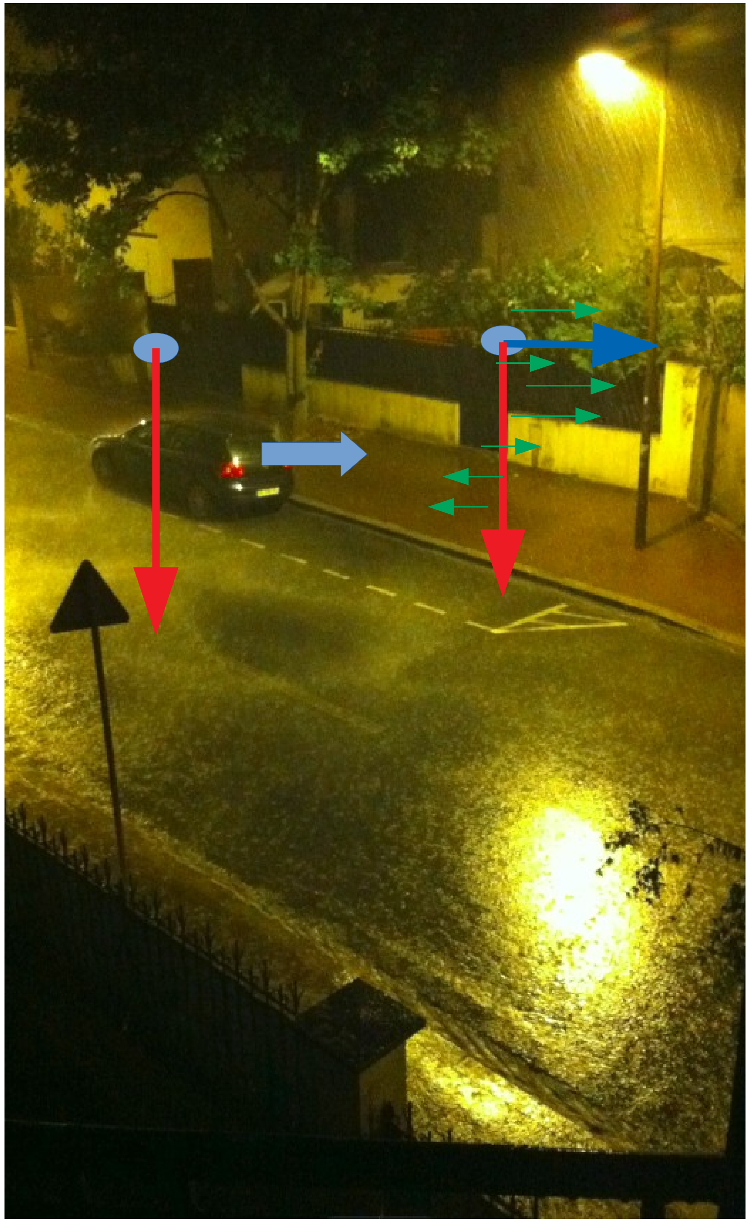
Temporal evolution of rain drops' velocities in a turbulent wind field

Auguste Gires[1], Ioulia Tchiguirinskaia[1], Daniel Schertzer[1]
(auguste.gires@enpc.fr)

(1) HM&Co, École des Ponts ParisTech, France

Introduction



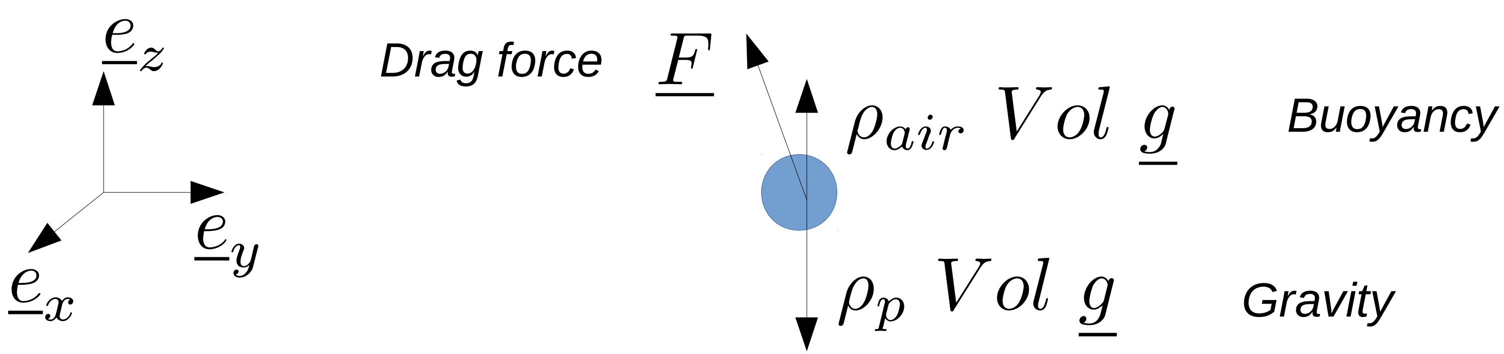
It is commonly assumed that a rain drop falls vertically at a speed equal to its so called "terminal fall velocity" which has been determined both empirically and theoretically by equating the net gravity force with the drag force due to the fact the drop is moving in the atmosphere. This velocity depends on the size of the drop, usually characterized by its equivolumic diameter.

