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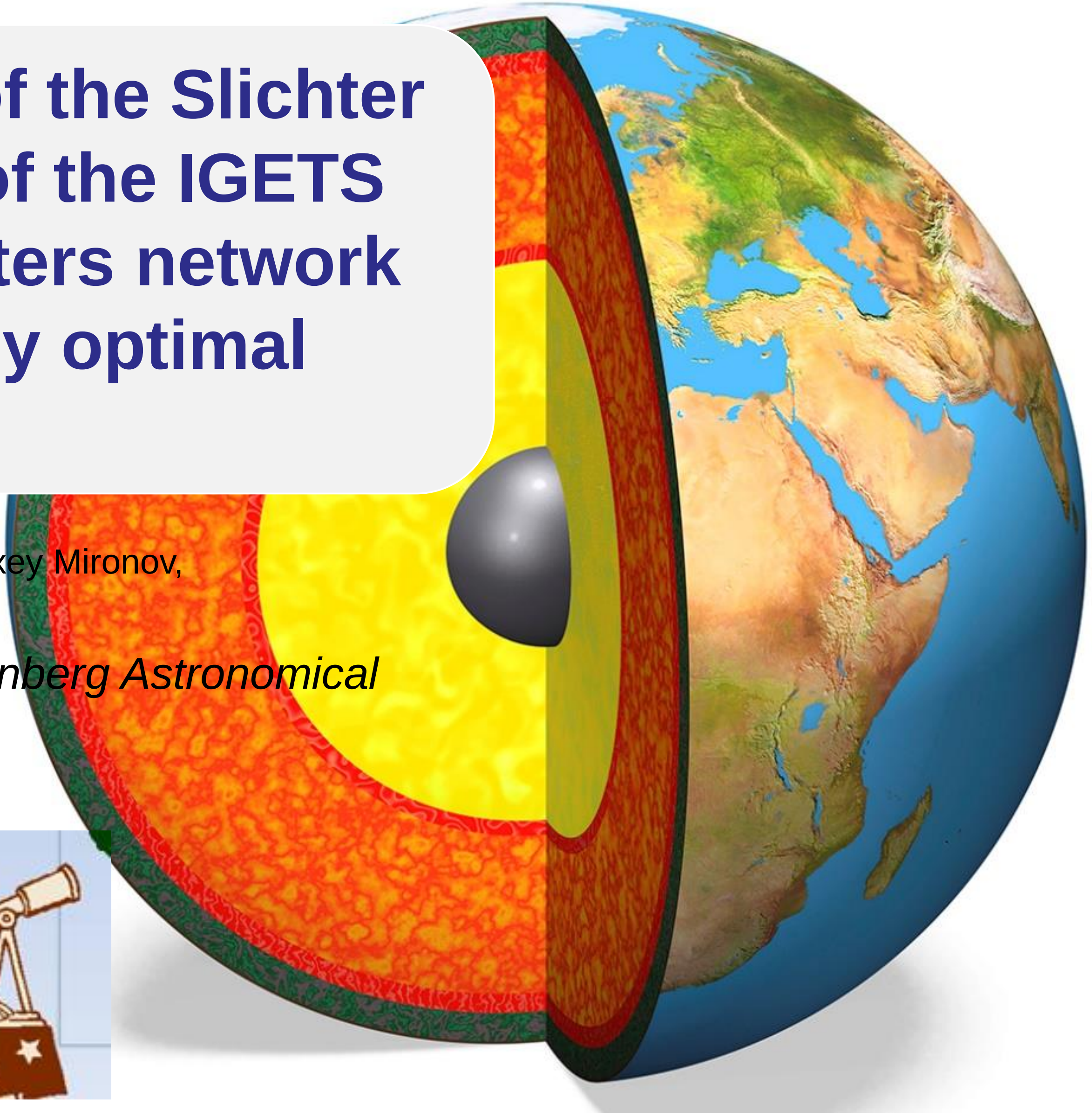
Detection and estimation of the Slichter mode based on the data of the IGETS superconducting gravimeters network using the asymptotically optimal algorithm



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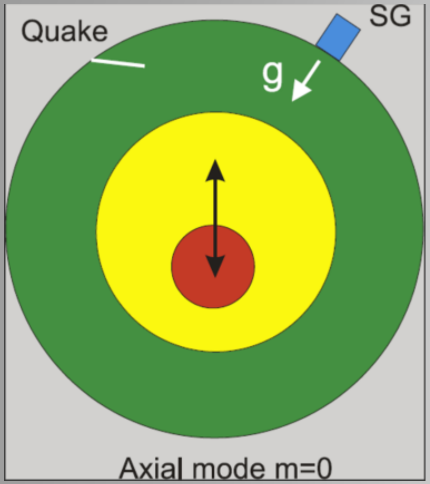
Introduction

Slichter mode, the long periodical oscillation of the Earth, 1S1, is caused by the translational oscillations of the solid inner core about its equilibrium position at the center of the Earth.

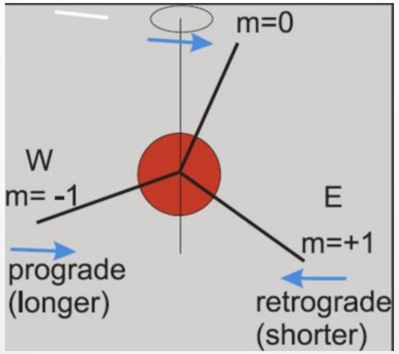


The preliminary estimation of its period was made by Louis Slichter in 1961. Up to now, the generally-accepted interpretation was that the frequency of the Slichter mode is principally controlled by the density jump between the inner (IC) and outer (OC) core, and the Archimedean force produced by the fluid outer core.

PREM:
ICB density jump $\Delta\rho = 0.6\ g/cm^3$
Periods (Crossley et al.)
4.767, 5.310, 5.979 hr
Q = 2000-5000

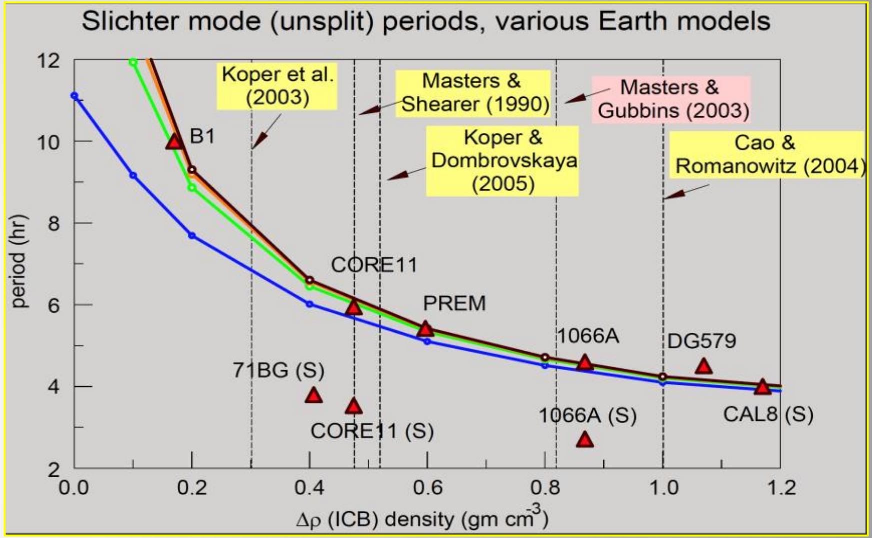
Splitting paramters [Dahlen&Sailor, 1978]:

$a = 15{,}306 \cdot 10^{-3}$;
$b = 98{,}380 \cdot 10^{-3}$;
$c = -0{,}554 \cdot 10^{-3}$;

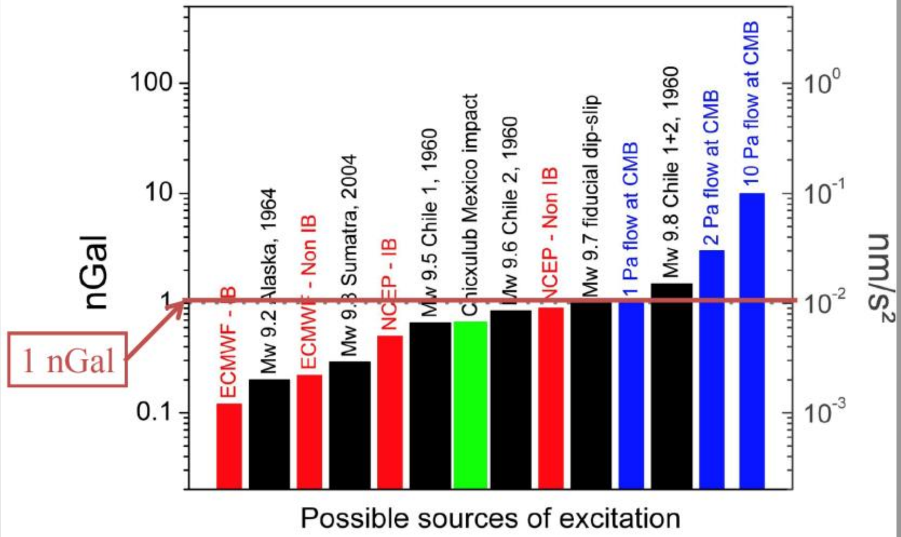


Search of Slichter mode is based on the SG data of the GGP network and for analysis is used different methods of data stacking from several stations. Up to now, there is no reliable knowledge about the experimental detection of the Slichter mode.

Slichter mode (unsplit) periods, various Earth models (Crossley, 2013)



Maximum surface amplitudes for the Slichter mode (Rosat, 2014)



Gravity Data

International Geodynamisc and Earth Tide service (IGETS) data base of superconducting gravimeters (SG) stations.

GFZ operates the IGETS data base of worldwide high precision SG records. We use so called "Level 3 products": Gravity data corrected for instrumental perturbations and after particular geophysical corrections (including solid Earth tides, polar motion, tidal and non-tidal loading effects), [Voigt et al, 2016].

For searching $_2S_1$ and $_1S_1$ modes we analyzed:

SG data from Sutherland station, South Africa (su037) after the earthquake in Peru (M = 8.4, June 23, 2001)

Detection algorithm

Search of Slichter mode is based on the SG data of the IGETS network and for analysis is used different methods of data stacking from several stations. Up to now, there is no reliable knowledge about the experimental detection of the Slichter mode.

The detection mode in the gravity records is a typical problem of detecting a weak signal against a noise background. If the noise is Gaussian, then the solution of this problem is matched filtering. However, real noise differs from Gaussian noise, especially after significant earthquakes that require a different approach. The authors proposed an asymptotically optimal algorithm for the simultaneous detection and estimation of Slichter mode parameters based on the maximum likelihood method [Vinogradov et al, 2019].

The essence of the method is to build so-called «Sufficient statistics». Sufficient statistics is a function of the observed random process which allows to find the optimal decision on the presence or absence of a signal. In the maximum likelihood method the sufficient statistics is the ratio of the probability densities of the random processes with and without a useful signal.

The noise properties for Non-Gauss process could be taken into account by the non-linear conversion of the original signal before the implementation of a matched filtering.

Detection algorithm

In the general case, we have four unknown parameters: the degenerate frequency f_d and three splitting parameters *a*, *b* and *c* . The splitting parameters determine the frequency offset of the individual singlets relative to the degenerate frequency:

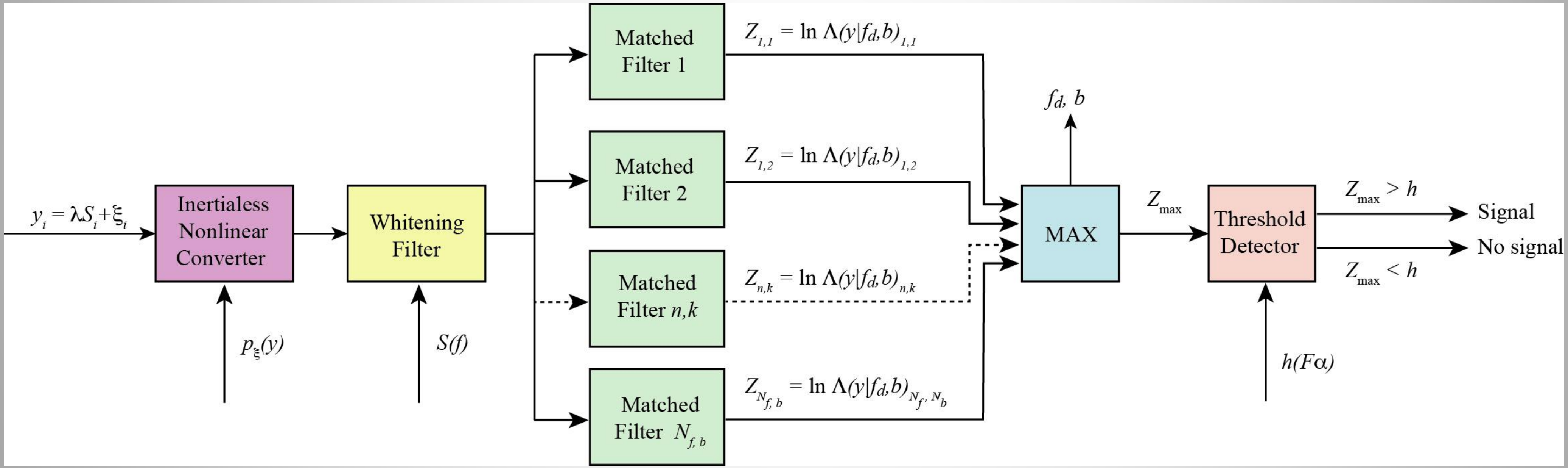
$$\delta f_{-1,0,1} = \begin{cases} f_d(1+a-b+c) \\ f_d(1+a) \\ f_d(1+a+b+c) \end{cases}.$$

Since the parameter *a* is included in all singlets with a constant sign, it affects only the constant correction to the degenerate frequency and does not affect the sufficient statistics, i. e. it can only shift the frequency estimate on the appropriate amount. The effect of the parameter *c* can be neglected in the first approximation, since its value is 30 times smaller than the parameter *a*, and almost 200 times smaller than the parameter *b*. Thus, the most crucial one is the parameter *b*, both because of the magnitude of its value and because it determines the distance between side singlets.

