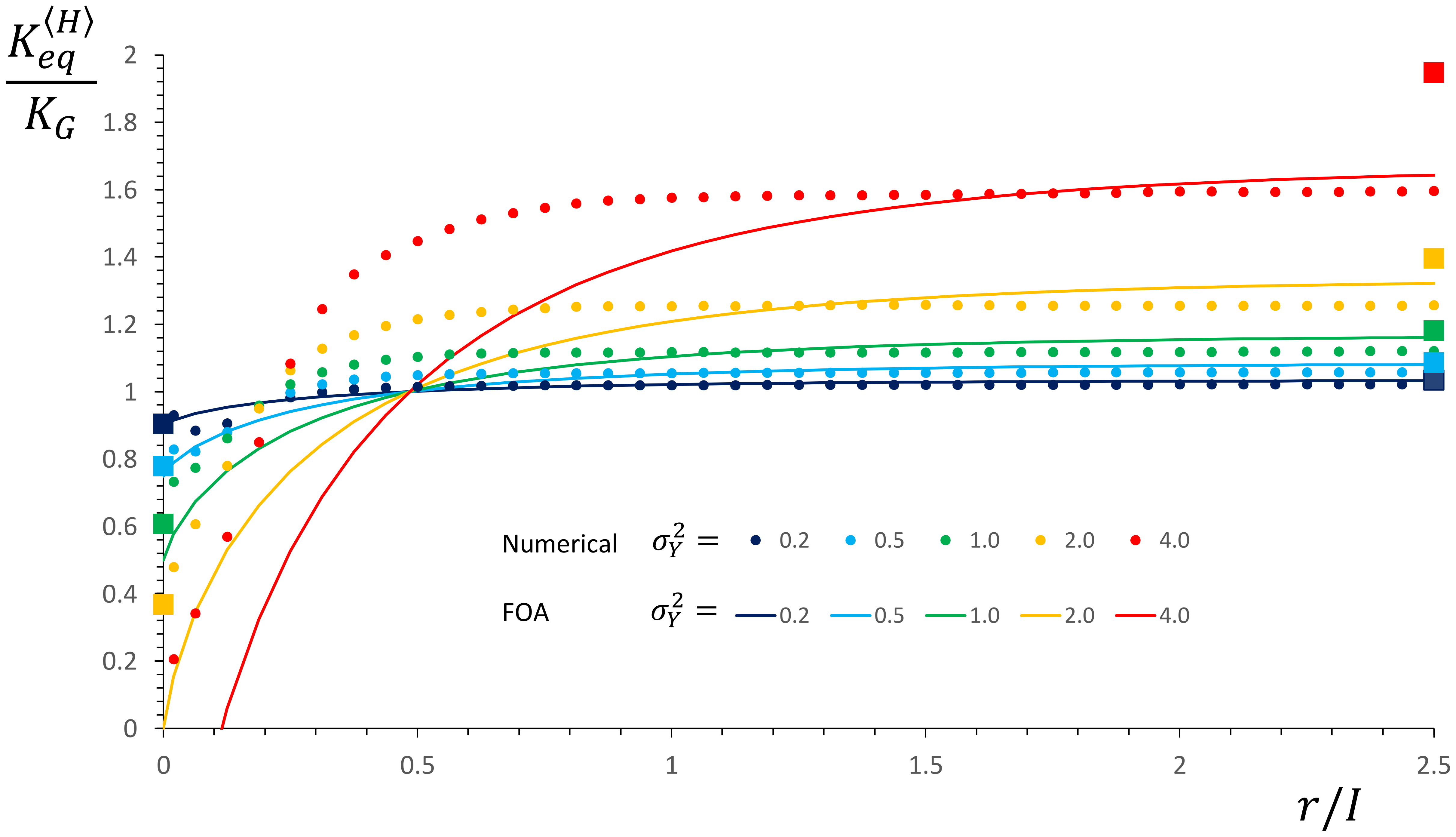
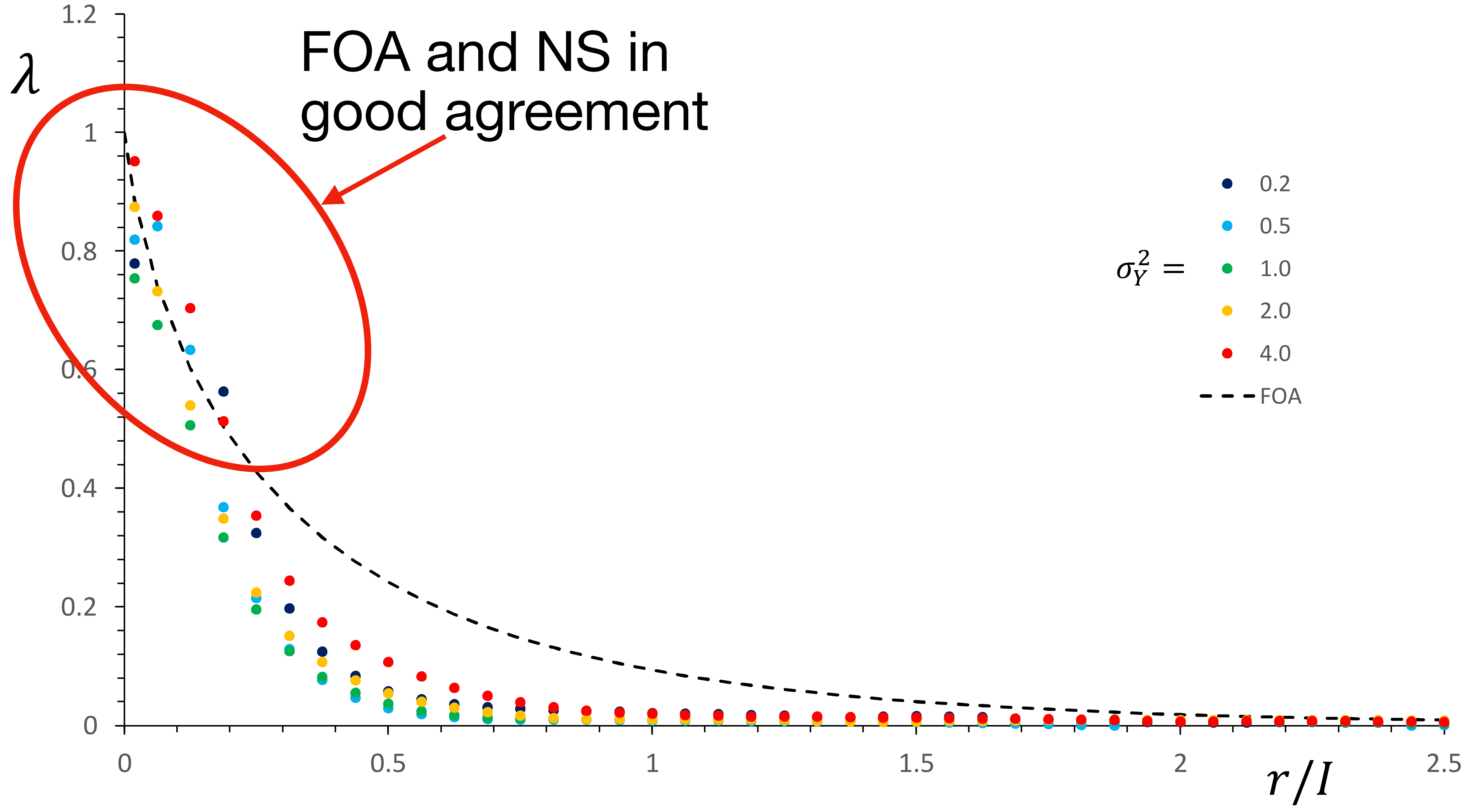
Dependence of $K_{eq}^{\langle H \rangle}$ on the distance r from the sink





