PRINCETON UNIVERSITY

Princeton Environmental Institute

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Optimality in landscape channelization and analogy with turbulence

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Collaborators:

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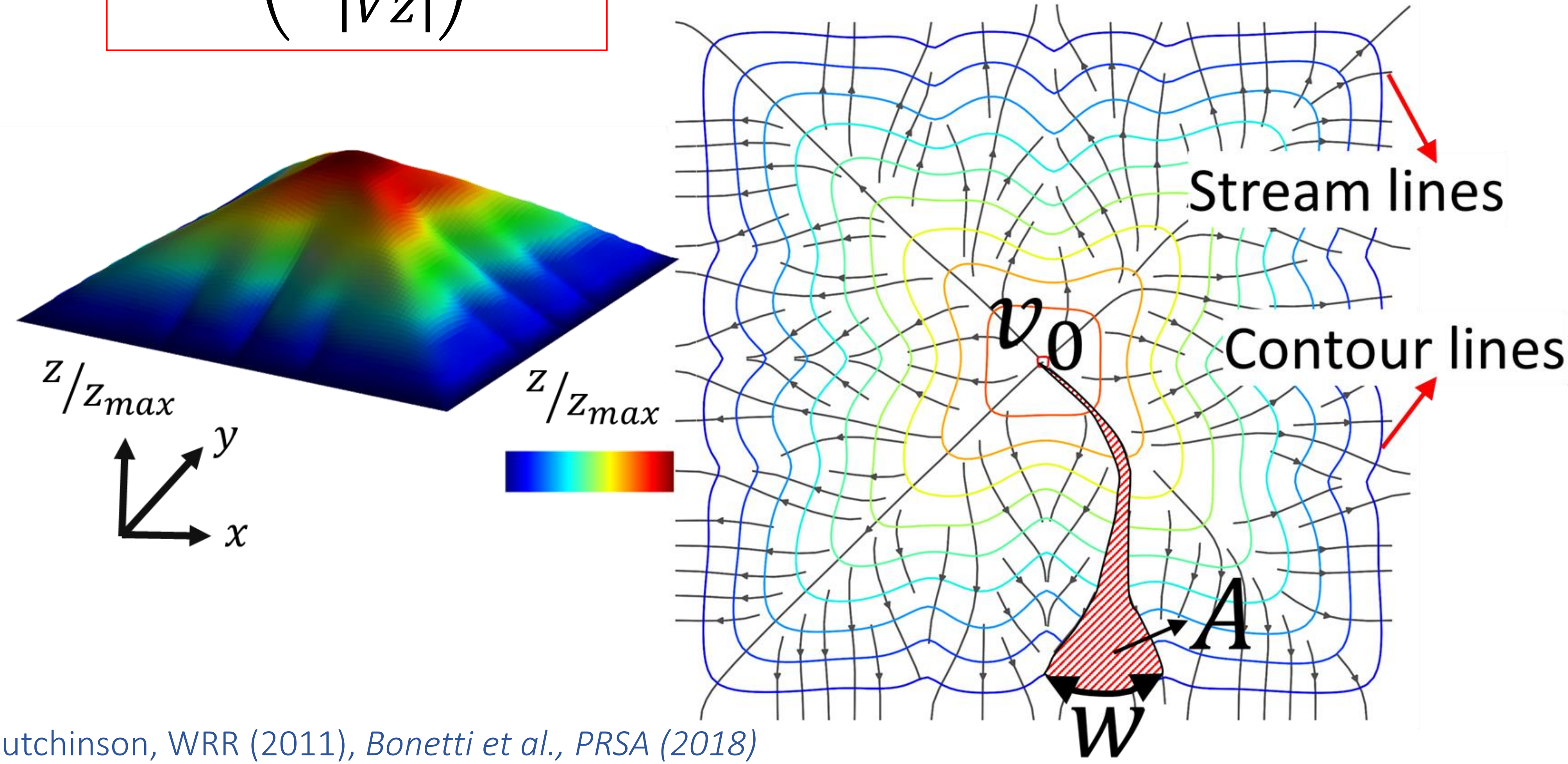
- Closed form PDEs → Variational formulation
- Initiation of channels and branching cascade
- Log-elevation profile

Equation for the specific drainage area

$$a = \lim_{w \to 0} \frac{A}{w}$$

$$-\nabla \cdot \left(a \frac{\nabla z}{|\nabla z|} \right) = 1$$



Gallant and Hutchinson, WRR (2011), *Bonetti et al., PRSA (2018)*

Minimalist Landscape-Evolution Model

$$\begin{cases} \dfrac{\partial z}{\partial t} = D\nabla^2 z - K a^m |\nabla z|^n + U \\ -\nabla \cdot \left(a \dfrac{\nabla z}{|\nabla z|} \right) = 1 \end{cases}$$

z → Elevation

a → Specific drainage area

U → Uplift rate

D, K, m, n → Model parameters

Channelization Index $\quad \mathcal{C}_{\mathcal{I}} = \dfrac{K\, l^{m+n}}{D^n U^{1-n}}$

Bonetti et al., PNAS (In press), Howard, WRR (1994)

