Geostatistical representation of multiscale heterogeneity of porous media through a Generalized Sub-Gaussian model

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EGU General Assembly 2020

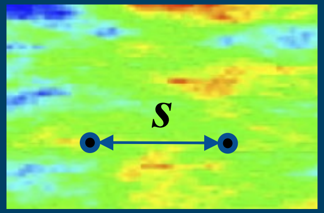
Sharing Geoscience Online

Key Observations

Many earth, environmental, physical, bio / ecological, financial, & other variables show

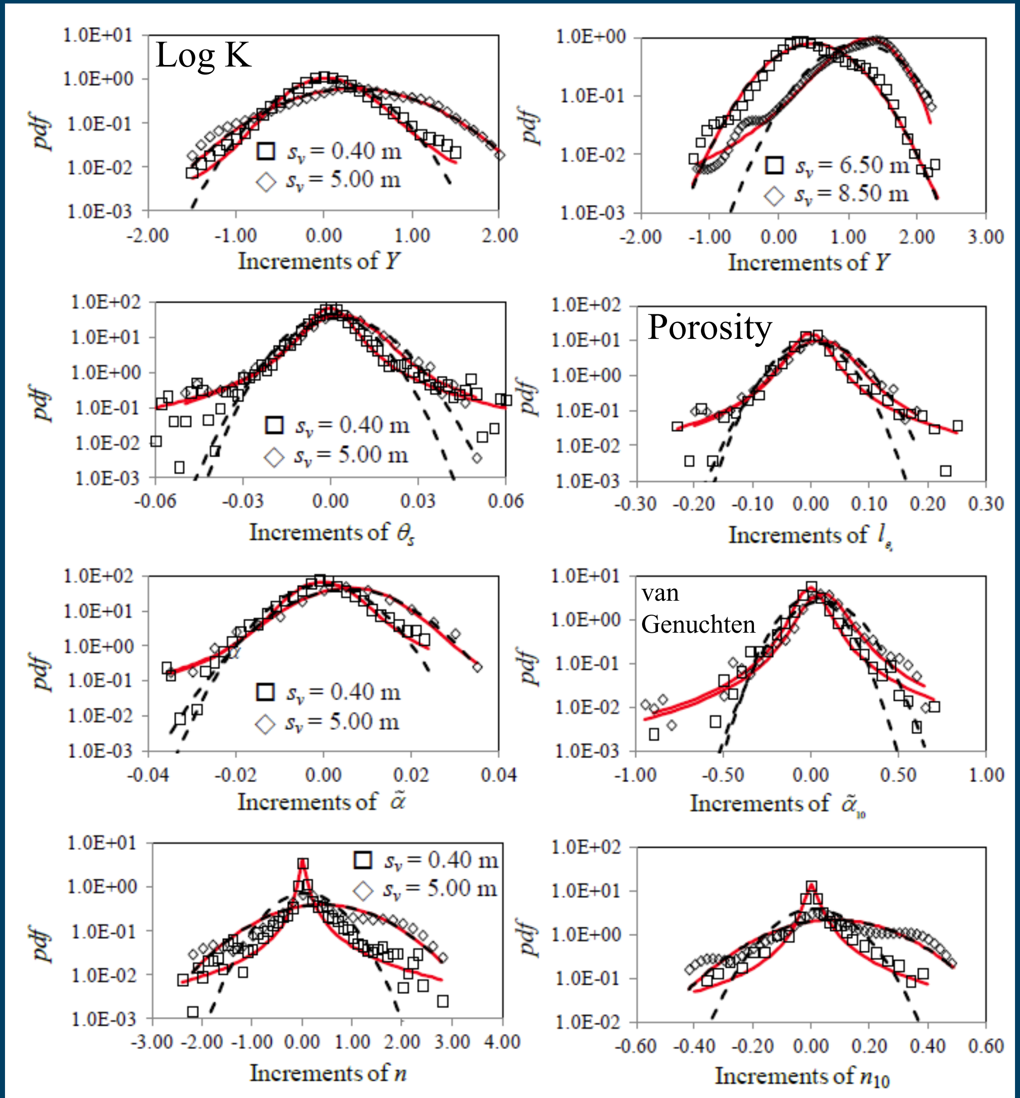
1. **Power-law scaling in midrange of separation scales (lags).**
   Breakdown in power-law scaling at small and large lags.
   Power-Law scaling extended to virtually all lags (ESS).

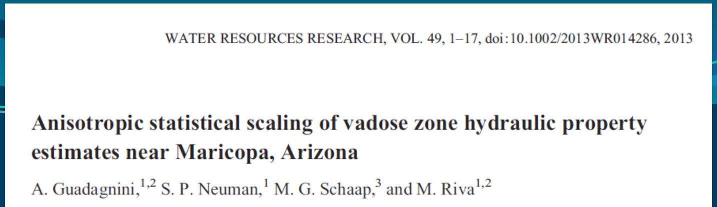
2. **Heavy-tailed frequency distributions of increments**
   A variable Y can be Gaussian or non-Gaussian
   Pdfs of spatial increments ΔY scale with lag
   Sharp peaks and heavy tails of pdf decaying with lag



$$\Delta Y(s) = Y(x+s) - Y(x)$$

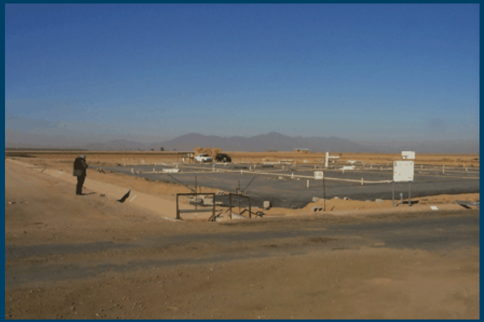
Observations




WATER RESOURCES RESEARCH, VOL. 49, 1–17, doi:10.1002/2013WR014286, 2013

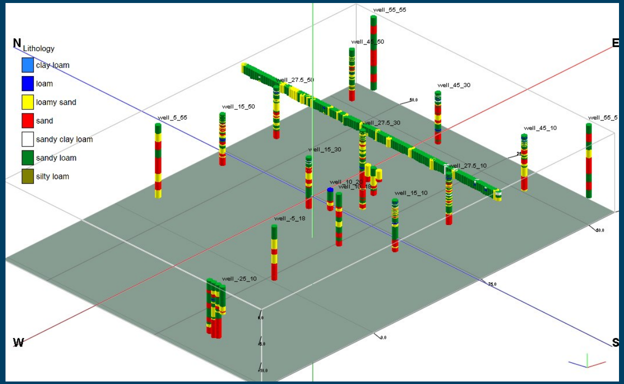
Anisotropic statistical scaling of vadose zone hydraulic property estimates near Maricopa, Arizona

A. Guadagnini,[1,2] S. P. Neuman,[1] M. G. Schaap,[3] and M. Riva[1,2]



Sharp Peaks and Heavy Tails of Increment Frequency Distributions Decaying with Lag

Neural network hydraulic property estimates of borehole samples at UA Maricopa Agricultural Station



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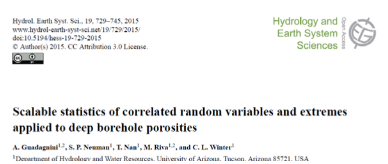
Model parameters and observation scale