FOA:

$$r' = r/I \to 0; K_{eq}^{\langle H \rangle}/K_G \to K_H/K_G = \exp(-\sigma_Y^2/2) \simeq 1 - 0.5\,\sigma_Y^2$$

$$r' > I; K_{eq}^{\langle H \rangle}/K_G \to K_{efu}/K_G \simeq \exp(\sigma_Y^2/6)$$

Matheron-Landau conjecture

$$\lambda(r') = \frac{K_{eq}^{\langle H \rangle} - K_{efu}}{K_H - K_{efu}}$$

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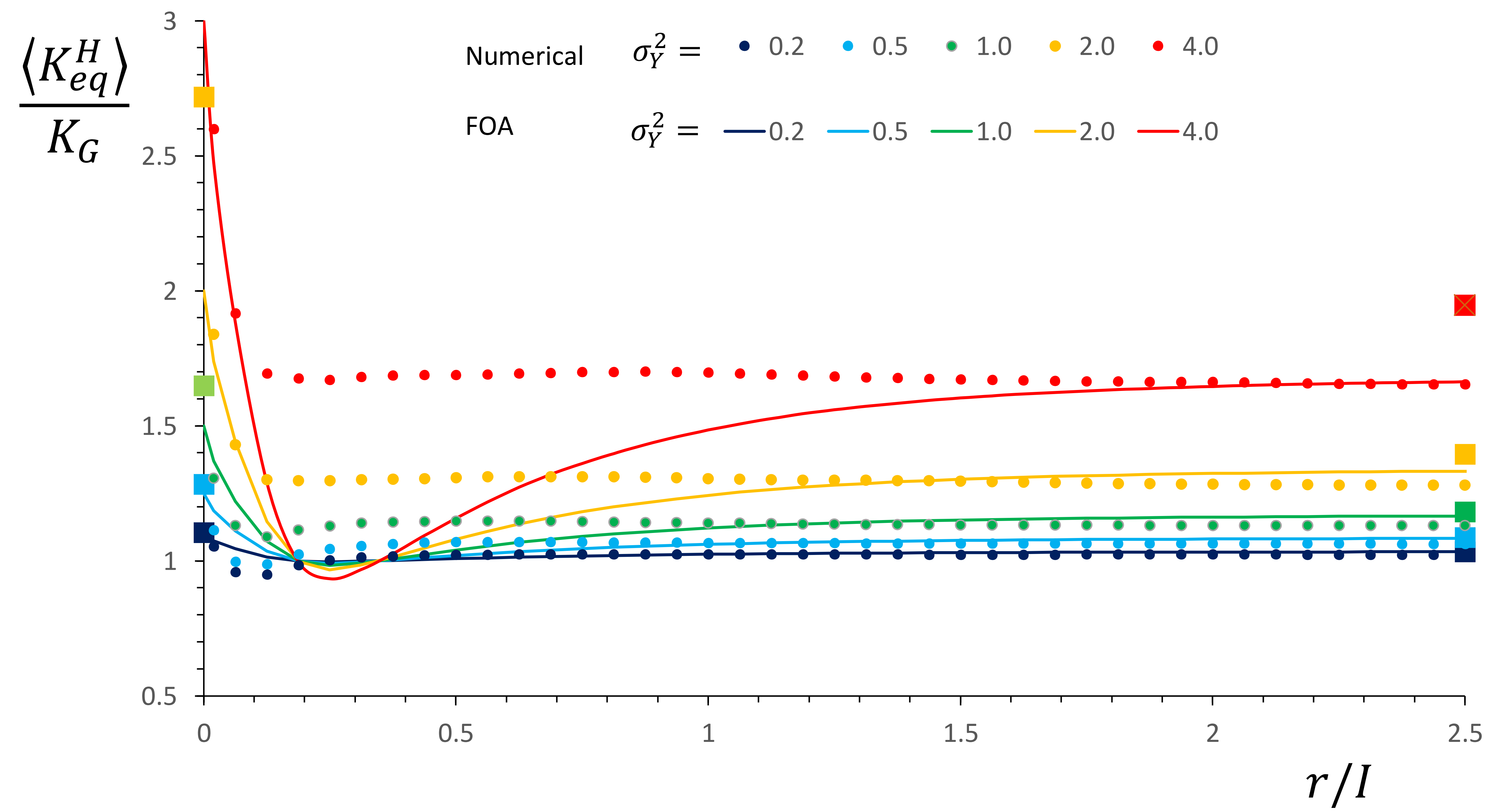
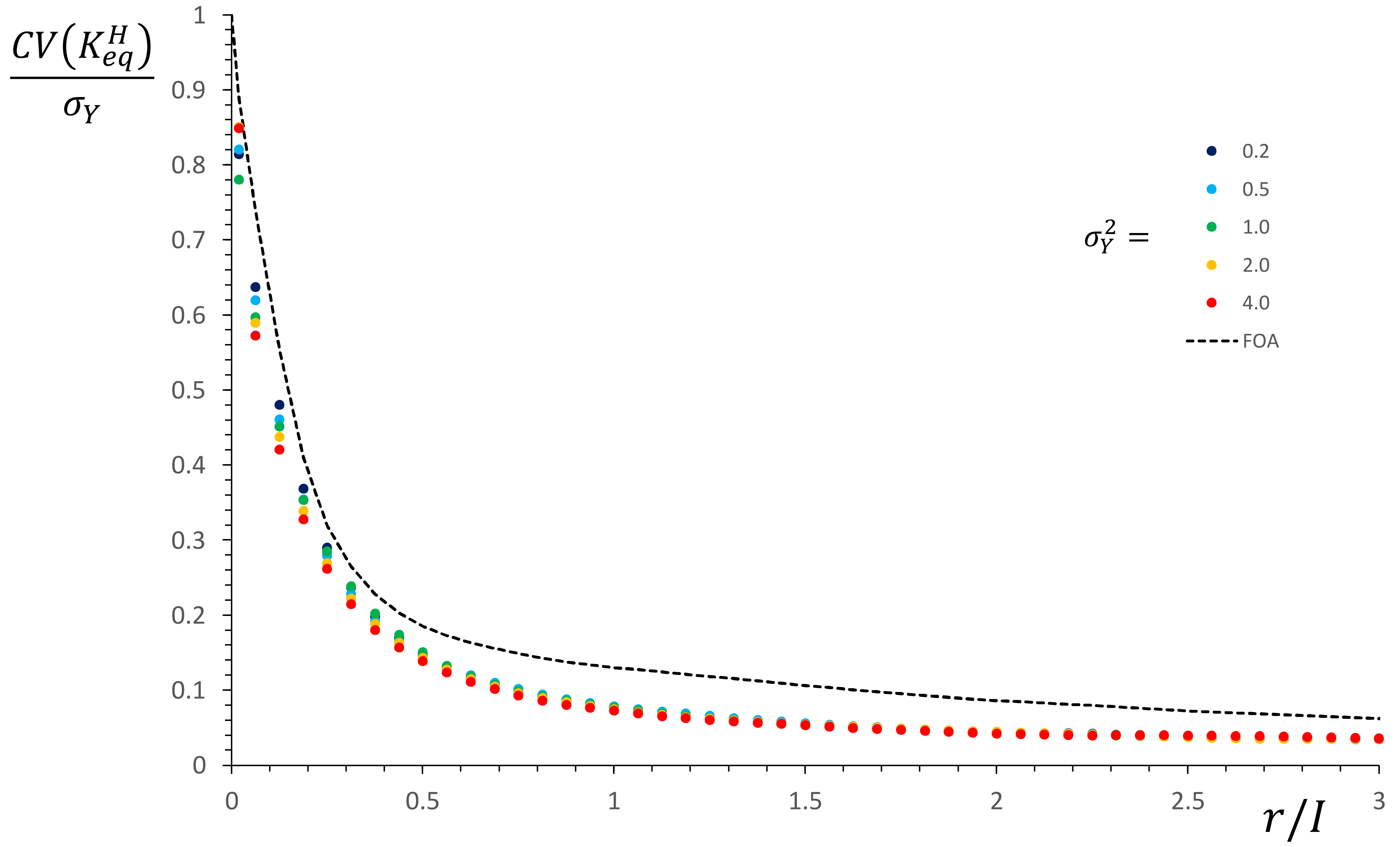
Dependence of $\langle K_{eq}^H \rangle$ on the distance r from the sink