Thus, the problem is to determine the degenerate frequency f_d and the splitting parameter *b.*

Detection algorithm

The optimal receiver circuit is shown in the figure. The input signal is the mixture of noise and possibly a useful signal (Slichter mode). At the output of the receiver we have the so-called Sufficient statistics *Z* as a function of two unknown parameters f_d and *b*. If the value of *Z* exceeds the threshold *h*, then a decision is made on the presence of a signal in the source data. Values of f_d and *b* that correspond to the maximum of Z, are taken as estimates of these parameters.



The characteristic of a Inertialess nonlinear converter is determined by the noise probability densities.

Because a priori these probability densities are unknown, then the Neumann-Pearson criterion is used as a decision rule. In this case the threshold value *h* depends on the false alarm probability F_α.

Algorithm Advantages

1. Optimality in terms of maximum likelihood (Providing maximum SNR at the output).
2. The ability to evaluate efficiency of detection (false alarm probability).
3. Accounting for non-Gaussian noise, which is especially important after large earthquakes.
4. Simultaneous estimation of the frequency and splitting parameters.
5. Universatility of the algorithm, allowing the use for estimates of any multiplets, as well as data of any instruments (gravimeters, strainmeters, seismometers etc).
6. Representation of the useful signal through a degenerate frequency and the splitting parameter *b* is significantly reduce the amount of computation processing (fewer filtering channels).
7. The ability for effective detecting of weak signal based on the data of one device/station (without stacking procedure).

Computer Modeling

Computer simulation was carried out to study the features of the algorithm. The synthetic useful signal simulating the Slichter mode (${}_1S_1$) was three cosine waves with parameters corresponding to the PREM model:

T_d = 5,42 h

$a = 15.306 \cdot 10^{-3}$;

$b = 98.380 \cdot 10^{-3}$;

$c = -0.554 \cdot 10^{-3}$.

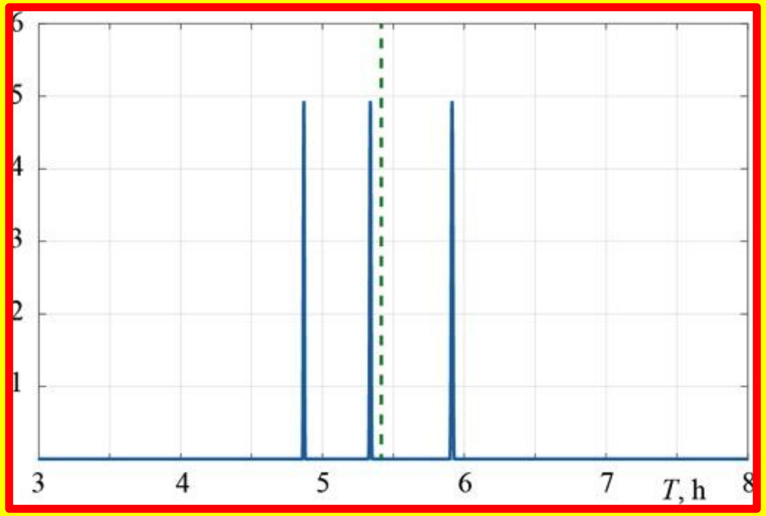
Sampling time = 30 min

N data points = 10 000

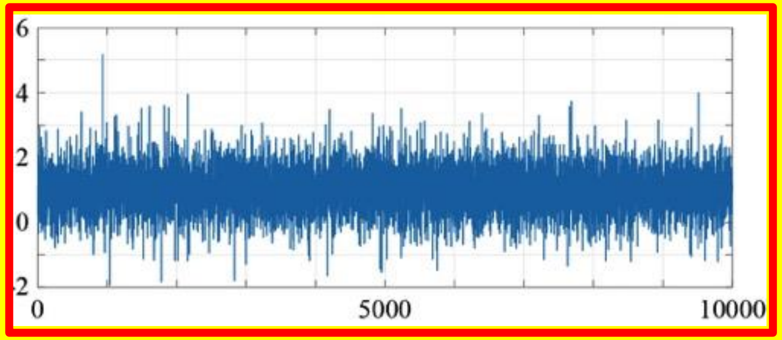
Total durability = 208 days

For modeling the noise, the t-Location Scale distribution was used, while the distribution parameters were determined by the real noise from the gravimeter records.

The amplitude of the useful signal («Slichter mode») was varied to obtain different signal-to-noise ratio (SNR) values; sufficient statistics were calculated for each SNR value