Variational formulation in the absence of diffusion

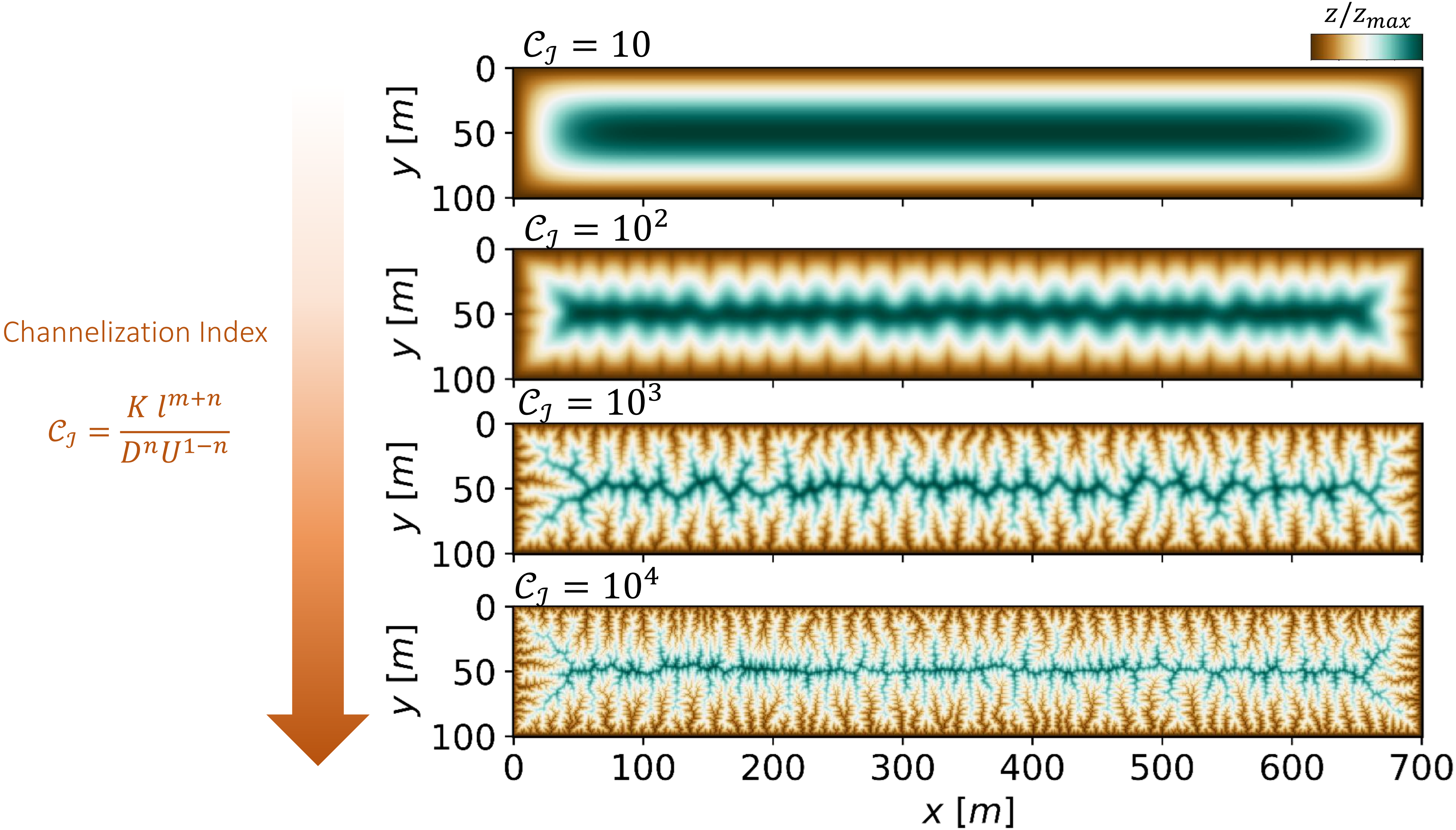
$$\begin{cases} -Ka^m|\nabla z|^n + U = 0 \\ -\nabla \cdot \left(a \dfrac{\nabla z}{|\nabla z|} \right) = 1 \end{cases} \Longleftrightarrow$$

$$\begin{cases} \min \displaystyle\int z\, ds \\ |\nabla z| \propto a^{-m/n} \end{cases} \qquad 0 < \frac{m}{n} < 1$$

$\int z\, ds$ = Cont. (Saddle) $\qquad \dfrac{m}{n} = 1$

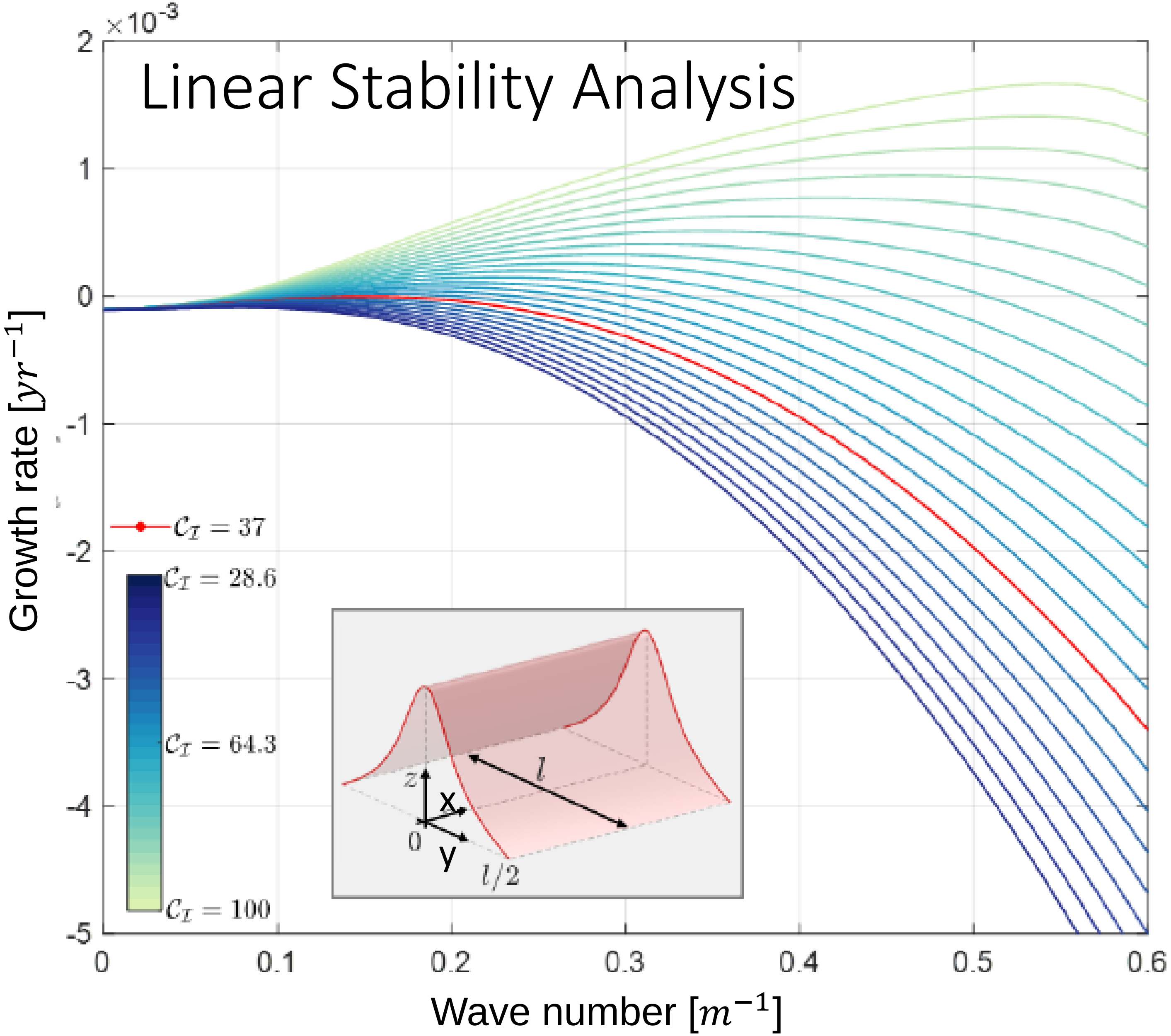
$$\begin{cases} \max \displaystyle\int z\, ds \\ |\nabla z| \propto a^{-m/n} \end{cases} \qquad 1 < \frac{m}{n} < 2$$