FOA solution: $\langle K_{eq}^H \rangle = K_G[1 - \sigma_Y^2\left(\psi - \frac{1}{2}\right) + \frac{\sigma_h^2}{H_0^2}]$ with

σ_h^2 obtained by NS





Inner zone (r small): $\langle K_{eq}^H \rangle / K_G = K_A/K_G = \exp(\sigma_Y^2/2)$

NS converge rapidly ($r \sim 0.2\, I$) to K_{efu}/K_G

$CV(K_{eq}^H) \to \sigma_{K_{eq}}/\langle K_{eq}^H \rangle = [\exp(\sigma_Y^2) - 1]^{1/2}$ for $r/I \to 0$

FOA estimate

$CV(K_{eq}^H)/\sigma_Y \cong 0.08$ for $r/I = 1$ and to 0.02 for $r/I > 3$

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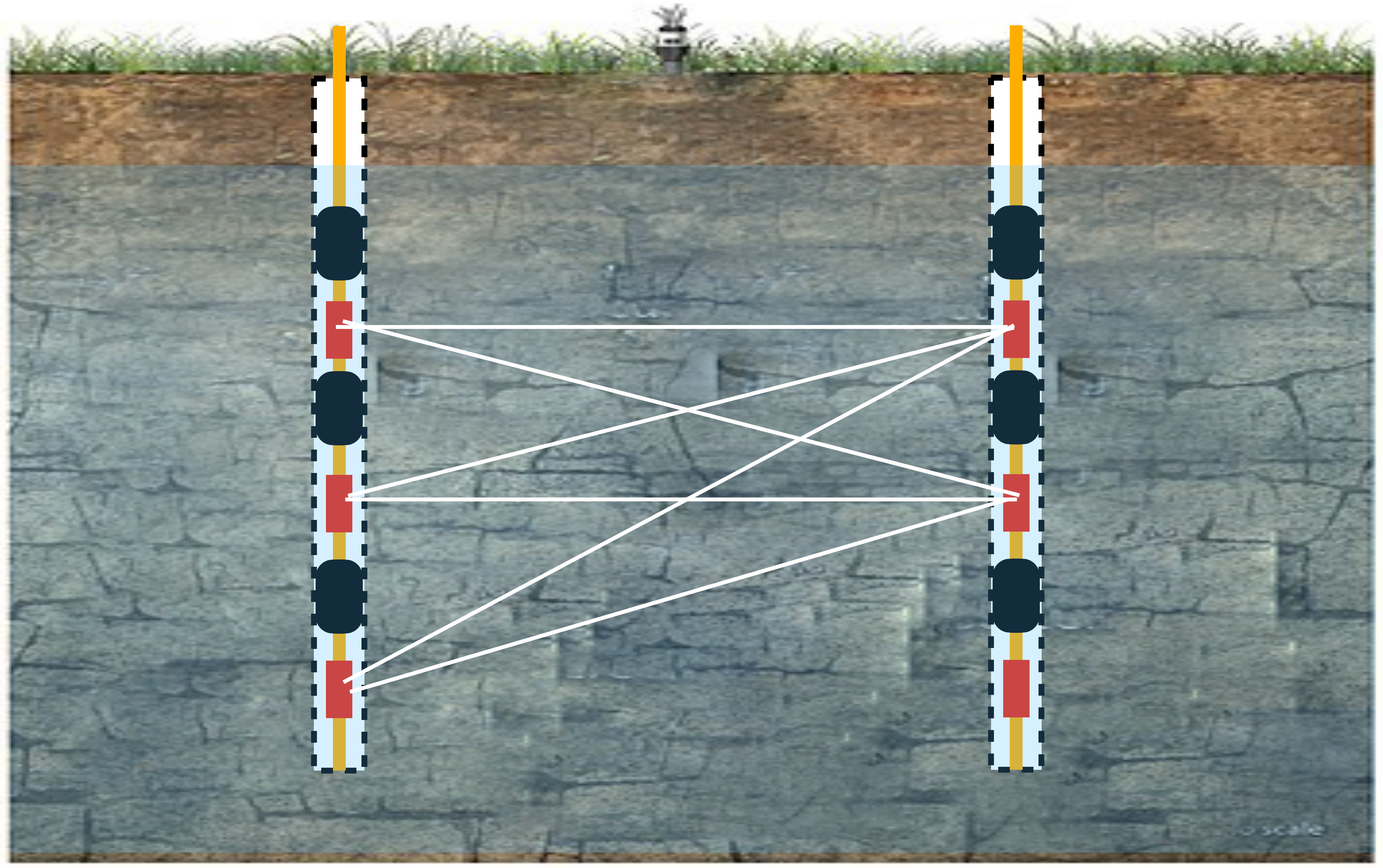
Identification of the structural (geostatistical) parameters