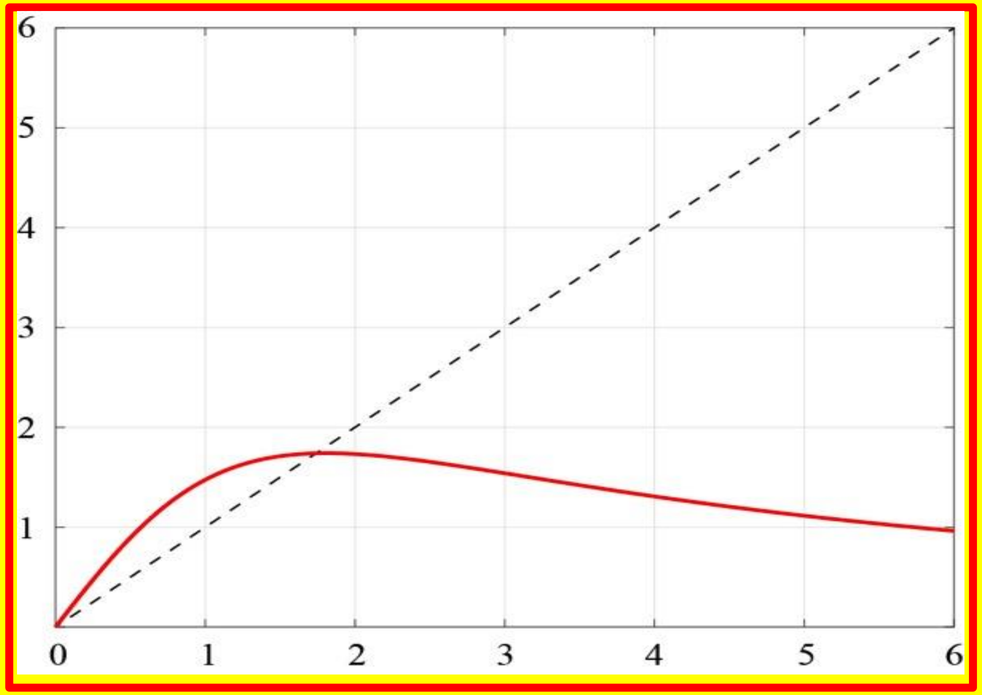


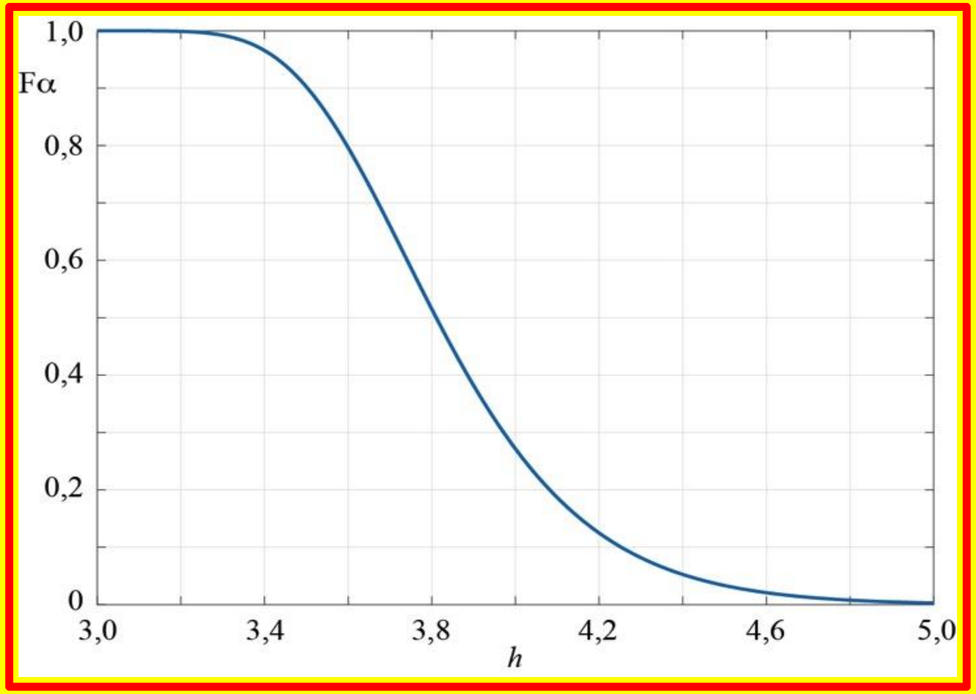
Spectrum of simulated Slichter mode. Degenerated period marked by doted line



Synthetic noise with t-location scale distribution

Computer Modeling

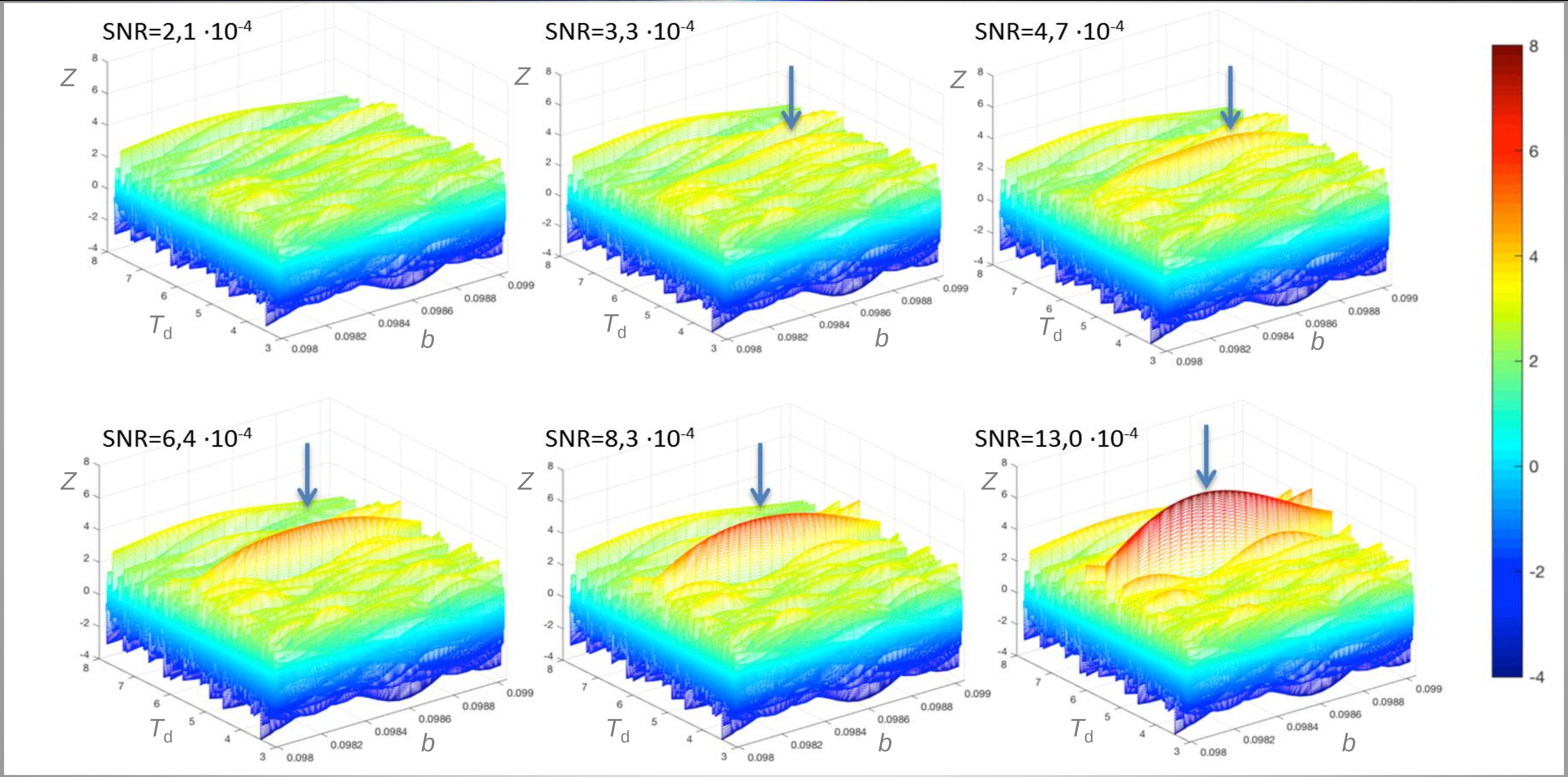




Inertialess nonlinear converter for gravity data:
the suppression of large amplitudes associated with noise is clearly visible