Hooshyar et al. (Under review)-Available in Arxiv



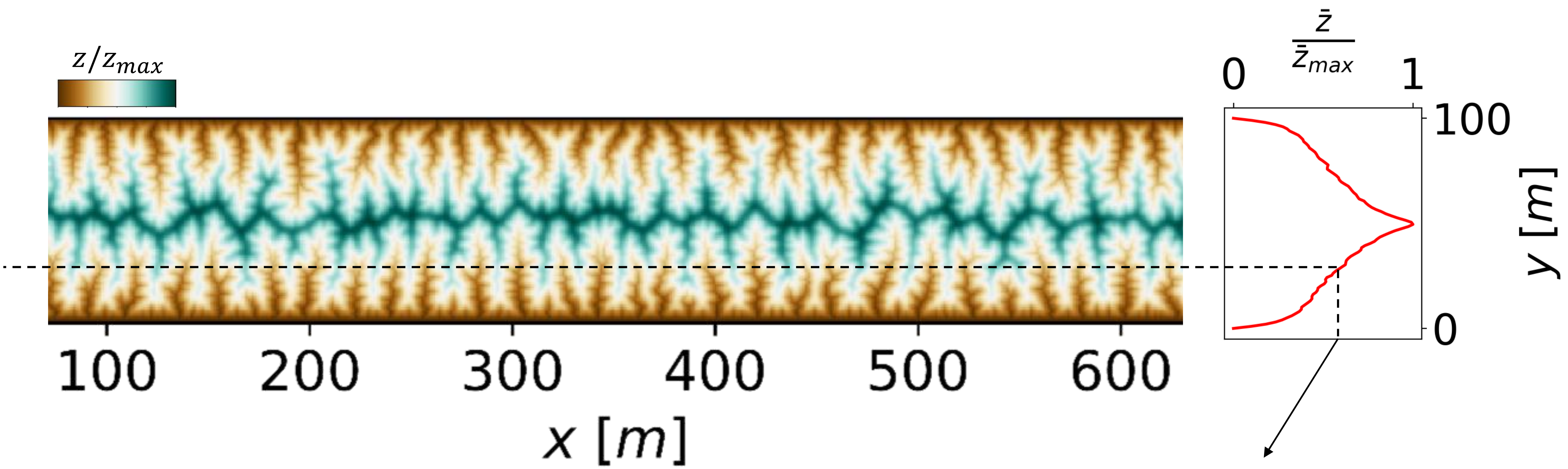
Channelization Index
$\mathcal{C}_\mathcal{J} = \frac{K\, l^{m+n}}{D^n U^{1-n}}$
z/z_{max}
$\mathcal{C}_\mathcal{J} = 10$
$\mathcal{C}_\mathcal{J} = 10^2$
$\mathcal{C}_\mathcal{J} = 10^3$
$\mathcal{C}_\mathcal{J} = 10^4$
y [m]
x [m]
0
50
100
200
300
400
500
600
700

Transition to Channelized State



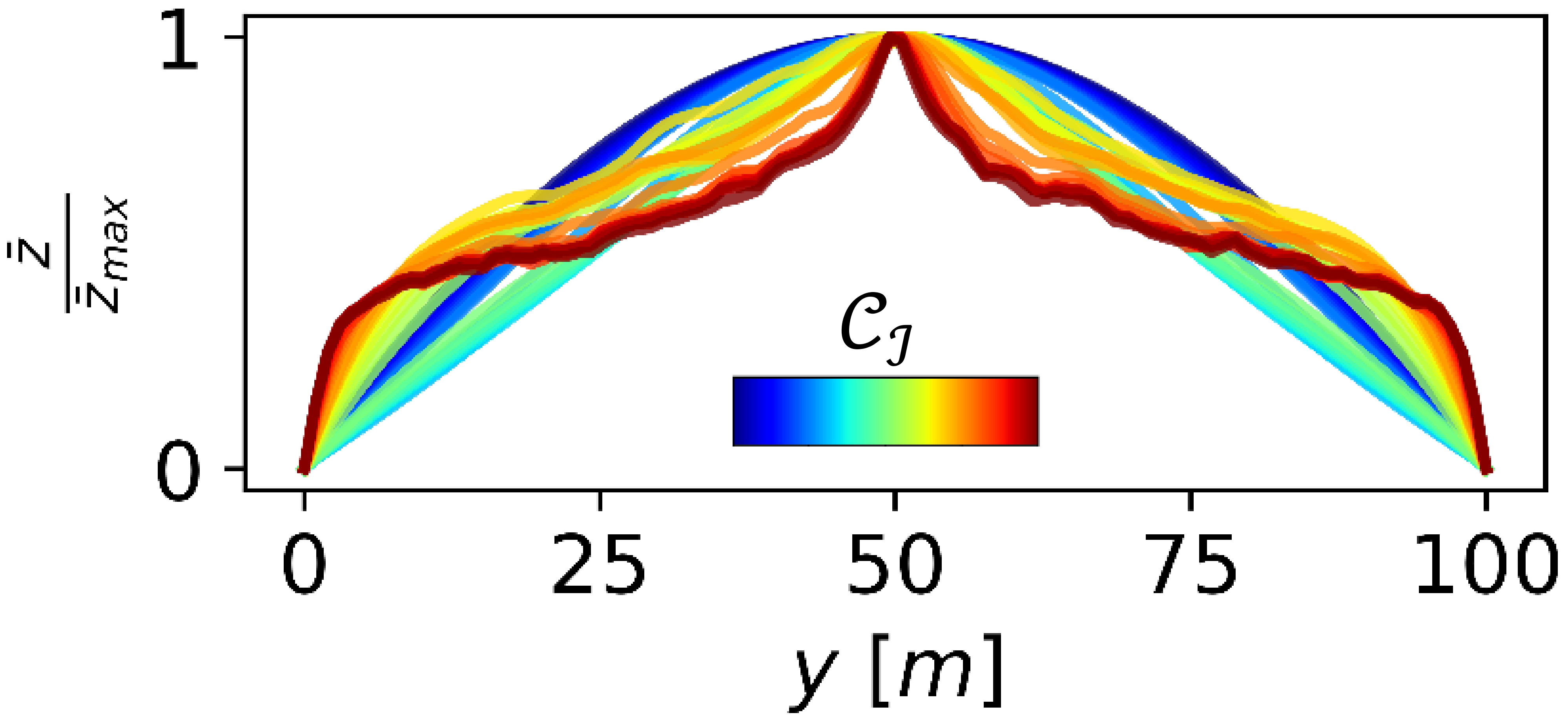
Bonetti et al., PNAS (2020)