Hydrol. Earth Syst. Sci., 19, 729–745, 2015
www.hydrol-earth-syst-sci.net/19/729/2015/
doi:10.5194/hess-19-729-2015
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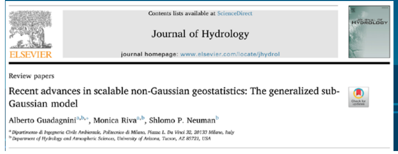
Hydrology and Earth System Sciences
Scalable statistics of correlated random variables and extremes applied to deep borehole porosities
A. Guadagnini[1,2], S. P. Neuman[1], T. Nan[1], M. Riva[1,2], and C. L. Winter[1]
[1]Department of Hydrology and Water Resources, University of Arizona, Tucson, Arizona 85721, USA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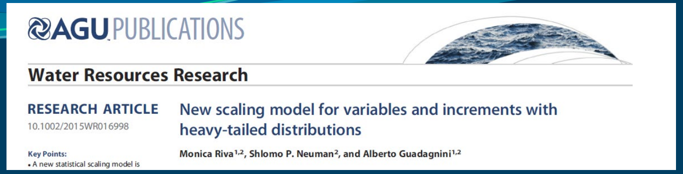
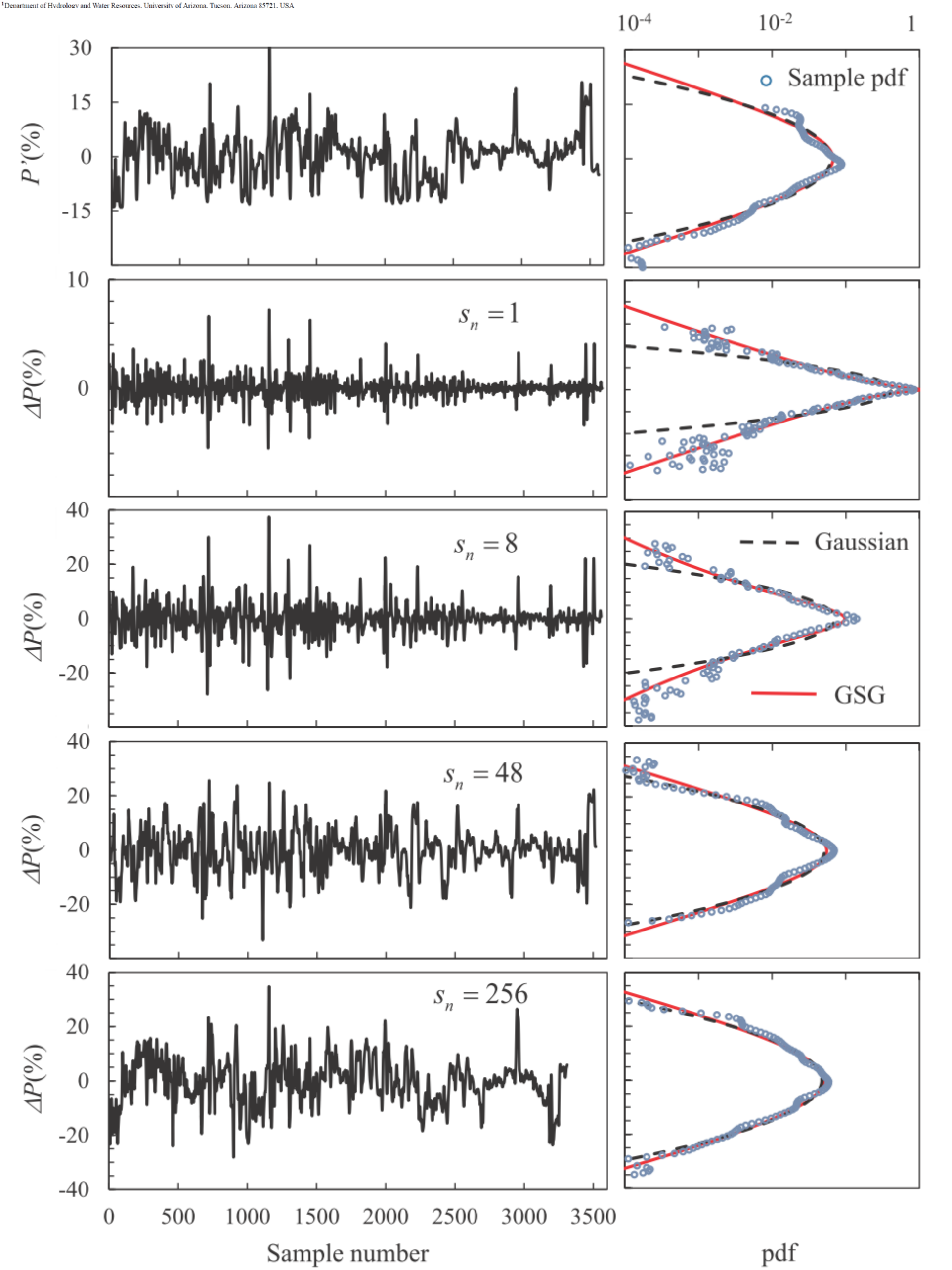
Journal of Hydrology
Review papers
Recent advances in scalable non-Gaussian geostatistics: The generalized sub-Gaussian model
Alberto Guadagnini, Monica Riva, Shlomo P. Neuman


AGU PUBLICATIONS
Water Resources Research
RESEARCH ARTICLE
10.1002/2015WR016998
New scaling model for variables and increments with heavy-tailed distributions
Monica Riva[1,2], Shlomo P. Neuman[2], and Alberto Guadagnini[1,2]



GSG-Generalized Sub-Gaussian Model

Consider $$Y'\left(x;\lambda_l,\lambda_u\right) = U(x)\underbrace{G\left(x;\lambda_l,\lambda_u\right)}_{\text{tfBm}}$$

$$Y'(x) = Y(x) - \langle Y \rangle$$

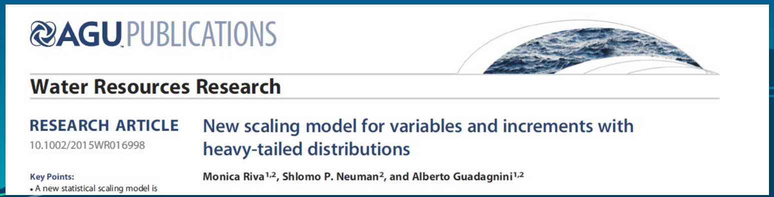
where $U(x) > 0$ **is iid subordinator**

N = 3567 Neutron Porosity data along a deep borehole

Mean= 14%; SD = 6.4%
Sampling interval: 0.15 m

data courtesy of Prof. Sahimi

- Several models for *U* (lognormal, Exponential, Gamma, ...)
- Subordinator *U* renders *Y'* a scale mixture of Gaussian RFs
- The scale mixture is non-Gaussian (distribution depends on *U*)


AGU PUBLICATIONS

Water Resources Research

RESEARCH ARTICLE

10.1002/2015WR016998

New scaling model for variables and increments with heavy-tailed distributions

Monica Riva[1,2], Shlomo P. Neuman[2], and Alberto Guadagnini[1,2]

Key Points:

• A new statistical scaling model is

GSG Model

Consider $Y'\left(x;\lambda_l,\lambda_u\right)=U\left(x\right)G\left(x;\lambda_l,\lambda_u\right)$ $\qquad Y'(x) = Y(x) - \langle Y \rangle$

where $U\left(x\right)>0$ **is iid subordinator.**

It renders Y

- Subordinated to G
- Scale mixture of G with random variances $\propto U\left(x\right)$

To start, lognormal (LN) subordinator

$$U = e^V \qquad V = \mathcal{N}\left\{0,\left(2-\alpha\right)^2\right\} \quad \alpha < 2$$

It renders Y' **normal-lognormal (NLN)**

$$f_{Y'}\left(y'\right)=\frac{1}{2\pi\left(2-\alpha\right)}\int_0^\infty \frac{1}{u^2}e^{-\frac{1}{2}\left(\frac{1}{\left(2-\alpha\right)^2}\ln^2\frac{u}{\sigma_G}+\frac{y'^2}{u^2}\right)}du$$

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GSG Model

$$f_Y(y) = \frac{1}{2\pi(2-\alpha)} \int_0^\infty \frac{1}{u^2} e^{-\frac{1}{2}\left(\frac{1}{(2-\alpha)^2}\ln^2\frac{u}{\sigma_G} + \frac{y^2}{u^2}\right)} du$$

Y increments are not NLN:

$$f_{\Delta Y}(\Delta y) = \frac{1}{2\sqrt{2}\pi^{3/2}(2-\alpha)^2} \int_0^\infty \int_0^\infty \frac{e^{-\frac{1}{2}\left[\frac{1}{(2-\alpha)^2}\left(\ln^2\frac{u_1}{\sigma_G} + \ln^2\frac{u_2}{\sigma_G}\right) + \frac{(\Delta y)^2}{u_1^2 + u_2^2 - 2u_1u_2\rho_G}\right]}}{u_2u_1\sqrt{u_1^2 + u_2^2 - 2u_1u_2\rho_G}} du_2 du_1$$

Scales with lag through $\rho_G(s)$
(coefficient of correlation of *G*)