In this investigation we study the temporal evolution of the velocity of a rain drop falling through multifractal turbulent wind field varying in space and time.

Methodology

The drop's governing equation

The forces at stake on the particle p (a spherical drop of water of diameter D) :



$$\underline{F} = \frac{1}{2}\frac{\pi D^2}{4} c_D \rho_{air} v_{rel} \underline{v}_{rel}$$

With :

$$\underline{v}_{rel} = \underline{v}_{wind} - \underline{v}_p$$

Relative velocity between the wind and the falling particle

$$c_D = 0.25 + \frac{24}{Re} + \frac{6}{1+\sqrt{Re}}$$

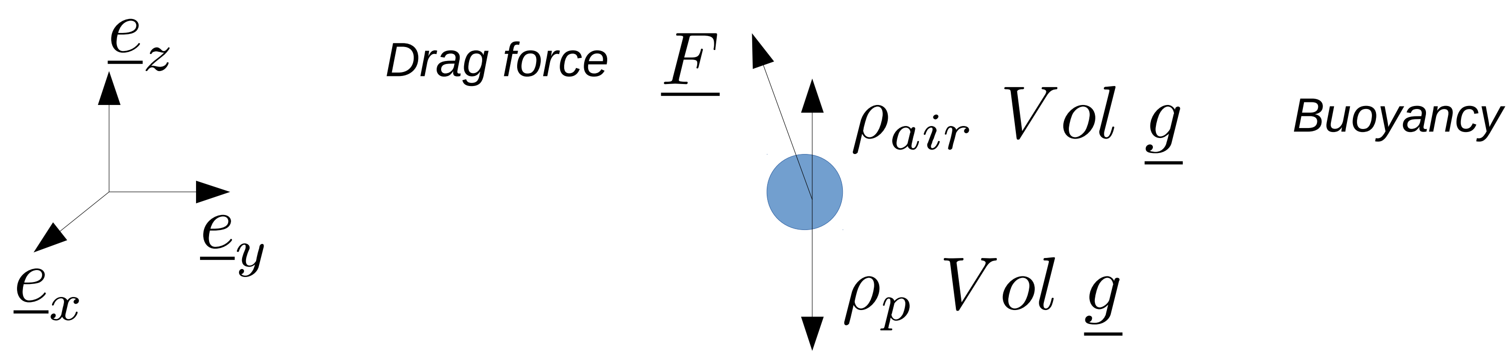
Drag coefficient (White 1974)

$$Re = \frac{\rho_{air} v_{rel} D}{\mu_{air}}$$

Reynolds number where μ_{air} is the absolute viscosity of air

Methodology

The drop's governing equation



$$\frac{d\underline{v}_p}{dt} = \frac{3}{4D} c_D \rho_{air} v_{rel} \underline{v}_{rel} + \underline{g}\frac{\rho_p - \rho_p}{\rho_{air}}$$

With : $\underline{v}_{rel} = \underline{v}_{wind} - \underline{v}_p$ — Relative velocity between the wind and the falling particle

$$c_D = 0.25 + \frac{24}{Re} + \frac{6}{1+\sqrt{Re}}$$

Drag coefficient (White 1974)

$$Re = \frac{\rho_{air} v_{rel} D}{\mu_{air}}$$

Reynolds number where μ_{air} is the absolute viscosity of air

Numerical solving through an explicit scheme with $\Delta t = 0.001\ s$

Methodology

Universal Multifractals

A physically based theoretical framework enabling characterization and simulation of geophysical fields exhibiting extreme variability over wide range of scales :

$$\langle \epsilon_\lambda^q \rangle \approx \lambda^{K(q)}$$

Scaling moment function

Conservative field

Resolution = $\lambda = \frac{L}{l}$

$$K(q) = \frac{C_1}{\alpha - 1}(q^\alpha - q)$$

$$\phi_\lambda = \epsilon_\lambda \lambda^{-H}$$

Non-conservative field

Conservative field ($K_c(q)$)

$$K(q) = K_c(q) - Hq$$

$$\beta = 1 + 2H - K_c(2)$$

$$E(k) \approx k^{-\beta}$$

Spectral slope

Power spectra

Wave number

Three exponents

- H: the degree of non-conservation (H=0 for a conservative field)
- C_1: the mean intermittency (how concentrated is the average field, C_1=0 for homogeneous field)
- α : the multifractality index (how fast the intermittency evolves when you slightly go away from the average field)