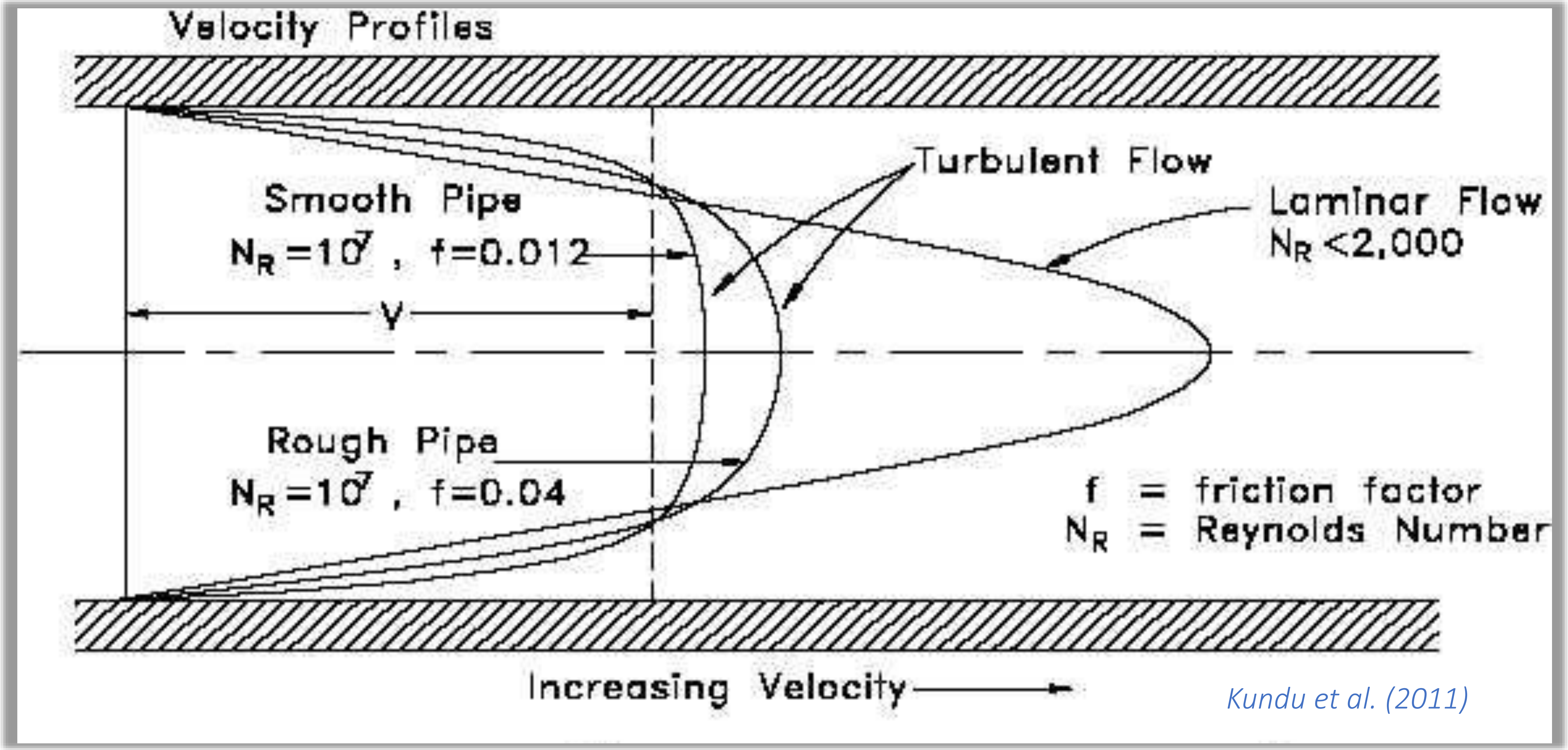
Mean-elevation profile



$$\bar{z}(y) = \frac{1}{l_x}\int_x z(x,y)\,dx$$

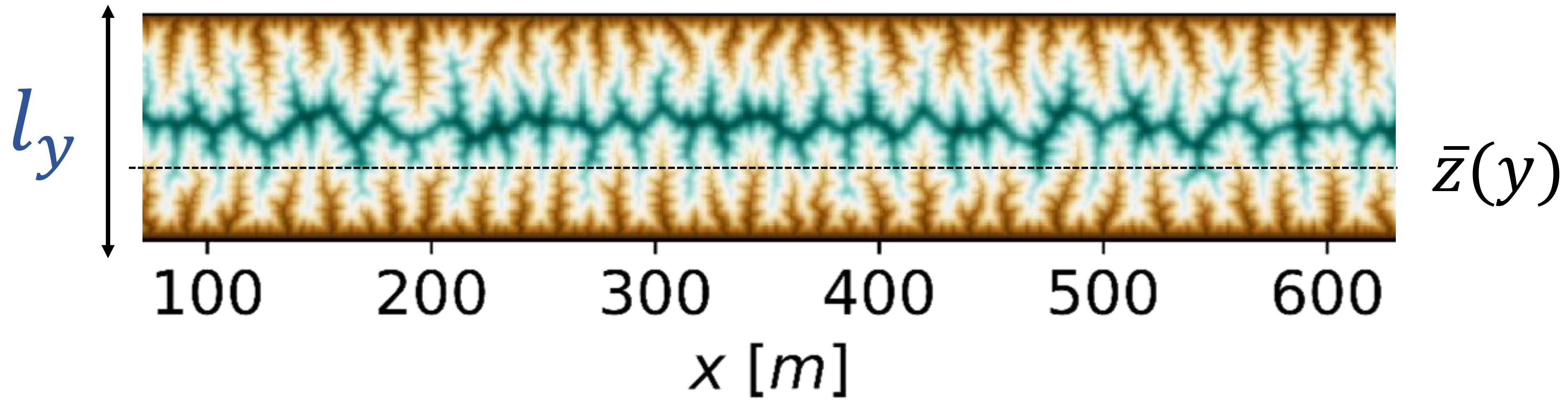
Flattening of the profile with increasing $\mathcal{C}_{\mathcal{J}}$





Kundu et al. (2011)