GSG Model

- ***Variogram of Y***

$$\gamma_Y = \sigma_G^2 e^{(2-\alpha)^2}\left(e^{(2-\alpha)^2} - 1\right) + e^{(2-\alpha)^2}\gamma_G$$

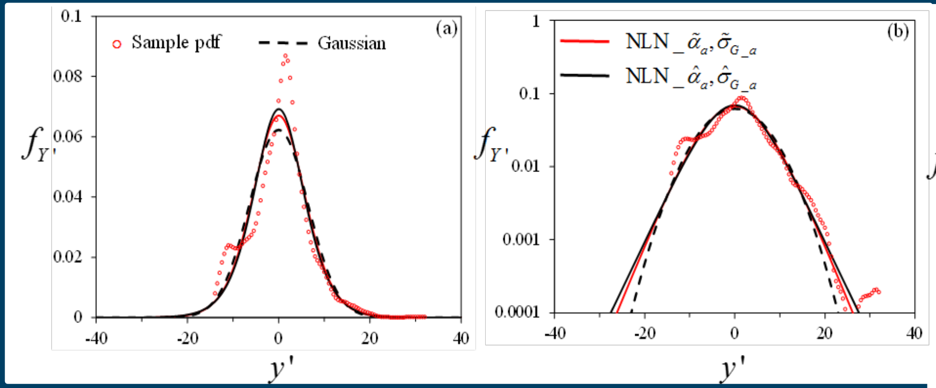
Observed nugget effects, attributed in the literature to variability of *Y* at scales smaller than the sampling interval and/or to measurement errors, may in fact be (at least in part) a symptom of non-Gaussianity.

- **Integral scale of *Y*** (subordination dampens and does not destroy covariance structure)

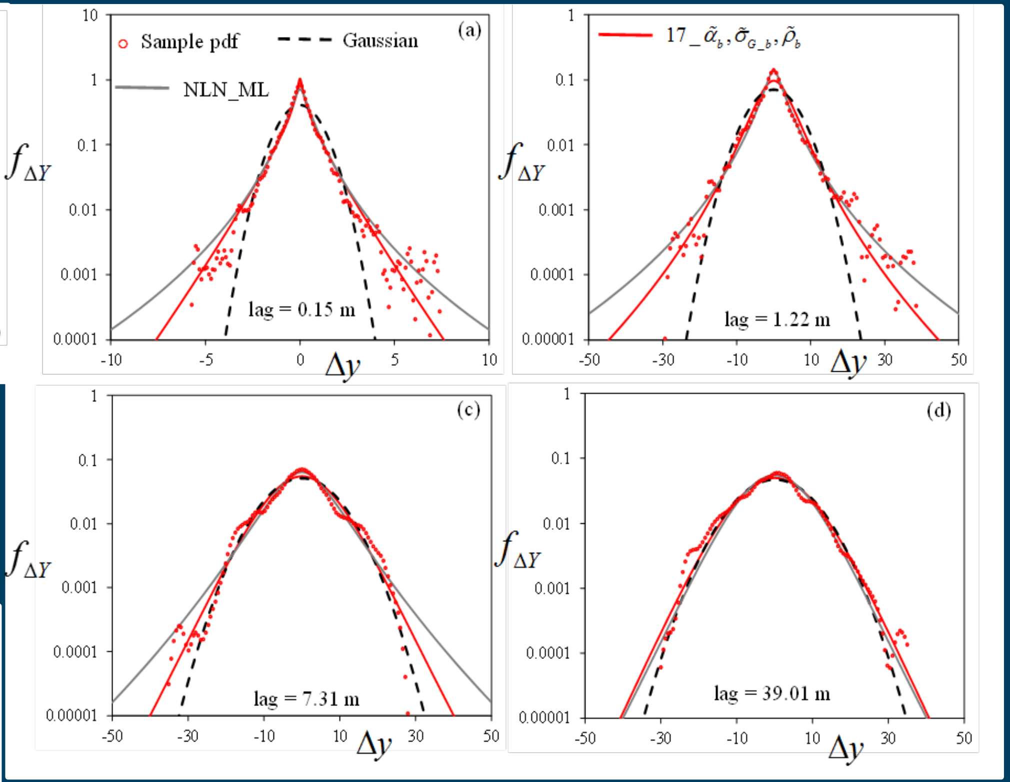
$$I_Y = e^{-(2-\alpha)^2} I_G$$

Deep Borehole Neutron Porosity data

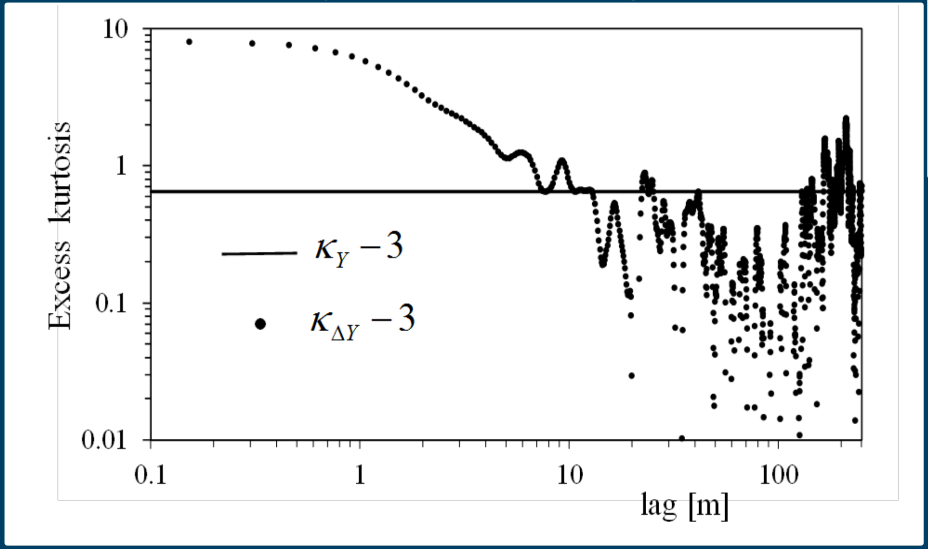
Sample pdf of neutron porosity data *Y'*

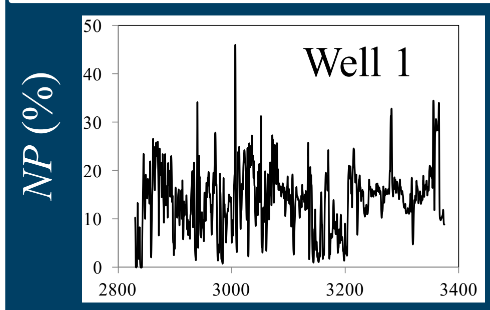


Sample pdf of increments of neutron porosity data



Excess kurtosis of mean-removed porosity data (continuous curve) and of porosity increments (symbols) versus lag





Dashtian et al. [2011]