ROC curve:
dependence of the threshold detection and probability of false alarm

Computer Modeling



Sufficient statistics $Z(T_d, b)$ for different SNR

Computer Modeling: Results

SNR	$2.08 \cdot 10^{-4}$	$3.25 \cdot 10^{-4}$	$4.68 \cdot 10^{-4}$	$6.37 \cdot 10^{-4}$	$8.33 \cdot 10^{-4}$	$13.0 \cdot 10^{-4}$
Z max	3.48	3.79	4.32	5.05	5.84	7.37
$F\alpha$	0.92	0.53	0.075	0.002	$< 1 \cdot 10^{-4}$	$< 1 \cdot 10^{-4}$
T_d, hours	6,208	5,419	5,419	5,420	5,420	5,420
b	0,0988	0,0987	0,0986	0,0984	0,0984	0,0984
Decision about Signal presence	No	Yes, but it is difficult to distinguish	Yes	Yes	Yes	Yes

The results show a reliable determination of the presence of the signal and the correct parameter estimation for SNR = 4 x10^{-4} and higher.

For SNR ~3 x10^{-4} we can talk about the possible presence of a signal, but its parameters can be estimated incorrectly. For lower SNR values the signal is not detected against the background noise.

$_2S_1$ mode

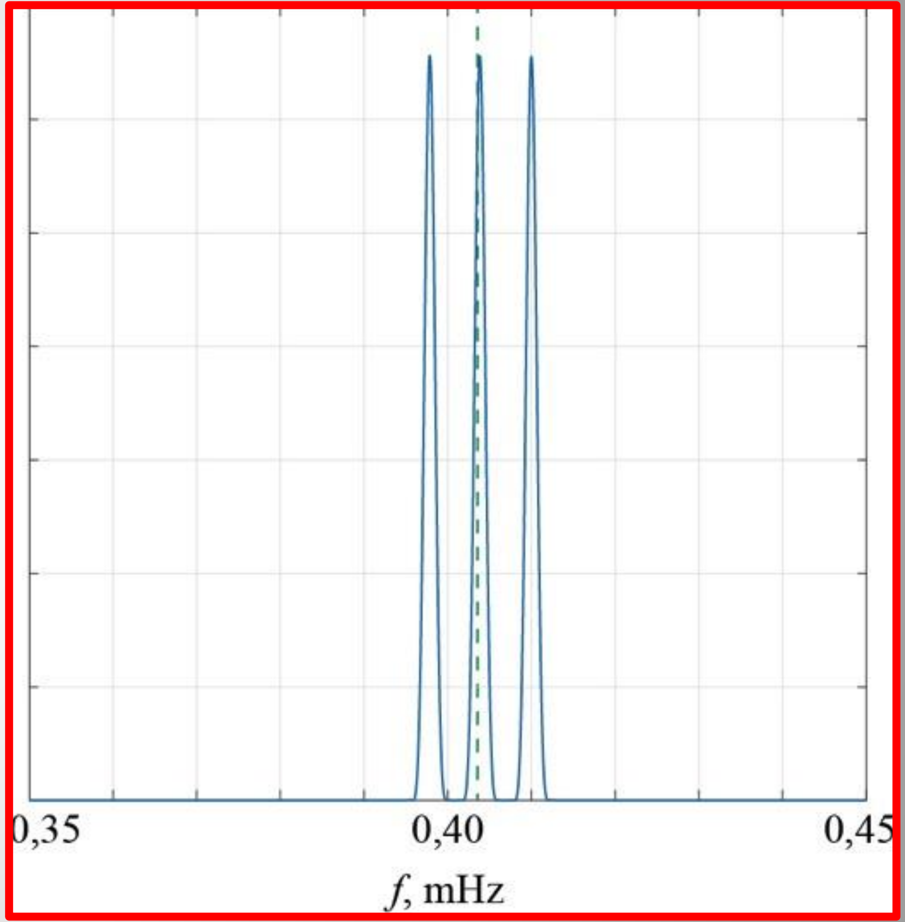
$_2S_1$ mode detection as a real test for the Slichter mode search algorithm

The $_2S_1$ mode is the first overtone of the Slichter mode. It corresponds to oscillation of the whole Earth's core. Like the Slichter mode, it should be observed as a triplet.

Theoretical calculations using the formulas given in [Dahlen and Tromp, 1998] show that the amplitude of the $_2S_1$ mode after earthquakes can be approximately 15 times larger than the Slichter mode one, which makes it easier to detect on gravimeters.

The first observation of $_2S_1$ mode was reported in [Rosat et al, 2003].

We chose exactly the same earthquake to search for $_2S_1$ and $_1S_1$ modes for comparing and demonstrating the features of the algorithm



Spectrum of model $_2S_1$ mode. Degenerated period marked by doted line. This model used as useful signal for matched filter.

Data Analysis for $_2S_1$

Search for $_2S_1$ mode after the earthquake in Peru (M = 8.4, June 23, 2001)