Mean-elevation profile



$$\frac{d\bar{z}}{dy} = f_1(y, l, z_*, D, K, U, m)$$

y, z_*, and D as dimensionally indep. variables + π-theorem:

$$(m+1)\eta\frac{d\varphi}{d\eta} = f_2(\eta, \mathcal{C}_\mathcal{J}, \xi, m)$$

$$\varphi = \frac{z}{z_*}$$

$$\eta = \frac{K\, y^{m+1}}{D}$$

$$\mathcal{C}_\mathcal{J} = \frac{K\, l^{m+1}}{D}$$

$$\xi = \frac{Ul^2}{Dz_*}$$

Low diffusion and far enough from center and boundary $\rightarrow$ intermediate region

$$\eta, \mathcal{C}_{\mathcal{J}}, \xi \gg 1$$

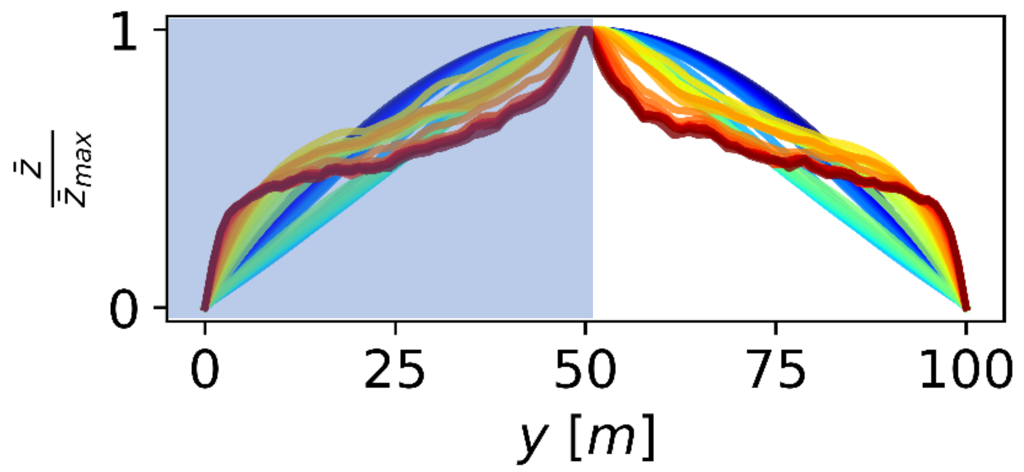
complete self-similarity

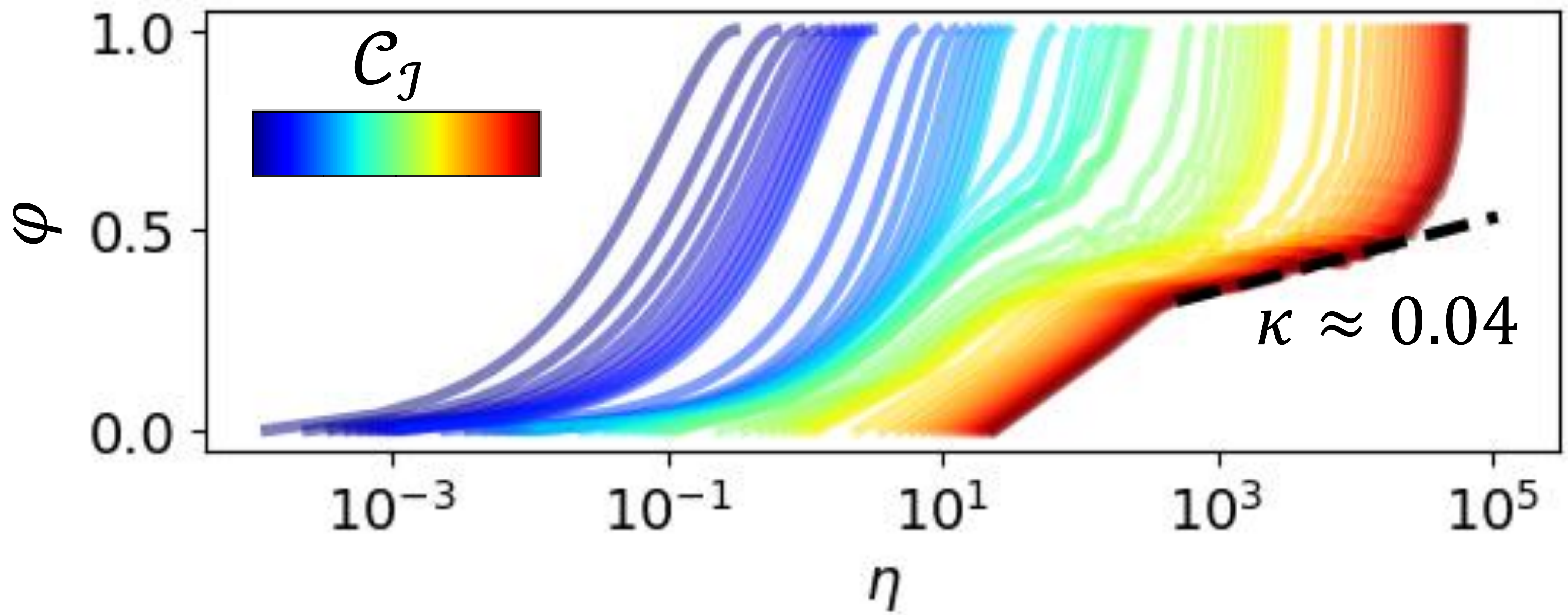
$$\eta \frac{d\varphi}{d\eta} = \kappa(m)$$