Southwest Iran
3,567 data
Minimum lag: 15 cm
Total depth: 543 m

$M_1^Y = 14\%$

Sample standard deviation = 6.4%

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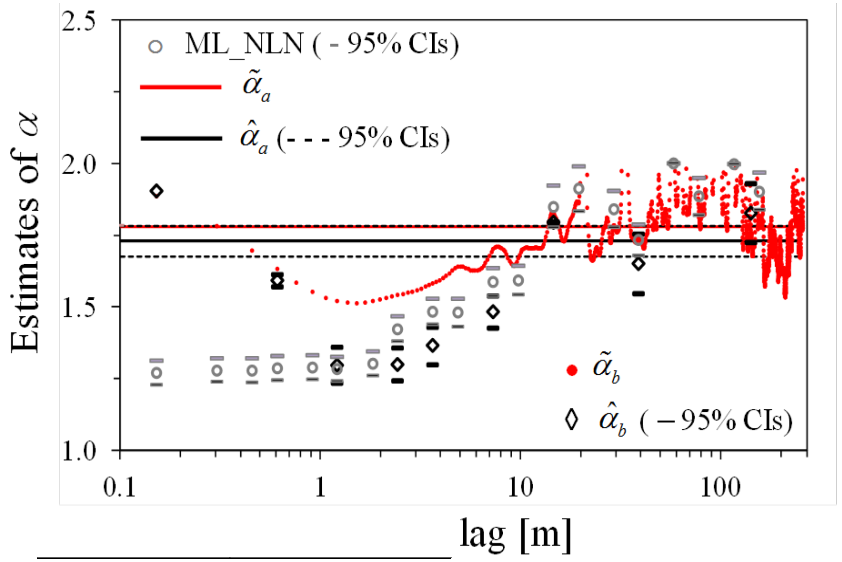
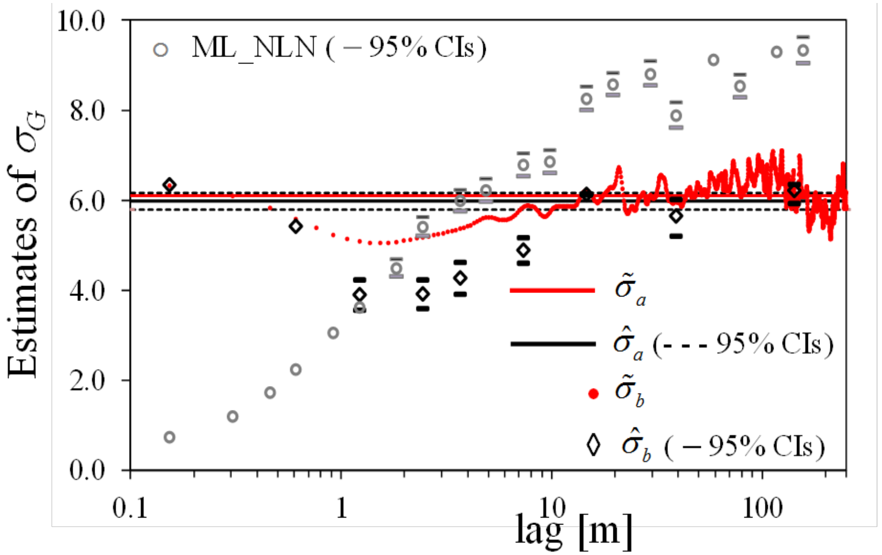
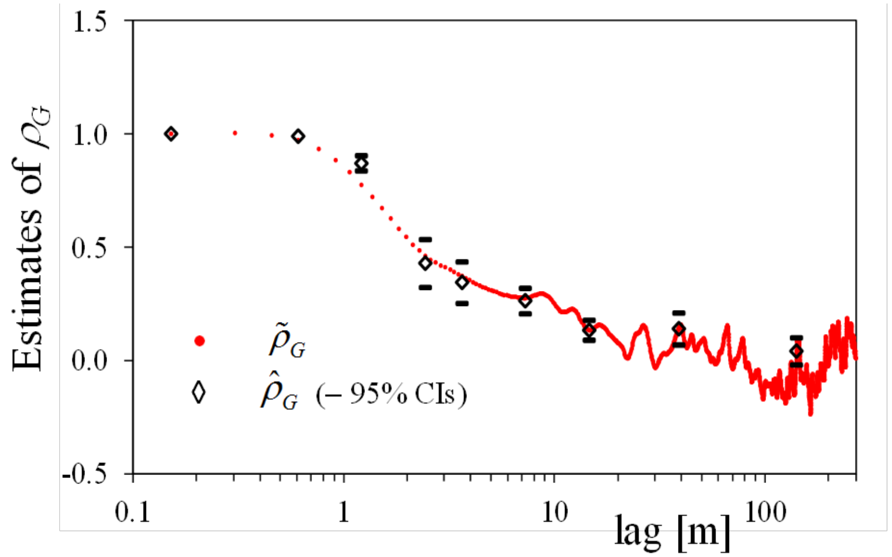
Deep Borehole Neutron Porosity data

Riva et al. (2015) WRR





$\tilde{\alpha}_a$	1.78
$\hat{\alpha}_a$	1.73 ± 0.05
Mean of $\tilde{\alpha}_b$	1.75
CV of $\tilde{\alpha}_b$	0.05
$\tilde{\sigma}_a$	6.1
$\hat{\sigma}_a$	5.98 ± 0.19
Mean of $\tilde{\sigma}_b$	6.15
CV of $\tilde{\sigma}_b$	0.07



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Spatial Statistics
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Statistical modeling of gas-permeability spatial variability along a limestone core

M. Siena [a,*], M. Riva [a,b], M. Giamberini [a], P. Gouze [c], A. Guadagnini [a,b]

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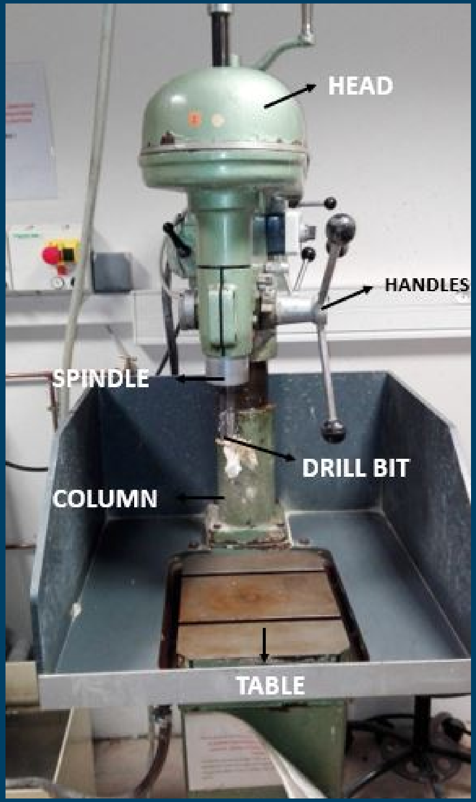
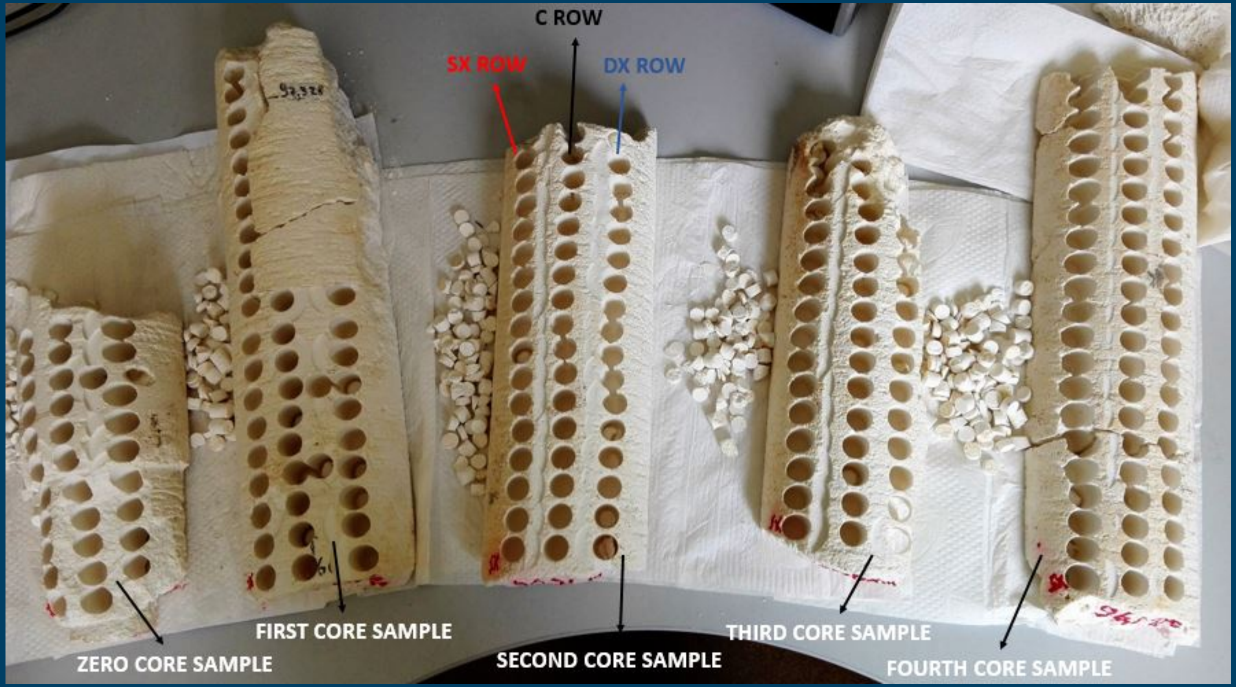
[b] Department of Hydrology and Atmospheric Sciences, University of Arizona, Tucson, AZ 85721, USA
[c] Géosciences Montpellier, CNRS-Université de Montpellier, Place Eugène Bataillon, 34095 Montpellier, France

- Drilling operation, using a drill press: multiple subsamples are obtained along three lines parallel to the core axis (left, central and right).