$$\lambda(r') = \frac{K_{eq}^{\langle H \rangle} - K_{efu}}{K_H - K_{efu}}$$ Fitted to the values of

$$K_{eq}^{\langle H \rangle}(r_i) = \frac{Q}{4\pi \langle H(r_i) \rangle}\left(\frac{1}{r_i} - \frac{1}{R}\right)$$

Computed by replacing $\langle H(r_i) \rangle$ with $\overline{H}_i$ obtained by averaging the measured heads at the same distance r_i.

		$\sigma_Y^2 = 0.5$		$\sigma_Y^2 = 1.0$	
		mean	SD	mean	SD
$N_w = 3$	$\widetilde{K}_G/K_G$	0.981	0.123	0.923	0.195
	$\widetilde{\sigma}_Y^2/\sigma_Y^2$	0.987	1.614	1.320	1.596
	$\widetilde{I}_Y/I_Y$	1.226	1.382	1.272	1.248
$N_w = 5$	$\widetilde{K}_G/K_G$	0.977	0.084	0.929	0.153
	$\widetilde{\sigma}_Y^2/\sigma_Y^2$	1.020	1.111	1.196	1.280
	$\widetilde{I}_Y/I_Y$	1.328	1.219	1.213	1.146
$N_w = 7$	$\widetilde{K}_G/K_G$	0.984	0.071	0.935	0.133
	$\widetilde{\sigma}_Y^2/\sigma_Y^2$	0.936	0.949	1.124	1.108
	$\widetilde{I}_Y/I_Y$	1.271	1.136	1.188	0.974

400 MC realizations

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Conclusions

- $K_{eq}^{\langle H \rangle} \to K_H$ near the source and then it grows with r to K_{efu} after a small transition zone ($r' = r/I \simeq 0.7$). This improves the chances to identify K_{efu} from field test, but makes the inference of the integral scale harder.

- A proof of concept has been proposed for inference of structural parameters (i.e., σ_Y^2, I, K_G) from cross well tomography;

- This approach may be useful to obtain a first estimate of the structural parameters to be improved by full numerical inversion, which may take advantage of the additional information provided by the transient regime (e.g., Liu et al., 2002; Illman et al., 2008; Cardiff et al., 2013; Illman et al., 2008; Castagna and Bellin, 2009; Castagna et al., 2011)

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