$$\varphi = \kappa(m) Log\, \eta + C$$

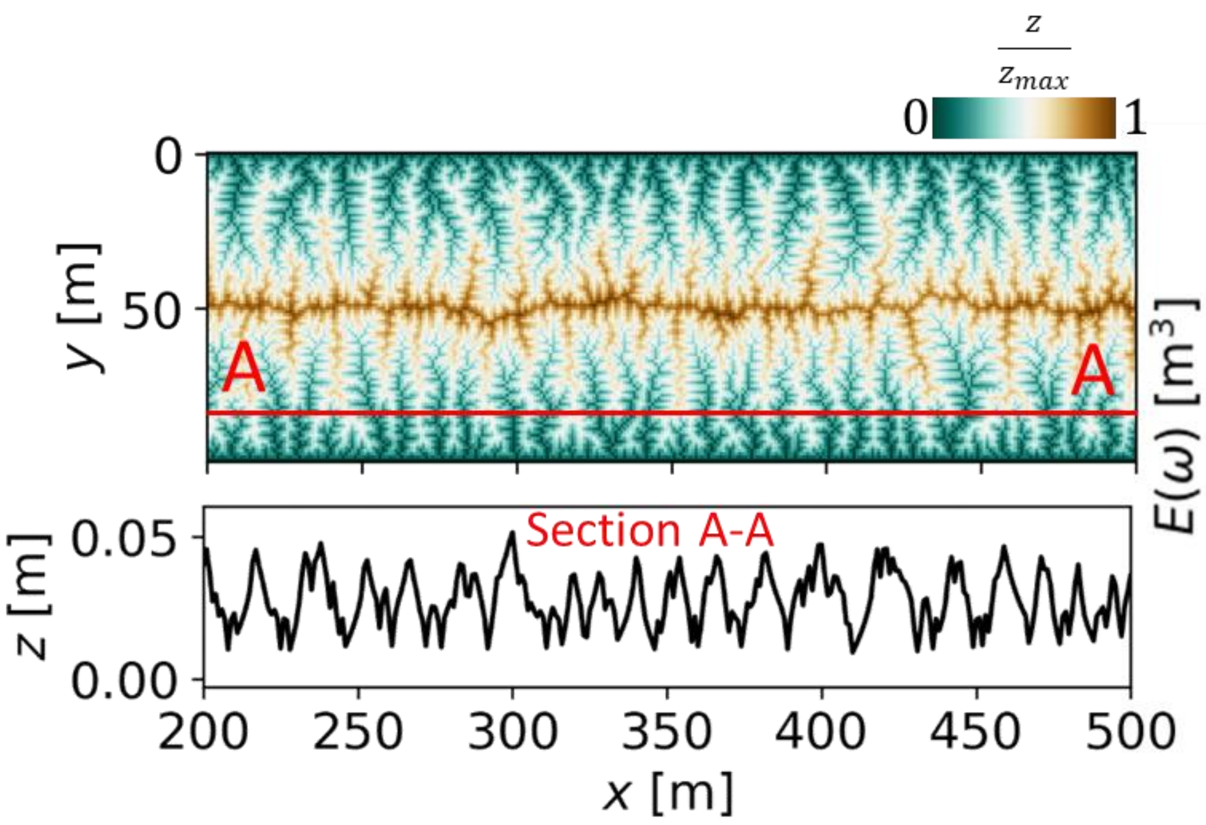
Barenblat, Scaling and similarity Cambridge (1996)