→ **222 samples** in total





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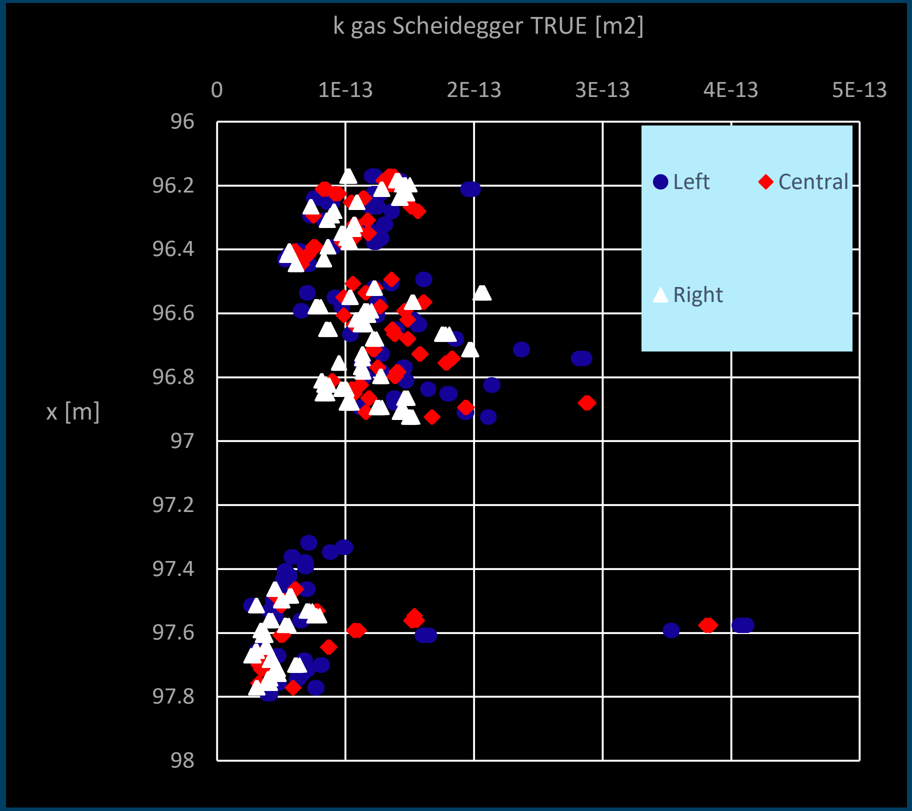
❖ One-dimensional variability



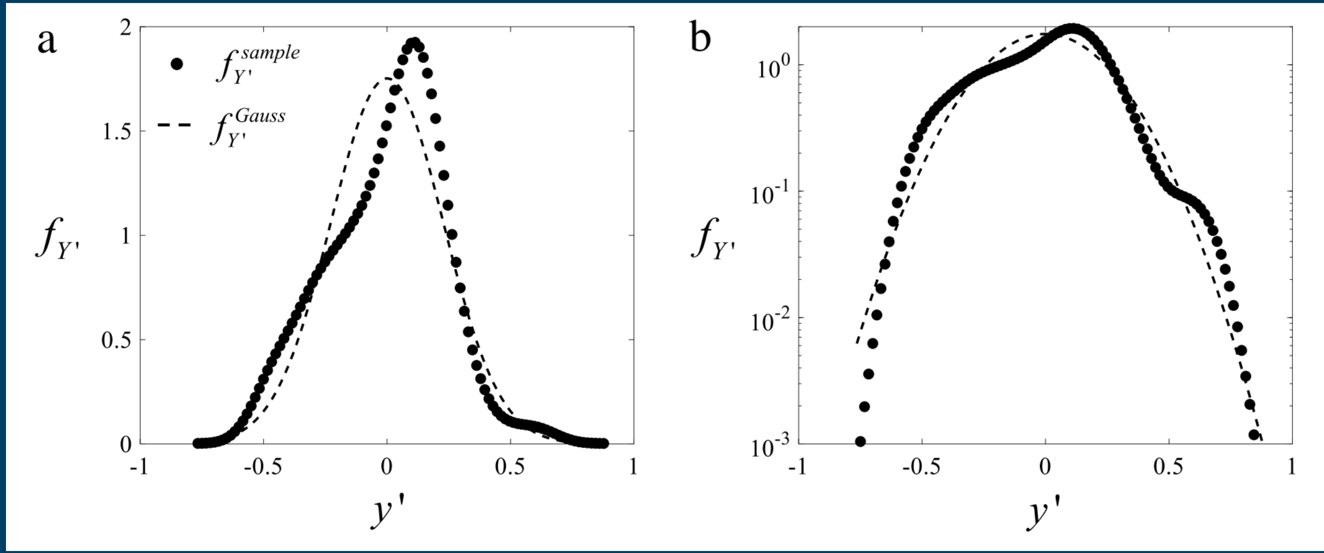


Fig. 3. Sample pdf of Y' data, $f_{Y'}^{sample}$ (symbols) on (a) arithmetic and (b) semi-logarithmic scale. A Gaussian pdf with variance equal to that of the Y' data, $f_{Y'}^{Gauss}$, is shown for comparison (dashed curve).

N. of observations	Mean	Standard deviation	Skewness	Excess kurtosis
220	-13.03	0.228	-0.208	-0.161

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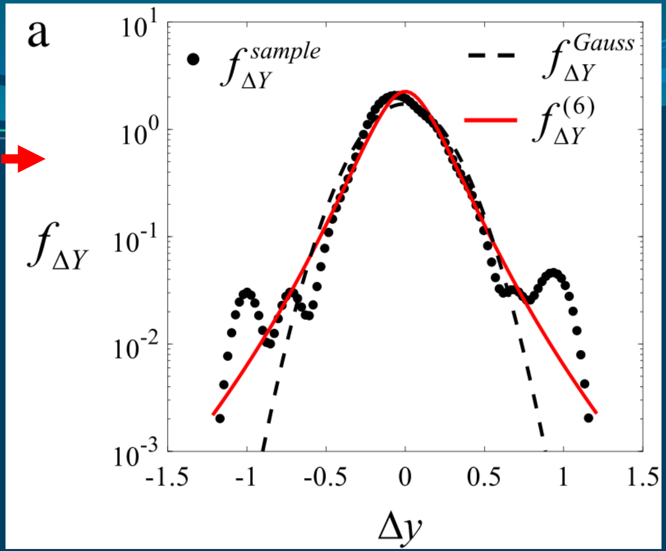
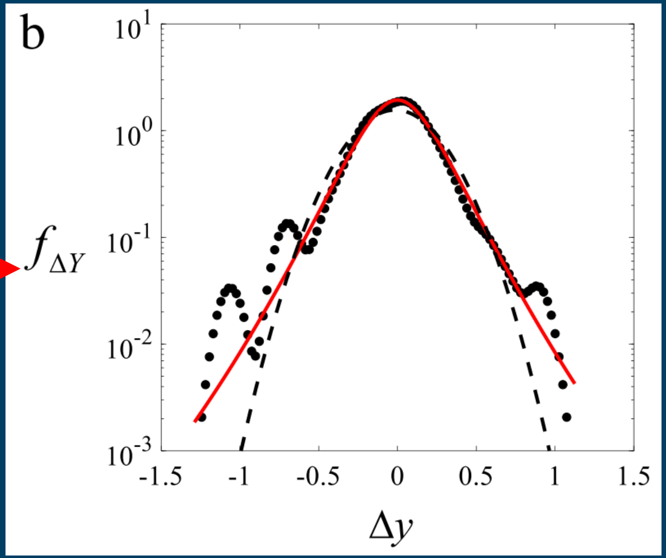
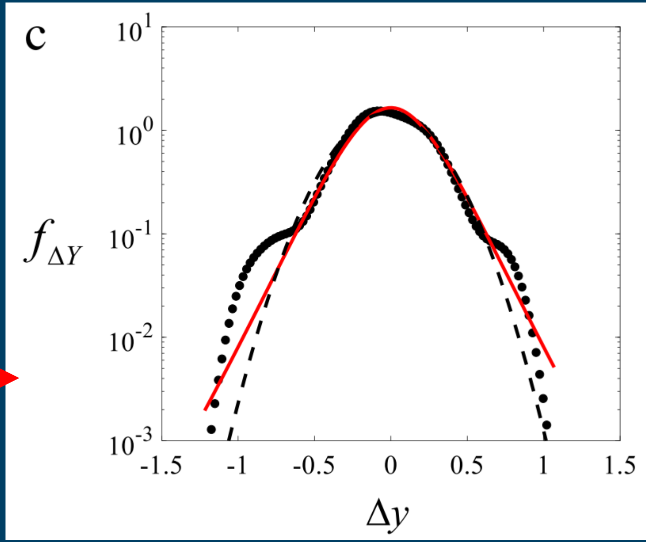
Table 2
Lead sample statistics of incremental data ΔY for the selected 14 classes of lags.