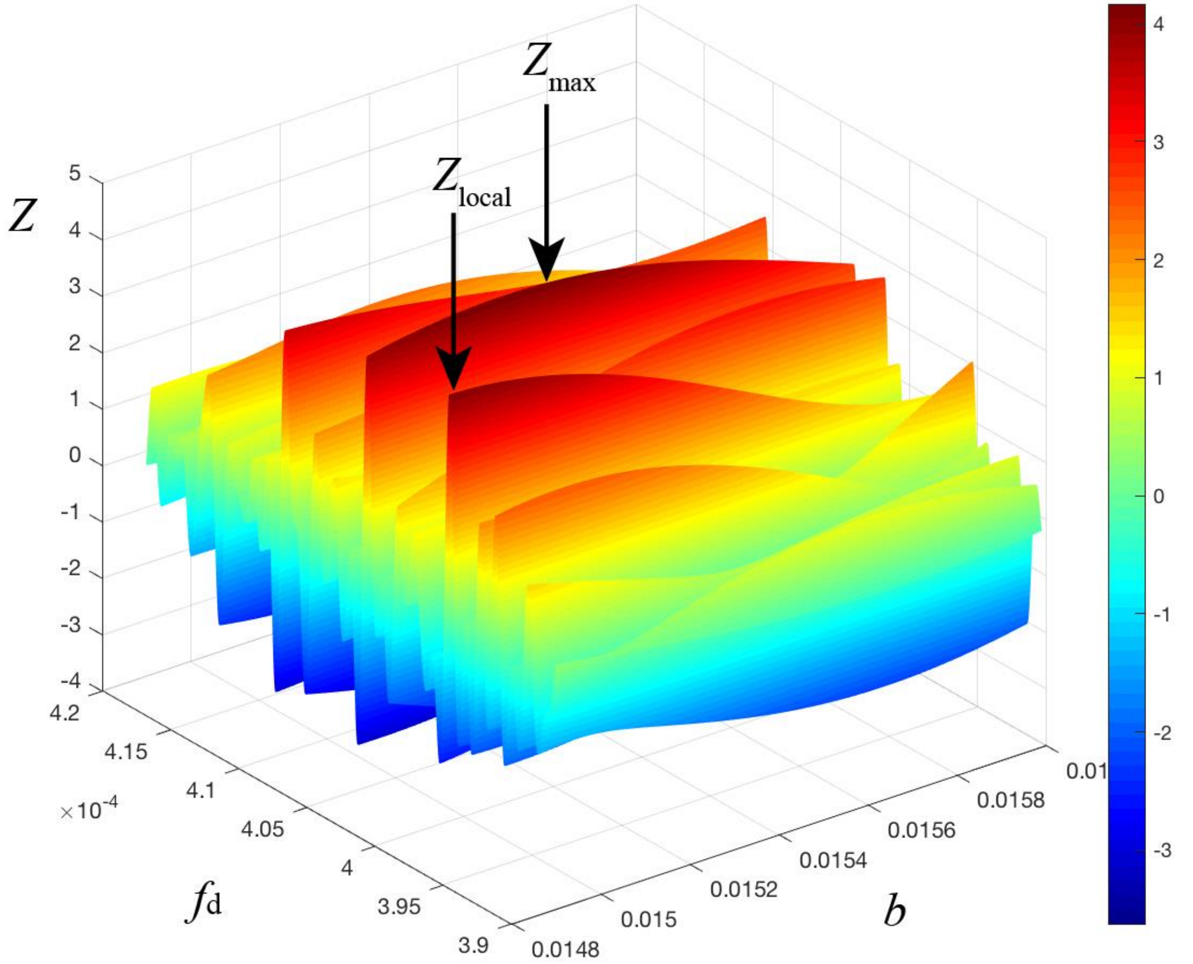
Theoretical data:
PREM degenerate frequency
f_d = 0,403881 mHz

Splitting parameters [Dahlen and Sailor, 1979]:
a = 2,094 $\cdot 10^{-3}$
b = 15,074 $\cdot 10^{-3}$
c = - 0,190 $\cdot 10^{-3}$

Original data:
Sutherland SG Station, su037-1, Level 3 data
N=16384 data points after Peru Earthquake
Sampling time = 1 min
Total durability = 11,3 days

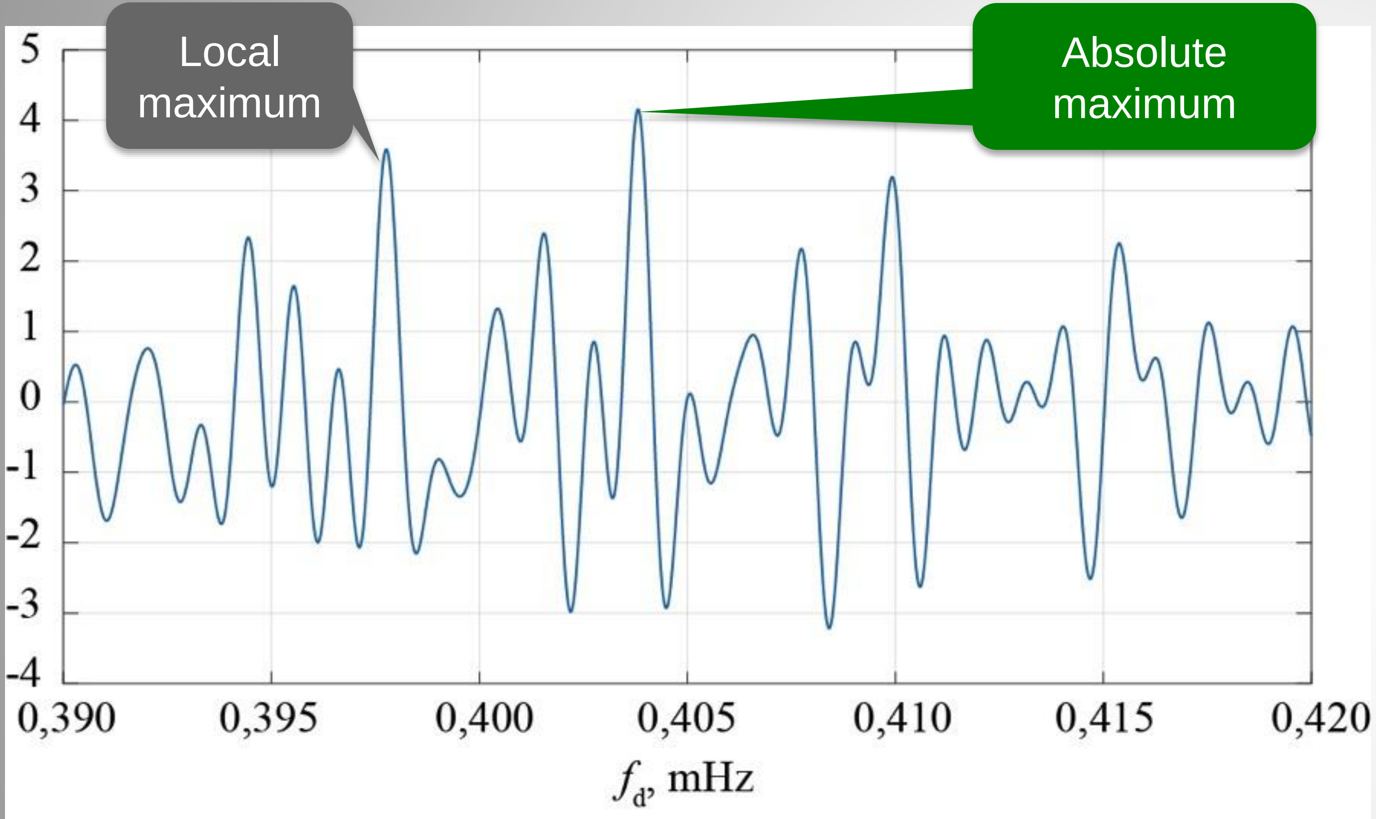
Data preprocessing:
Low pass filtering

Noise parameters (t-Location Scale distribution):
m = 0.0534
s = 0.2238
n = 1.8993