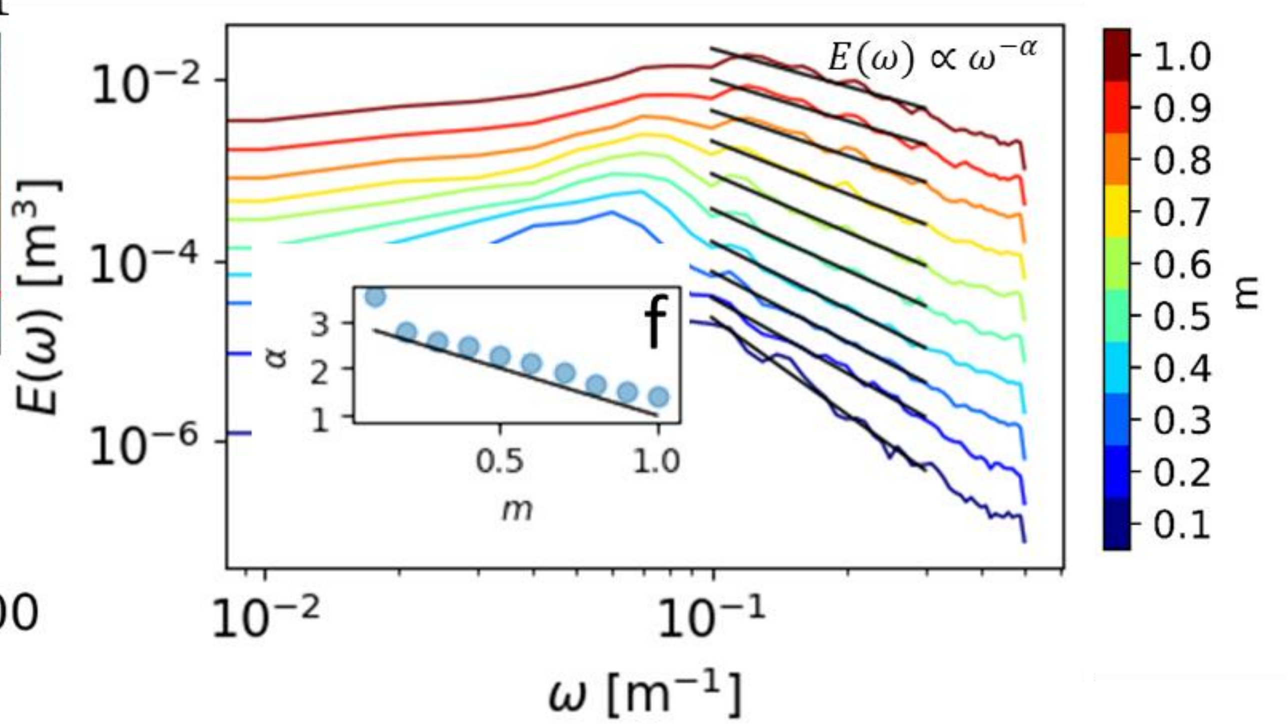
Logarithmic profile



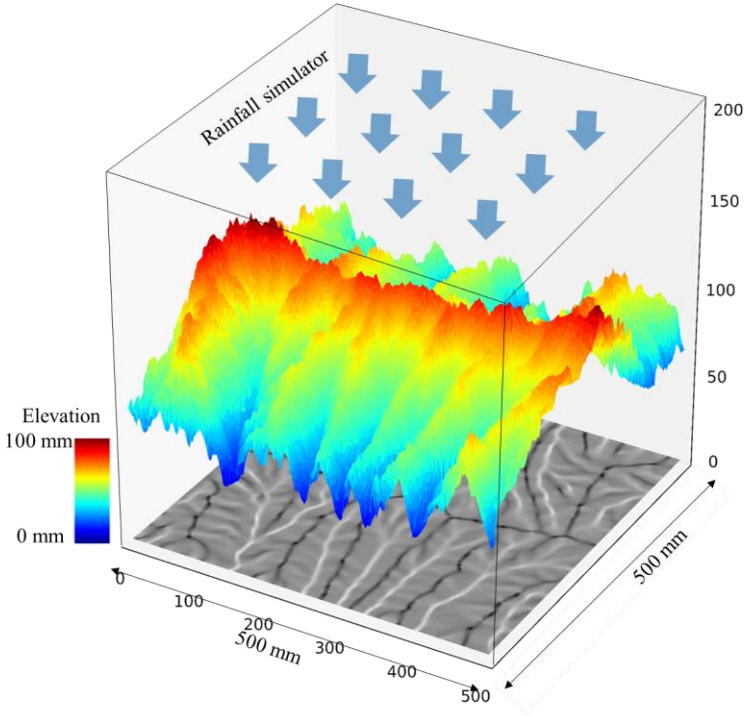


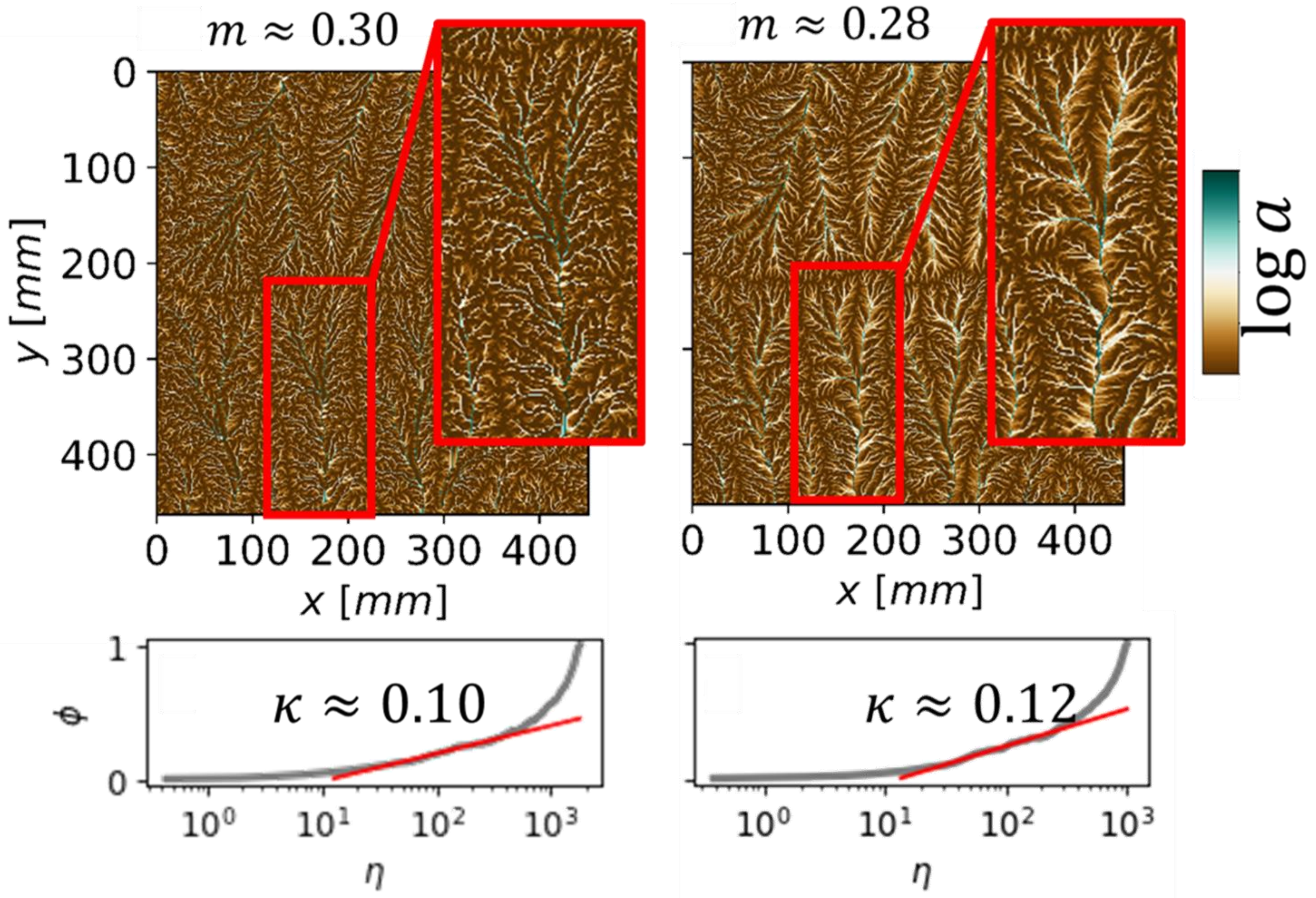
Power spectra of elevation profiles (in intermediate region)