s_k	N	Mean	Standard deviation	Skewness	Excess kurtosis
1	154	−0.01	0.13	−0.18	0.39
2	211	−0.01	0.17	0.38	1.97
3	189	−0.01	0.21	0.27	3.44
4	182	−0.01	0.23	0.41	4.03
5	177	−0.01	0.27	0.24	4.28
6	176	0.00	0.28	0.11	3.64
7	165	−0.01	0.27	0.03	3.44
8	159	−0.02	0.26	−0.14	2.54
9	156	−0.01	0.26	−0.33	2.77
10	156	−0.02	0.26	−0.26	2.96
11	149	−0.03	0.26	−0.11	2.08
12	152	−0.04	0.26	−0.07	2.15
13	138	−0.03	0.28	−0.32	2.04
14	134	−0.02	0.27	−0.21	1.23







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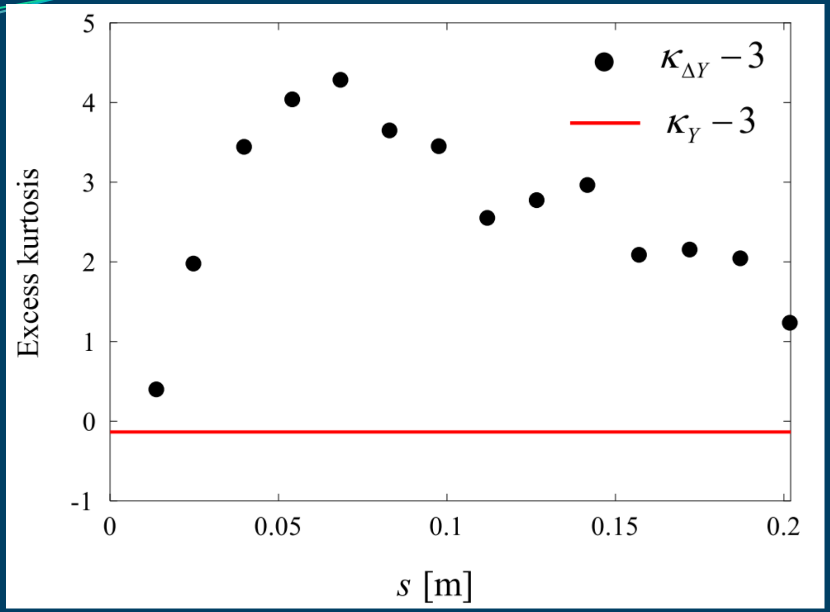