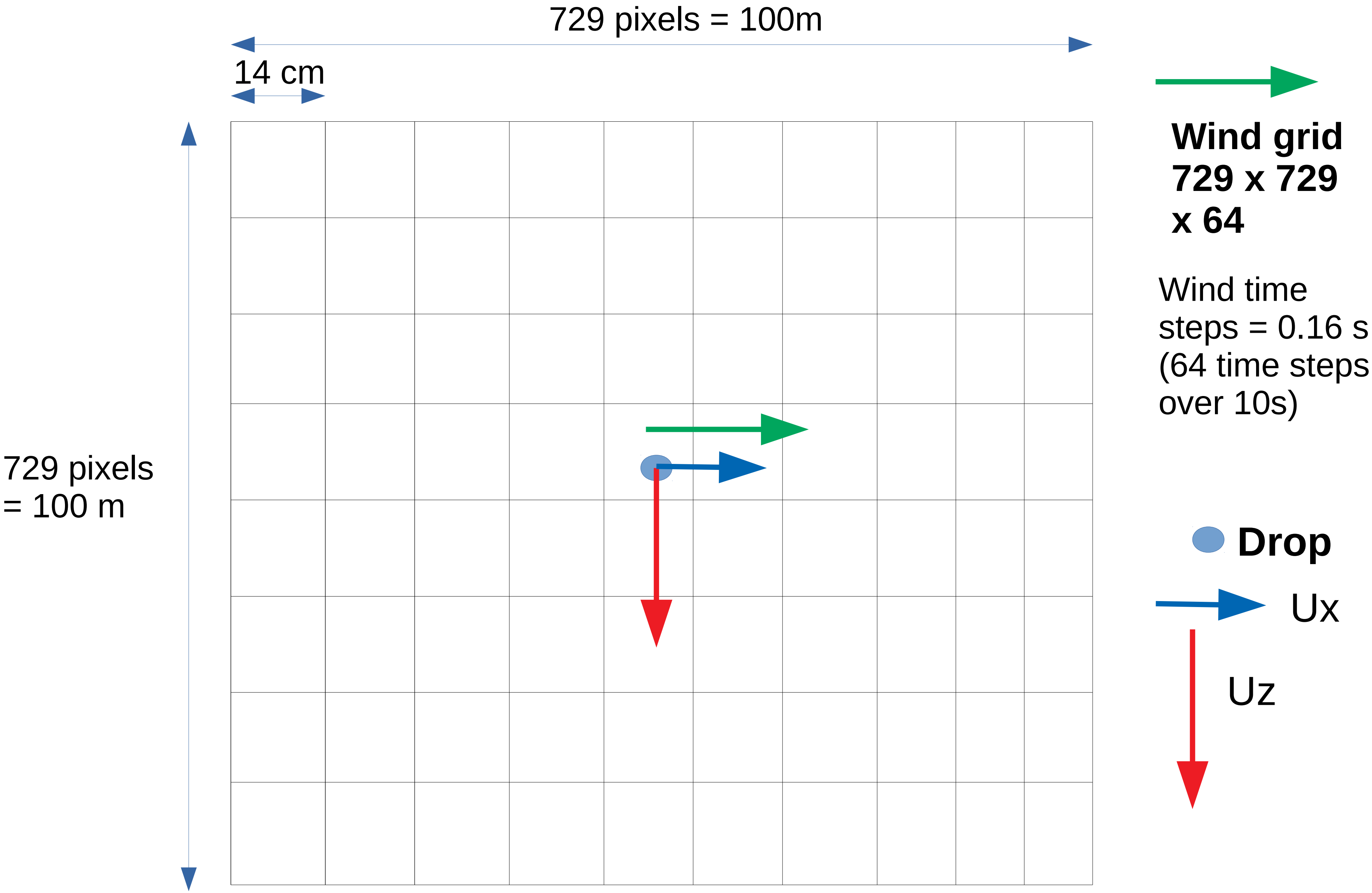
With straightforward consequence on the extremes