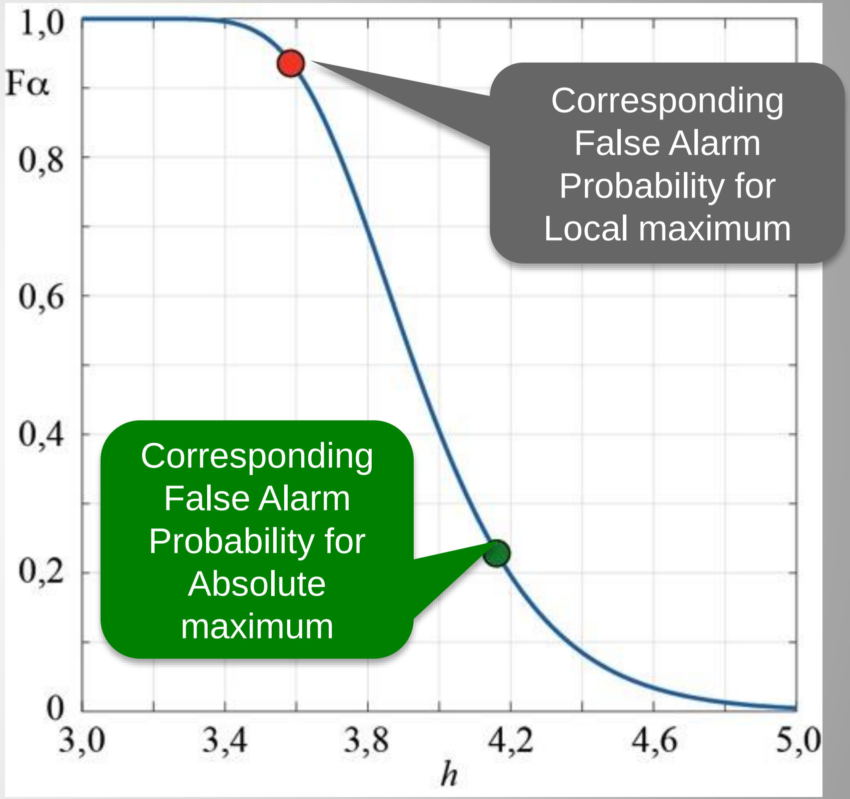


Data Analysis for $_2S_1$

Search for $_2S_1$ mode after the earthquake in Peru (M = 8.4, June 23, 2001)



Z as function f_d for maximizing b = 0,01521



ROC curve (F_a as function of threshold h)

The absolute maximum of sufficient statistics Zmax = 4.156 is achieved at a frequency f_d=0,4038. The corresponding probability of false alarm Fa = 0,23.

Decision: $_2S_1$ is detected. Mode parameters estimation: f_d = 0,40381; b = 0,01521

Data Analysis for $_2S_1$

Search for $_2S_1$ mode after the earthquake in Peru (M = 8.4, June 23, 2001)

Parameter	Model PREM	Model 1066A	Rosat et all, 2003	This work
f_1, mHz	0,398750	0,398708	0,398600	0,398590
f_0, mHz	0,404727	0,404690	0,404900	0,404656
f_{-1}, mHz	0,410948	0,410880	0,411100	0,410874
b	0,015069	0,015039	0,015436	0,015210
f_d, mHz	0,403881	0,403844	0,404054	0,403810
T_d, minute	41,266	41,270	41,249	41,274

Comparison with theoretical and previous experimental results

Data Analysis for ${}_1S_1$

Search for ${}_1S_1$ mode after the earthquake in Japan (M = 8.4, June 23, 2001)

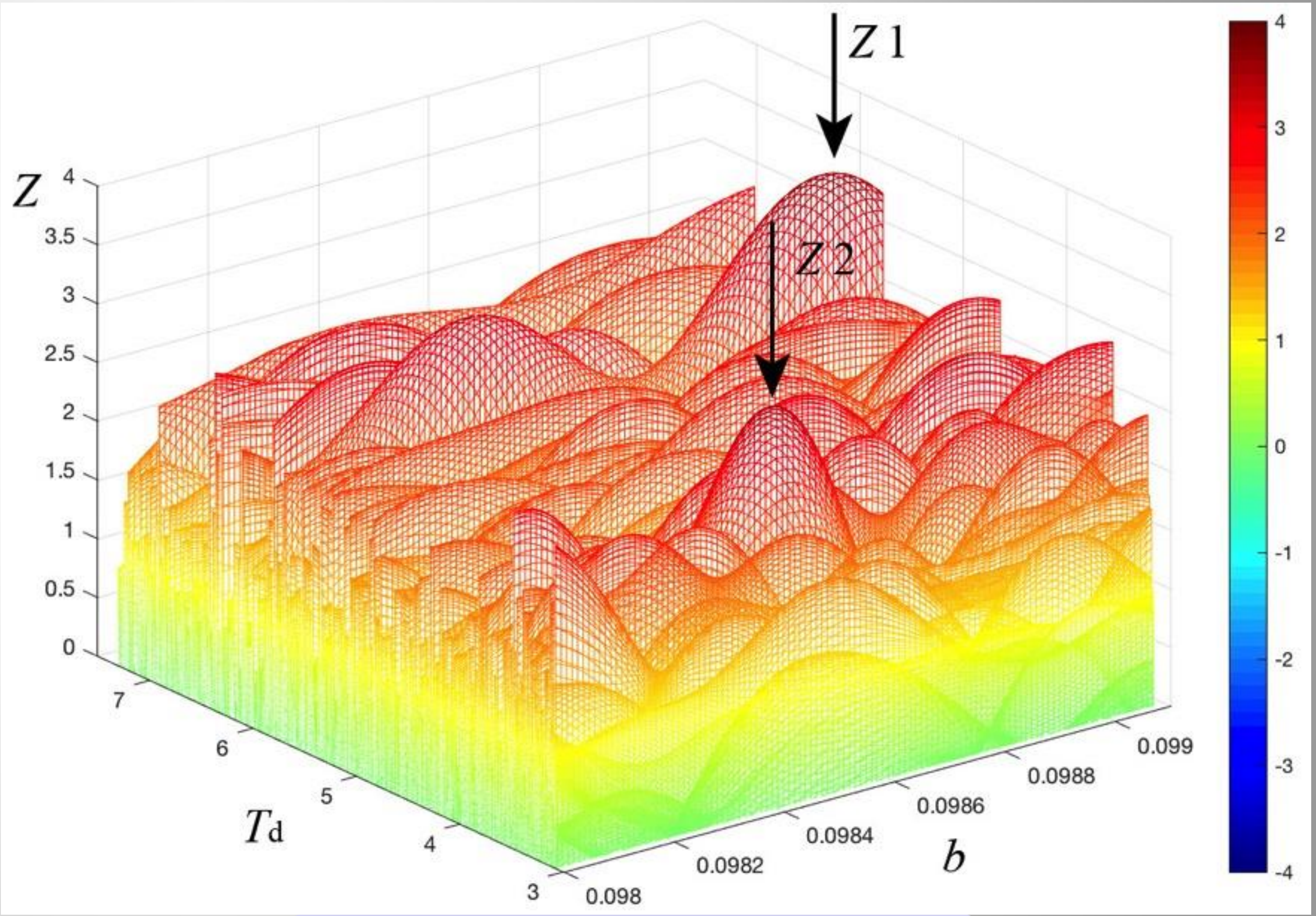
Theoretical data:
PREM degenerate frequency
T_d = 3.5…7.0 hour (depends from model and density jump between inner and outer core)