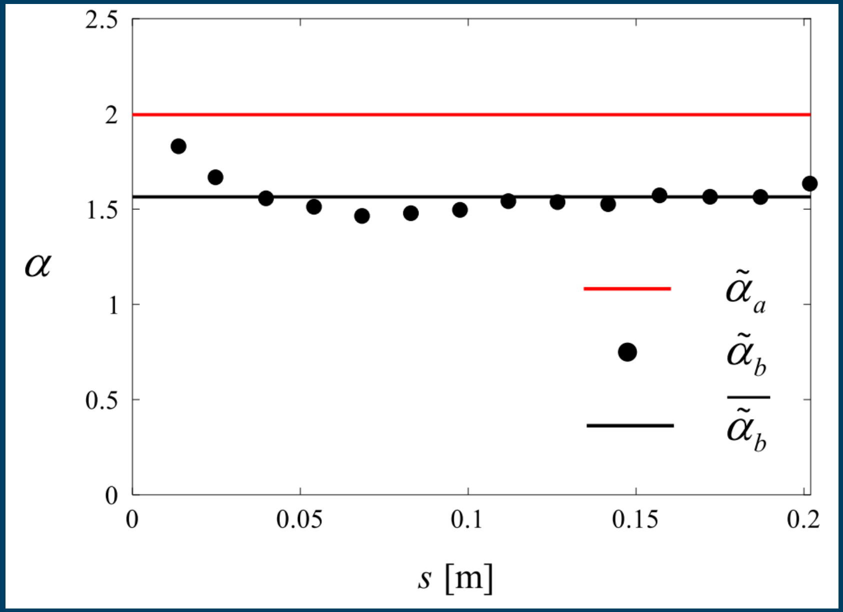

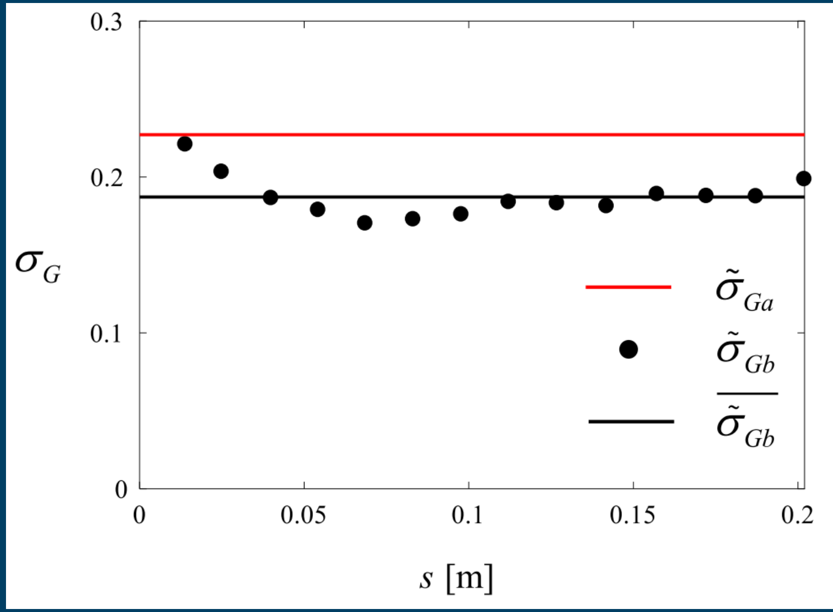

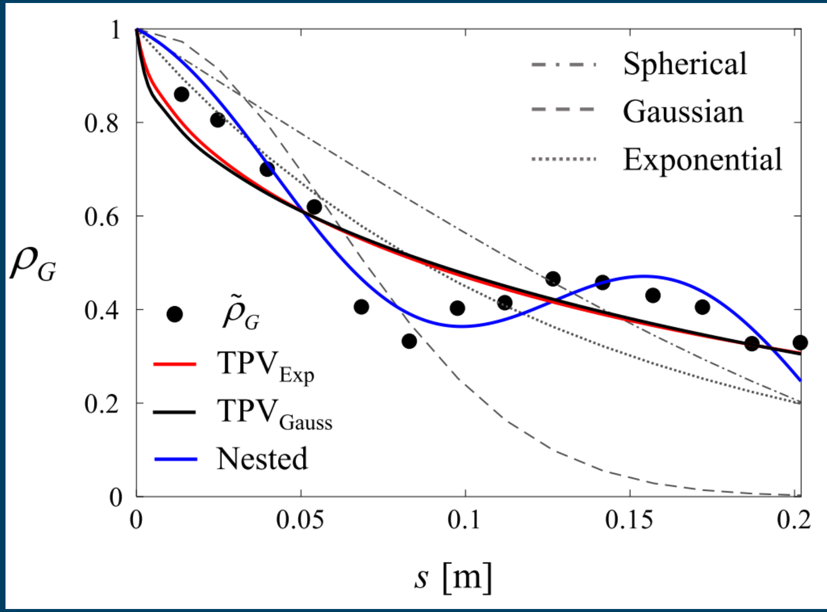

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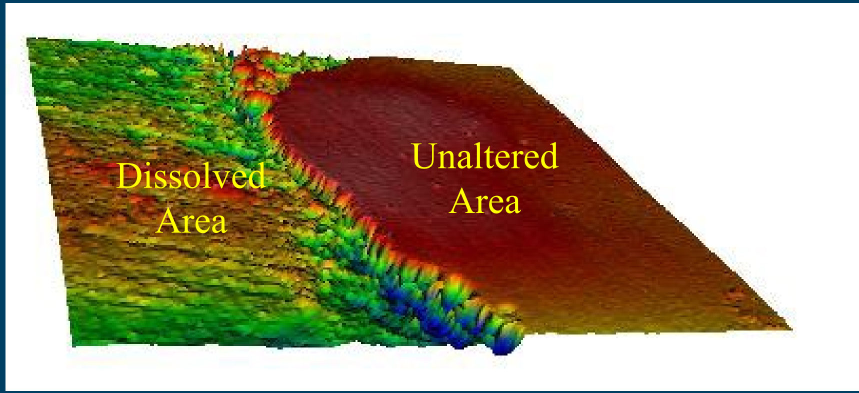
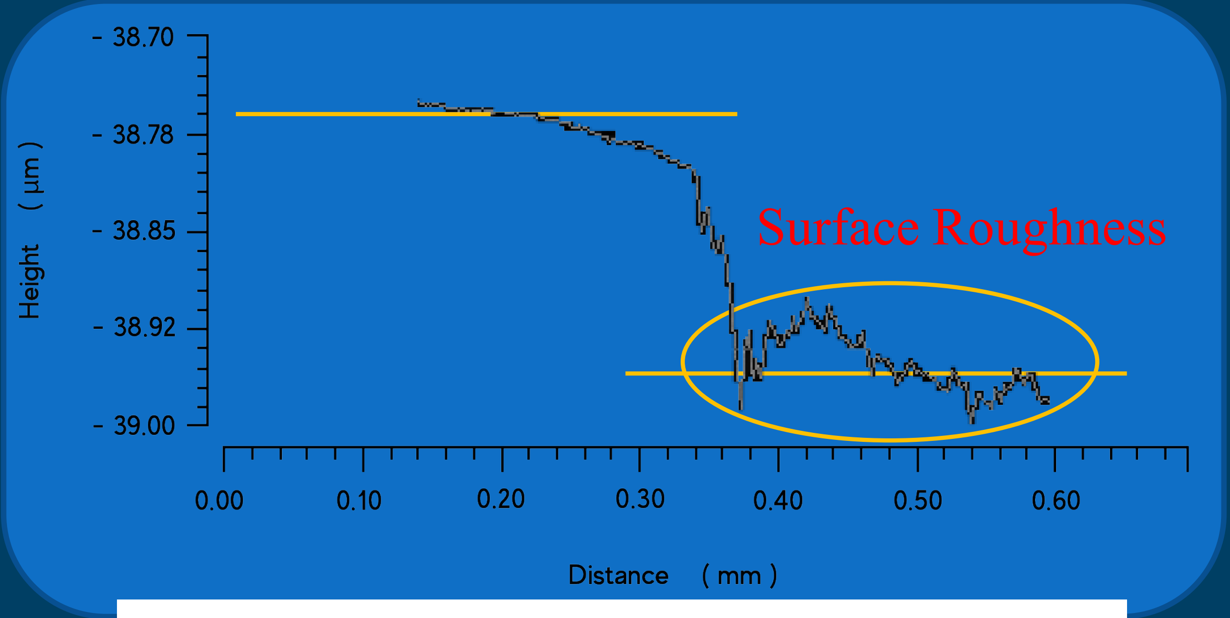
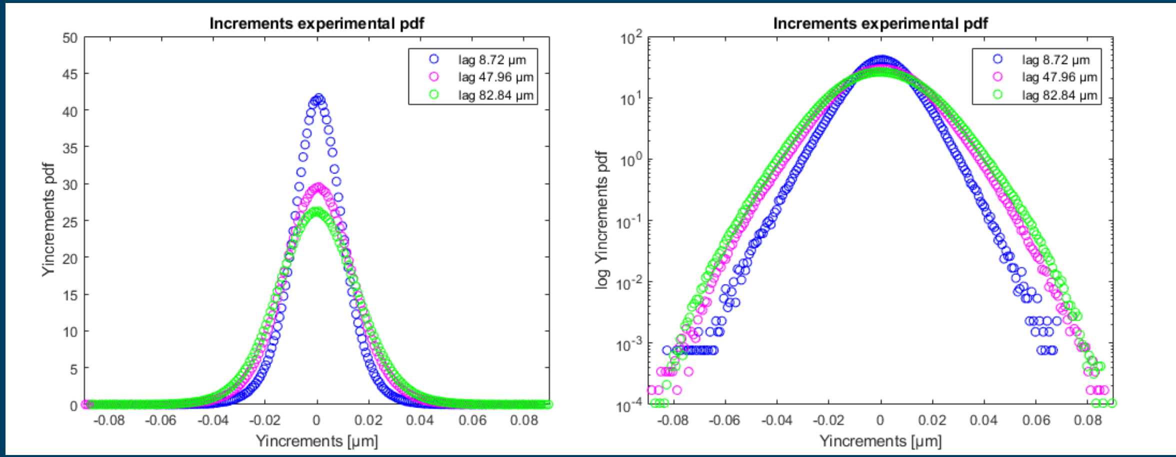
Application to geochemical data – assessment of calcite dissolution

Joint work with Philippe Ackerer and Damien Daval (Université de Strasbourg-CNRS ENGEES/EOST)

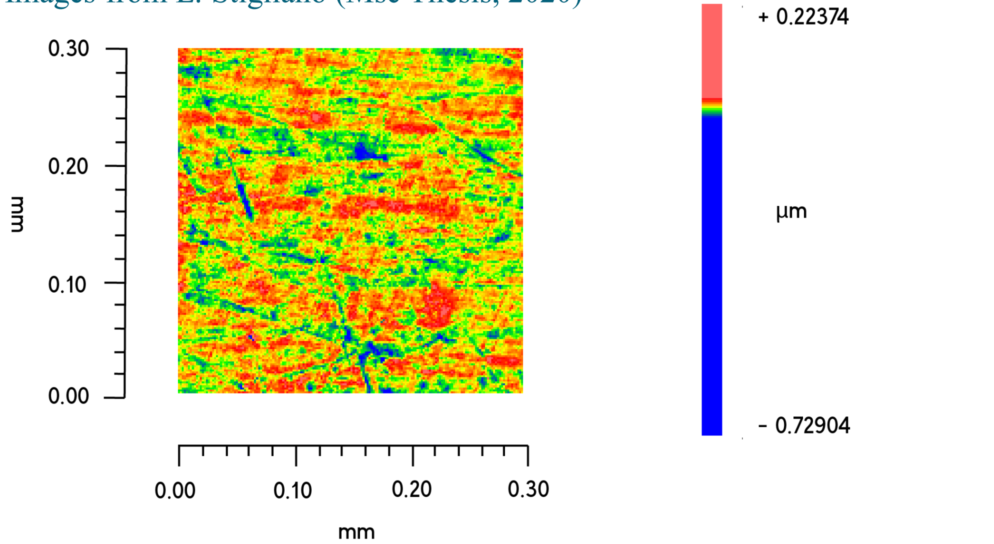
- Topography images of calcite crystals







Images from L. Stigliano (Msc Thesis, 2020)