- Large α and C_1 → strong extremes
- Little α and C_1 → low extremes

Schertzer and Lovejoy (1987)

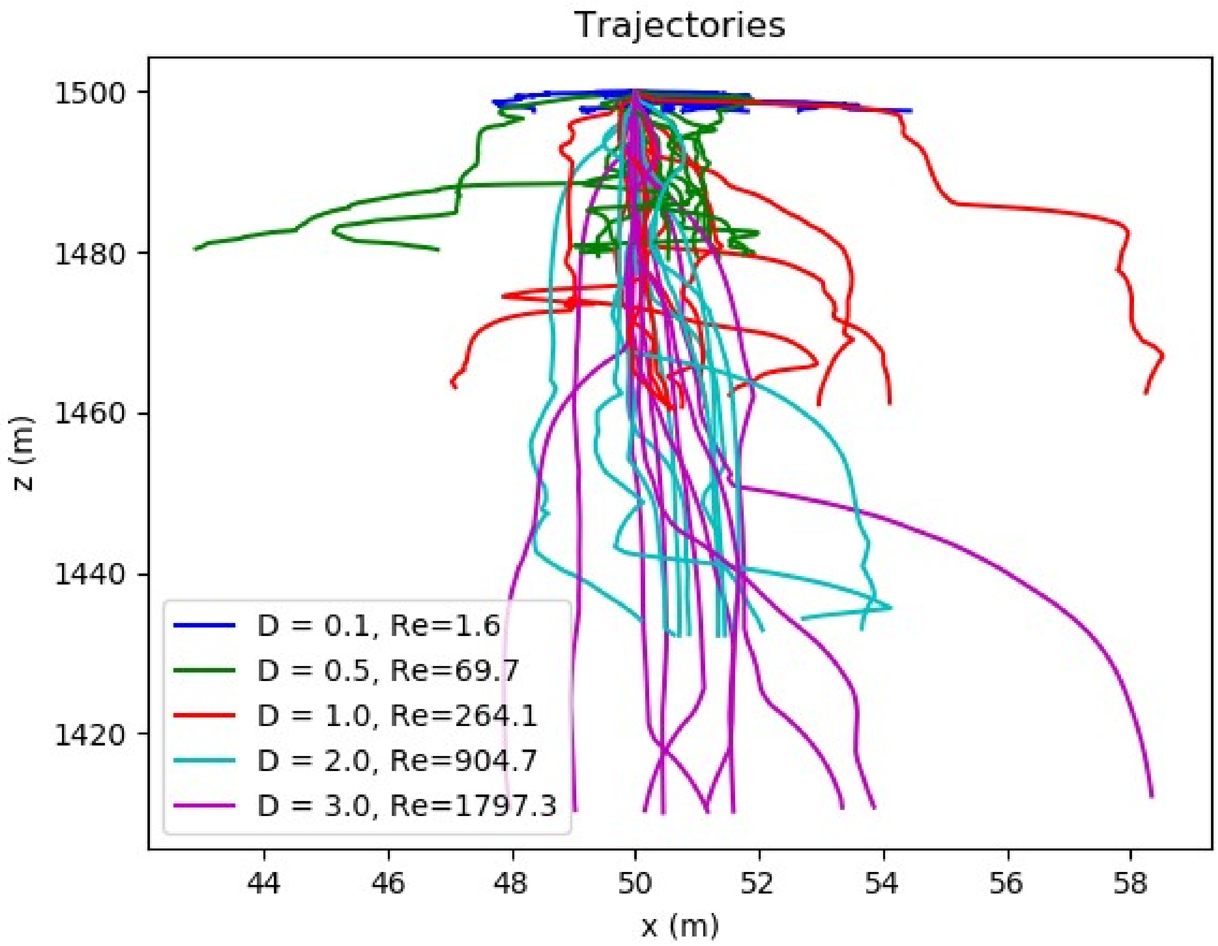
Methodology

A wind grid simulated with space-time discrete UM cascades



Results

Trajectories for various wind samples



- Trajectories of drops of various diameter for 5 different wind fields (α=1.7; C_1=1.2)