Splitting parameters [Dahlen and Sailor, 1979]:
a = 15,306 $\cdot 10^{-3}$;
b = 98,380 $\cdot 10^{-3}$;
c = - 0,554 $\cdot 10^{-3}$;

Original data:
Sutherland SG Station, su037-1, Level 3 data
N = 10 000 data points after Peru Earthquake
Sampling time = 30 min
Total durability = 208 days

Data preprocessing:
Band pass filtering

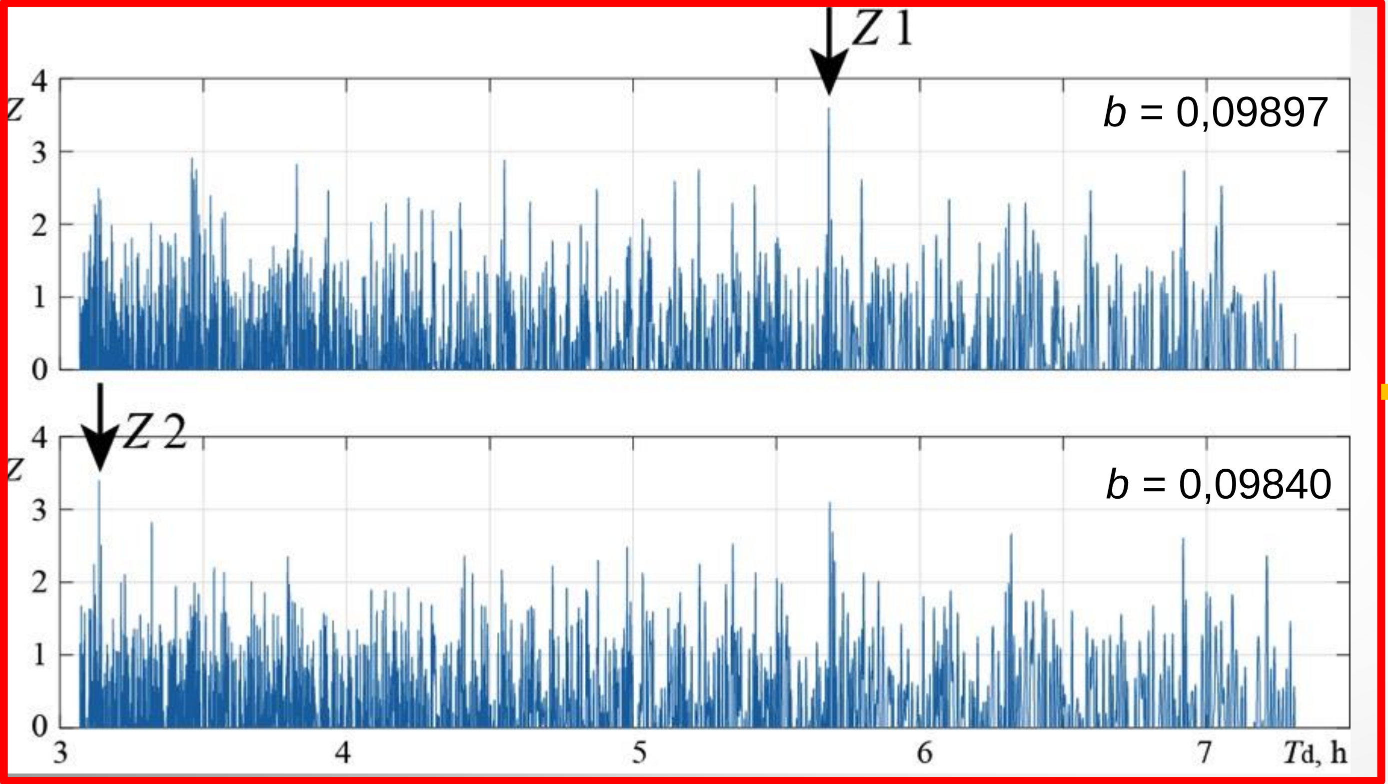
Noise parameters (t-Location Scale distribution):
m = 0
s = 0.5714
n = 10.026



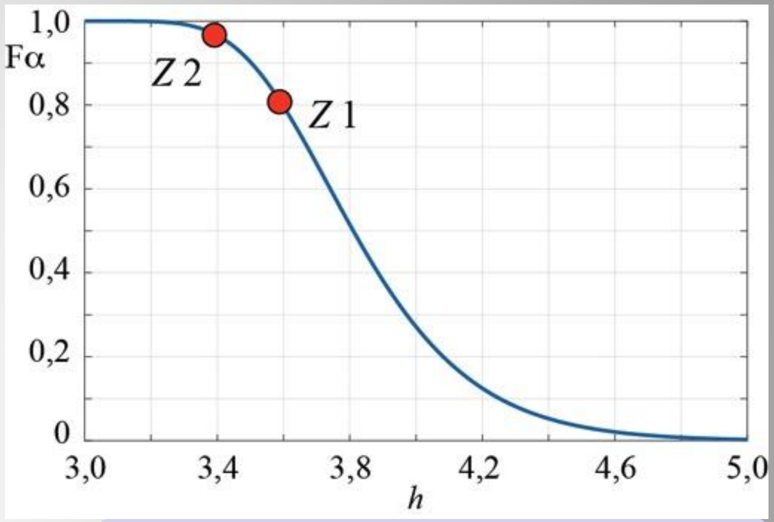
Normalized sufficient statistics $Z(f_d, b)$

Data Analysis for $_1S_1$

Search for $_1S_1$ mode after the earthquake in Peru (M = 8.4, June 23, 2001)



Z as function f_d for maximizing *b*



ROC curve (F_a as function of level *h*)

	Z1	Z2
Amplitude	3.59	3.39
Fα	0.81	0.97
T_d, hours	5,682	3,137
b	0,09897	0,09840
Decision about signal presence	No	No

Decision: $_1S_1$ is not detected.

Conclusion

1. The optimal algorithm for detecting the Slichter mode in the presence of non-Gaussian noises and estimating mode parameters is proposed.
2. The presence of the ${}_2S_1$ mode in the gravimetric SG-data recorded at the Sutherland station after the earthquake in Peru in 2001 was confirmed.
3. The degenerate frequency and splitting parameter of the ${}_2S_1$ mode are determined, the frequencies of the mode triplet are calculated based on the data of one instrument. The results are close to the theoretical values and experimental values by stacking on 5 gravimeters [Rosat, 2003].
4. ${}_1S_1$ mode (the Slichter mode) was not detected
   (SNR $<4\mathrm{x}10^{-4}$; false alarm probability for presence of Slichter mode > 0.8).

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