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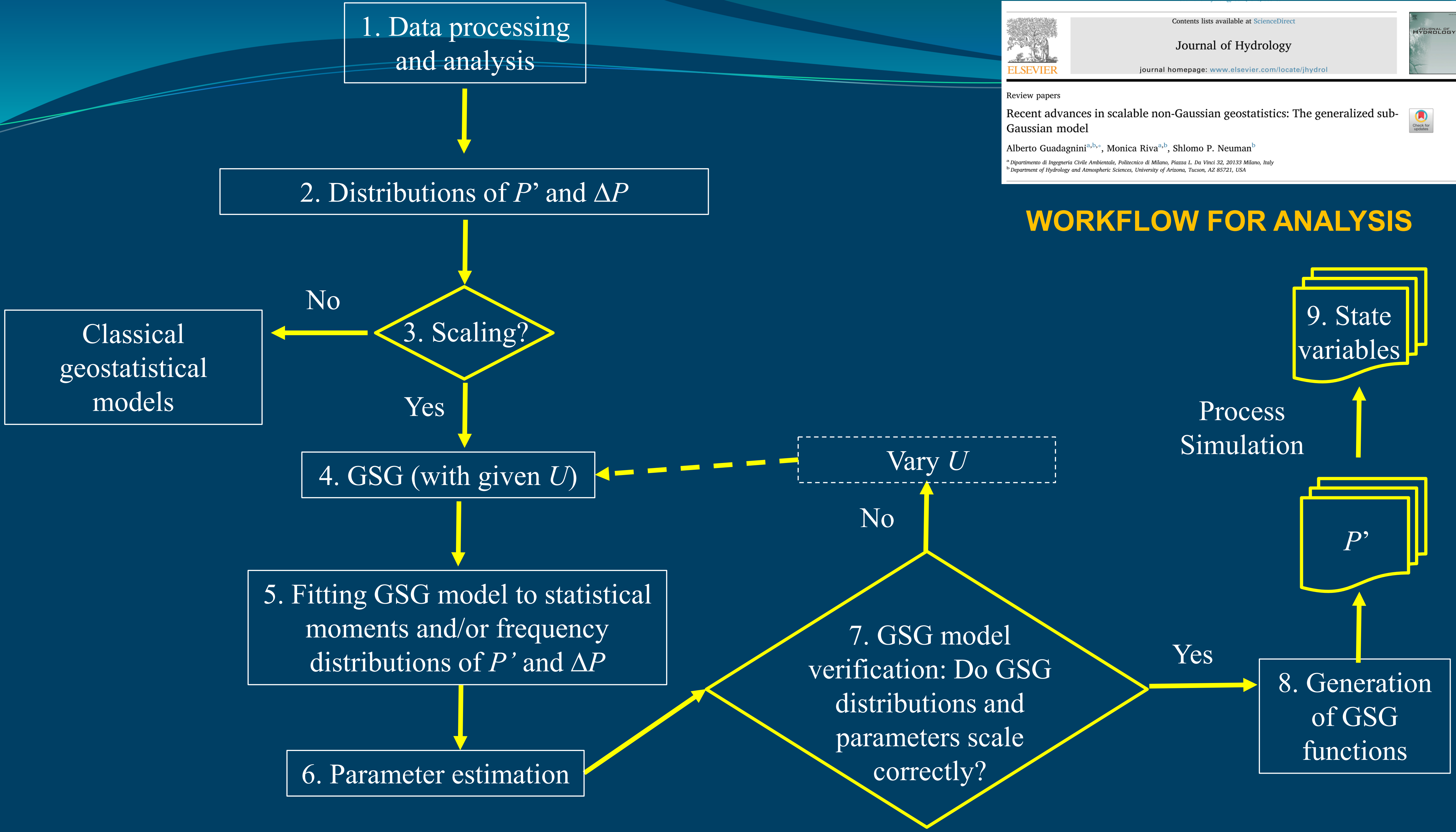
Review papers

Recent advances in scalable non-Gaussian geostatistics: The generalized sub-Gaussian model

Alberto Guadagnini[a,b,*], Monica Riva[a,b], Shlomo P. Neuman[b]

[a] Dipartimento di Ingegneria Civile Ambientale, Politecnico di Milano, Piazza L. Da Vinci 32, 20133 Milano, Italy
[b] Department of Hydrology and Atmospheric Sciences, University of Arizona, Tucson, AZ 85721, USA

WORKFLOW FOR ANALYSIS



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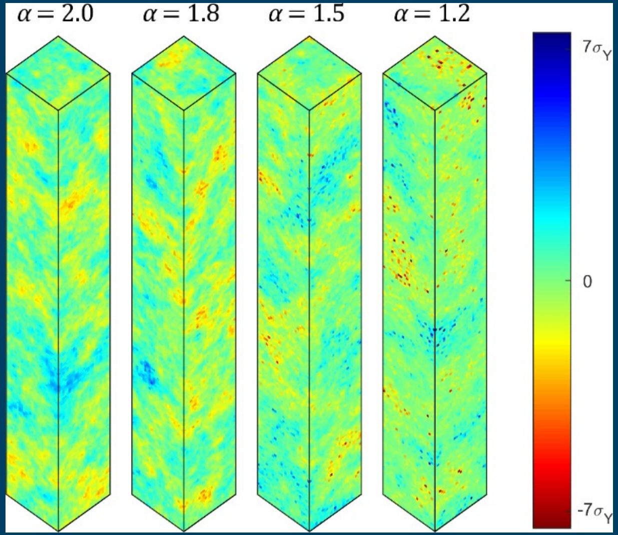
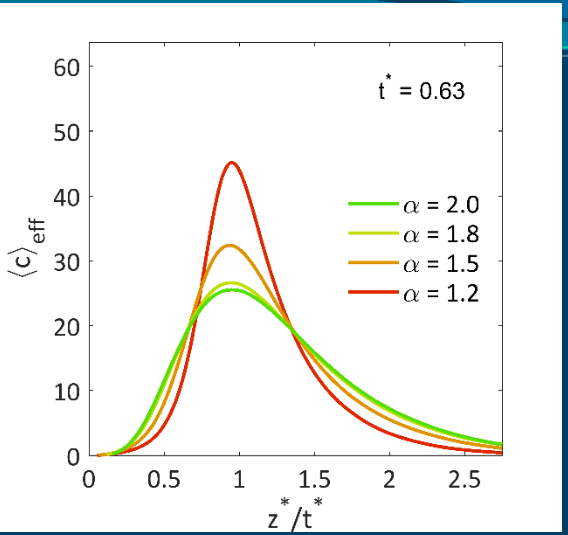
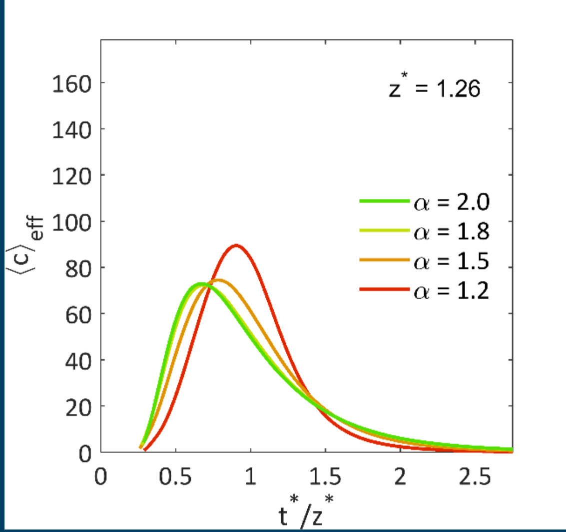
3D-flow/transport problems



8×10^6 cells





What we have seen so far!

- Many variables with scalable statistics exhibit
  - Heavy-tailed increment frequency distributions with sharp peaks at small lags
  - Peak and tail decay with increasing lag

- These aspects are captured by a GSG model based on sub-Gaussian subordination
  - Simple ways to estimate parameters

- We developed a methodology to generate unconditional and conditional realizations of such random fields
  - Ready to be employed for Monte Carlo flow and transport simulations in randomly heterogeneous systems (see *Libera*, *A*., *F. de Barros*, *M. Riva*, and *A. Guadagnini*, JCH, 2018).

- Lead-order analytical solutions for a simple non-Gaussian case of flow and transport in an unbounded unimodal GSG log permeability field Y under mean uniform flow in two spatial dimensions

Summary

- GSG model can reconcile several aspects exhibited by many (environmental and other) variables
  - Power-law scaling of structure functions in midrange of lags
  - Breakdown in such scaling at small / large lags
  - Heavy-tailed increment frequency distributions with sharp peaks at small lags
  - Peak and tail decay with increasing lag
  - Extended power-law scaling or ESS
  - Linear / nonlinear scaling of power exponent
- Simple ways to estimate model parameters
- Lead-order analytical solutions for simple flow/transport settings
- Numerical flow/transport solutions
- Conditional GSG