Physical experiment

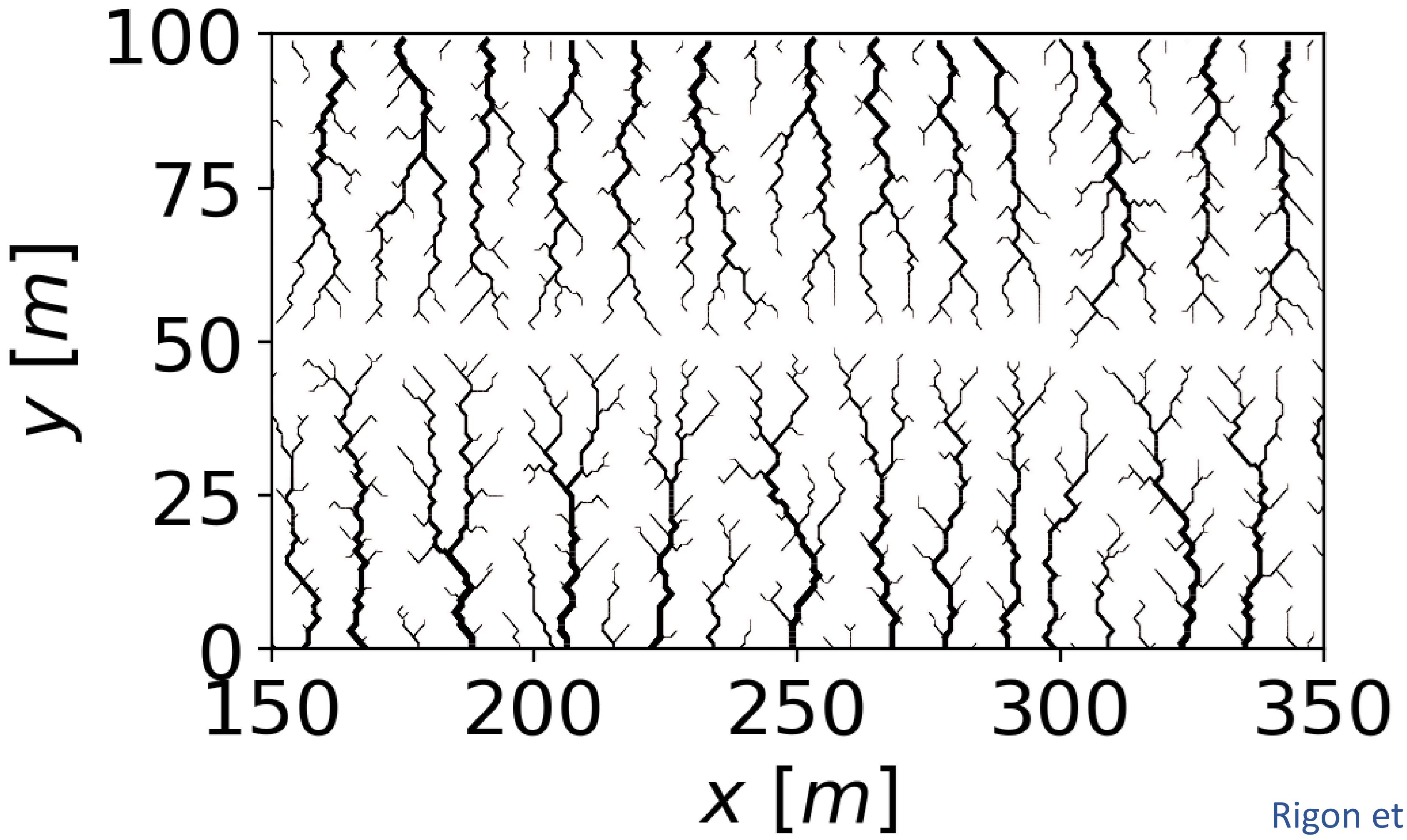




Analogy with turbulence as a guide to analyze landscape evolution

- Transition to channelization regime
- Scaling of mean elevation profiles (link to OCNs and variational analysis)
- Elevation fluctuations: Reynolds stresses and power spectra

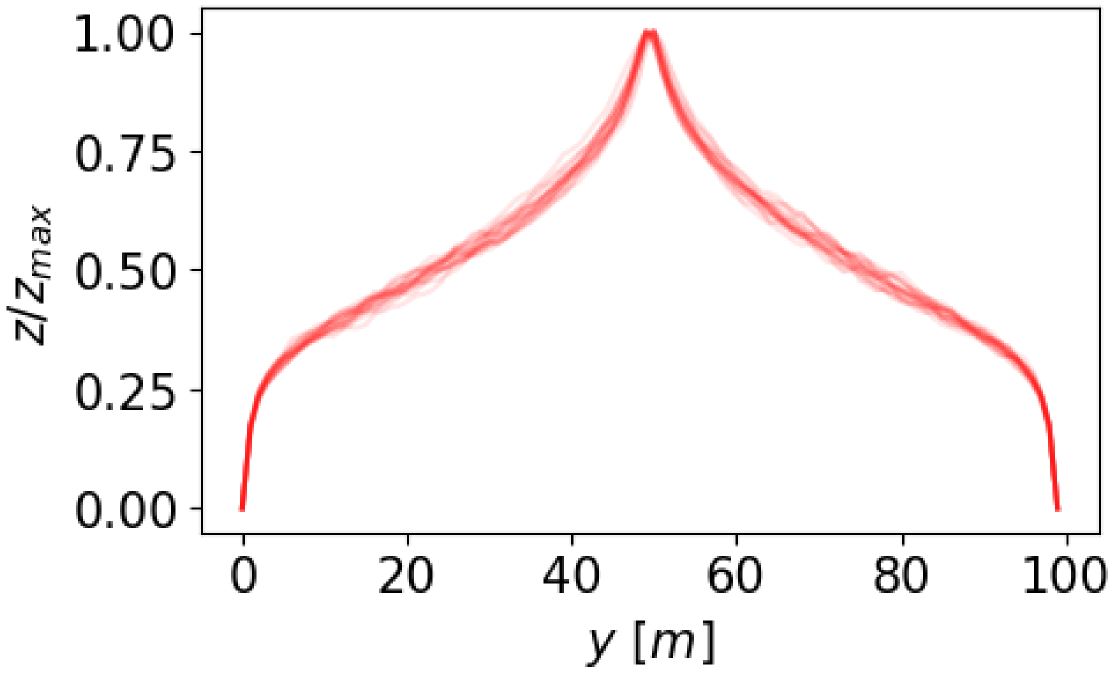
Optimal Channel Networks



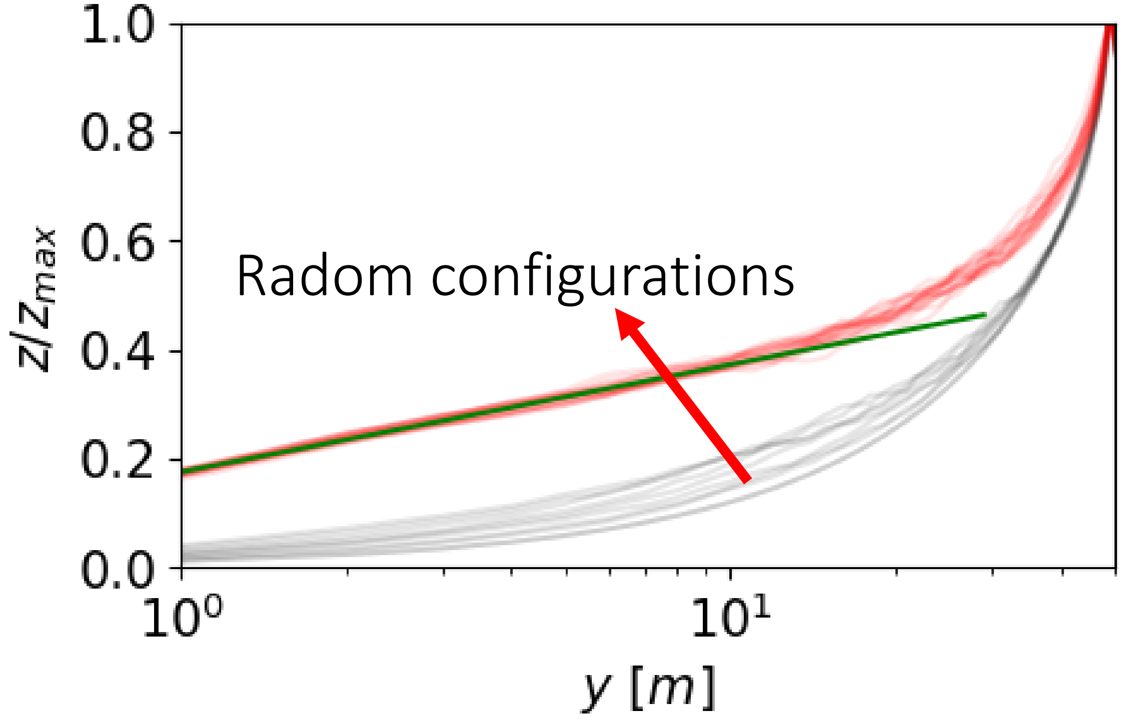
Rigon et al. WRR (1993)

Logarithmic profile in OCNs

Mean profile of optimal surfaces



Logarithmic profile of optimal surfaces



- Bonetti et. al “Channelization cascade”, PNAS, 2020.
- Hooshyar et. al “From turbulence to landscapes: the logarithmic profiles in complex bounded systems”, Under review in PRL, available on arXiv.
- Hooshyar et. al “Variational analysis of landscape elevation and drainage networks”, Under review in Proc. R. Soc. A, available on arXiv.