- All drops starting at z = 1500 *m*; evolution over 10 s studied

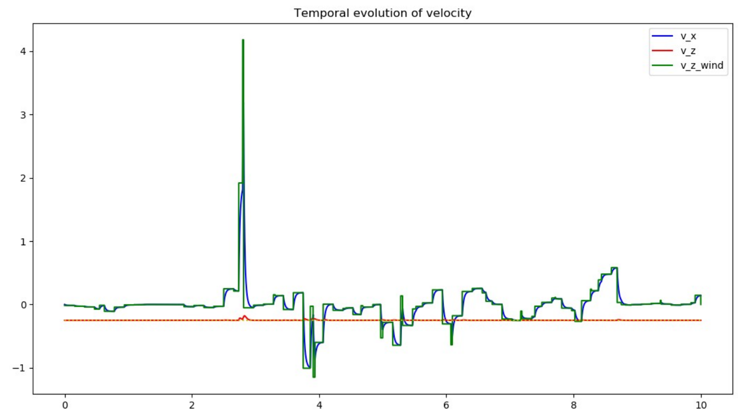
- Obviously, larger drops fall quickly

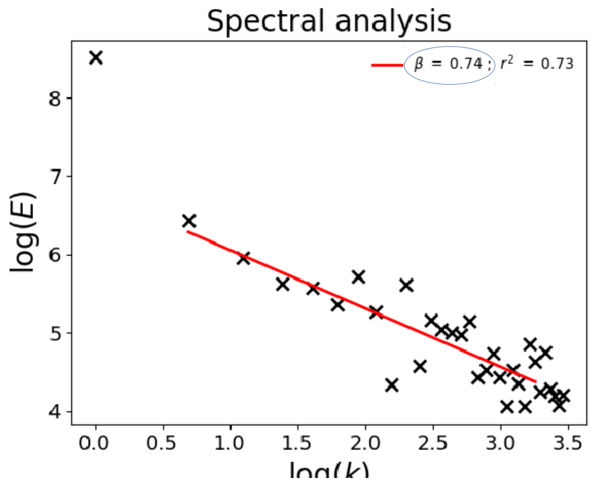
- Some horizontal dispersion is noticed, slightly more pronounced for smaller drops

Results

Scaling properties of horizontal drop velocity

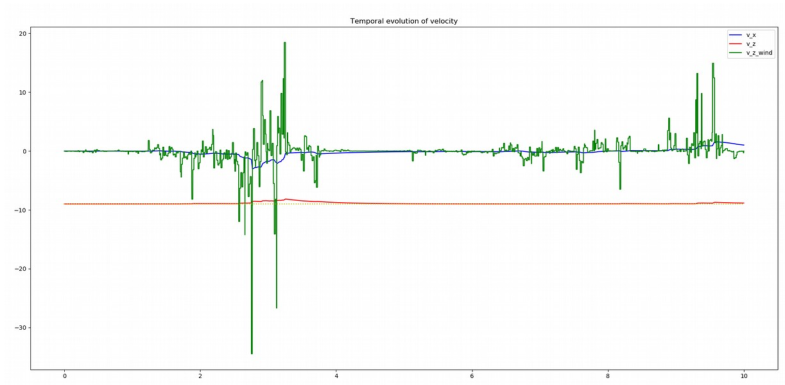
For D = 0.1 mm

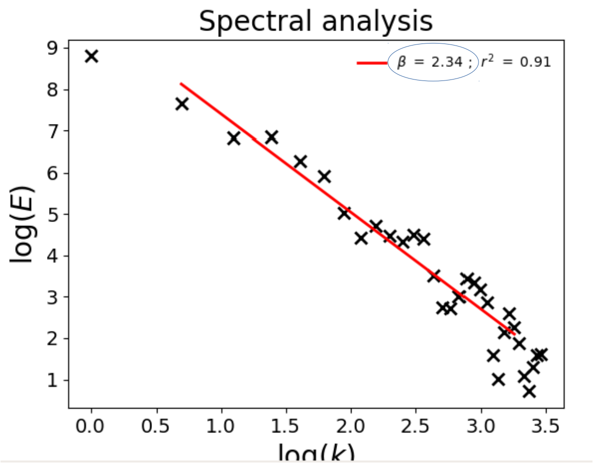




- v_x closely following wind fluctuations
- v_x exhibiting scaling behaviour
- Some minor impact on vertical fall velocity

For D = 3 mm





- v_x following only large scale wind fluctuations
- v_x exhibiting scaling behaviour with β much greater
- Some minor impact on vertical fall velocity

Preliminary conclusions :

- Horizontal drop velocity is basically a fractional integration of the horizontal wind input (some UM parameters, increasing *H*)
- *H* increases with the drop diameter
- Some significant dispersion of drops in their fall simply due to turbulent wind fluctuations

Future work and perspectives :

- Include a vertical component of the wind accounting for correlation with the horizontal wind and anisotropy with continuous UM
- Test various possible wind UM parameters
- Simulate a larger area to investigate the dispersion of drops between 1500 m and ground, for applications to rainfall radar measurements