Using an antidiffusive transport scheme for vertical transport

A promising path towards solving the long-standing problem of excessive vertical diffusion in Eulerian models

Sylvain Mailler, Mathieu Lachâtre and Laurent Menut

Laboratoire de Météorologie Dynamique, CNRS, École Polytechnique, École Nationale des Ponts et Chaussées

May 7, 2020

Outline

Abstract

We propose here a solution to the long-standing problem of excessive diffusion in the vertical direction in Eulerian chemistry-transport models

- Excessive numerical diffusion is a painful problem for eulerian models, hindering their use for simulating long-range transport of concentrated plumes, *e.g.* volcanic plumes, biomass burning plumes, mineral dust plumes, plumes from industrial accidents, etc.
- For this purpose, Lagrangian models usually are extremely useful, but they are comparatively not well-suited for the representation of phenomena such as chemistry, radioctive decay, etc.
- We argue that choosing specific antidiffusive schemes such as the Després and Lagoutière 1999 scheme for vertical transport permits to substantially reduce this problem (Lachatre et al. 2020)

Transport strategy

- The advection equation for a tracer species is solved with directional splitting in a 2d-domain.
- Godonov, Van Leer 1977 and Colella and Woodward 1984 schemes are tested fr the evaluation of tracer fluxes. These ate the schemes currently implemented in the CHIMERE 2017 chemistry-transport model (Mailler et al. 2017)
- The Després and Lagoutière 1999 scheme presented hereinafter is tested for vertical transport.

The Després and Lagoutière 1999 scheme

The Després and Lagoutière 1999 advection scheme is a order 1, antidiffusive scheme. Its idea is to reduce numerical diffusion as much as possible while preserving monotonicity of the scheme. The formulation of the scheme is simple:

$$\bar{\alpha}_{s,k+\frac{1}{2}} = \alpha_{s,k} + \frac{1-\nu}{2}\mathrm{Max}\left[0, \mathrm{Min}\left(\frac{2}{\nu}\frac{\alpha_{s,k}-\alpha_{s,k-1}}{\alpha_{s,k+1}-\alpha_{s,k}}, \frac{2}{1-\nu}\right)\right],$$

The previous equation is similar to the Van Leer 1977 scheme with slope limiter I, but *the slope estimate is prescribed in priority by the limiter instead of the gradient estimate*. This approach sacrifices order-2 acccuracy but, reduces numerical diffusion by design.

Test case design and domain configuration

Case definition

- Initially vertical plume confined between $z = 4500\,\mathrm{m}$ and $z = 7500\,\mathrm{m}$ and horizontally on one domain cell ($25\,\mathrm{km}$ wide)
- Zonal wind is constant in time, zonally uniform and vertically sheared with $u(x,z,t) = U_0\frac{2z}{H}$ where H is domain height
- Vertical wind defined as $w(x,z,t) = w_0 \cos(\omega T)$ with $w_0 = 5\,\mathrm{cm\,s^{-1}}$

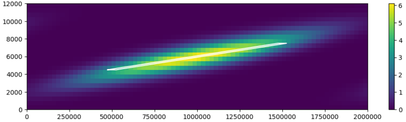
Domain

- 2d, x-z grid, $2000\,\mathrm{km} \times 10000\,\mathrm{m}$
- Resolution $25\,\mathrm{km} \times 250\,\mathrm{m}$ (80×40 grid cells)
- Periodic boundary condition in the horizontal

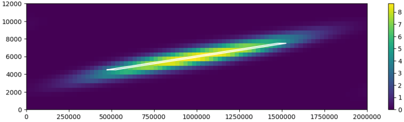
This is an idealized representation of formation of a thin layer formation due to wind shear.

4 simulations are performed for 48 hours, corresponding and $2000\,\mathrm{km}$ advection. All the simulations use Colella and Woodward 1984 for horizontal advection. Vertical advection with Godunov, Van Leer 1977, Colella and Woodward 1984 and Després and Lagoutière 1999 has been tested.

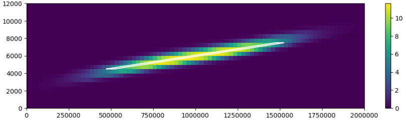
Results: Visual performance for representing fine layer formation under the effect of wind shear



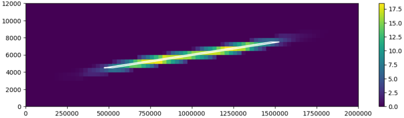
(a) upwind



(b) Van Leer

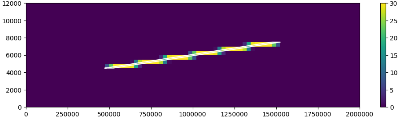


(c) PPM



(d) Després and Lagoutière 1999

Reduced numerical diffusion and improved comparison to the exact solution is clearly visible. Fine layer structure is represented better with Després and Lagoutière 1999 than with other tested schemes.



(e) exact solution (discretized)

Results: Objective performance for representing fine layer formation under the effect of wind shear

	Exact	Upwind	VL	PPM	DL99
Max. Mixing Ratio	30.0	6.10	8.69	11.6	**18.5**
% error (norm 1)	0.	157.	140.	122.	**87.**
% error (norm 2)	0.	86.1	80.4	73.9	**60.3**
% mass in envelope	100.0	23.3	33.2	44.4	**64.7**

Table 1: Performance indicators for of simulations performed with the Upwind, VL, PPM and DL99 vertical advection schemes

Table 8 shows a spectacular overperformance of Després and Lagoutière 1999 compared to the other schemes. As visible on previous slide, the tracer stays very much contained in the desired envelope.
The maximal mixing ratio is preserved much better and error metrics compared to the exact solution (computed from exact Lagrangian trajectories in this simple flow) are weaker.

Discussion

From the present experiment and others (not shown), we conclude that:

- Using the Després and Lagoutière 1999 scheme on the vertical yields results superior to higher-order schemes in representing formation and long-range advection of tracers
- This results holds only if the vertical resolution is *insufficient* to resolve smoothly the desired features. Otherwise, as expected from theory, higher-order schemes perform better (*not shown*)
- Therefore, Després and Lagoutière 1999 antidiffusive transport scheme is extremely promising for the simulation of sharp plumes in chemistry-transport models. Already tested successfully in CHIMERE (Lachatre et al. 2020) in the context of Etna volcanic eruptions.

Perspectives

- Is this antidiffusive formulation an option for operational use ? -> Thorough testing in CHIMERE including routine simulations of boundary layer pollution.
- Is this antidiffusive formulation adapted for transport inweather/climate models ?
- Using Després and Lagoutière 1999 for both horizontal and vertical direction introduces uncontrollable spurious effects (not shown). Is it possible to find a successfull antidiffusive formulation for 2d horizontal transport ?

References

Colella, P. and P. R. Woodward (1984). "The piecewise parabolic method (PPM) for gas-dynamical simulations". In: *Journal of Computational Physics* 11, pp. 38–39.

Després, Bruno and Frédéric Lagoutière (1999). "Un schéma non linéaire anti-dissipatif pour l'équation d'advection linéaire". In: *Comptes Rendus de l'Académie des Sciences - Series I - Mathematics* 328.10, pp. 939 –943. ISSN: 0764-4442. DOI: https://doi.org/10.1016/S0764-4442(99)80301-2. URL: http://www.sciencedirect.com/science/article/pii/S0764444299803012.

Lachatre, M. et al. (2020). "New strategies for vertical transport in chemistry-transport models: application to the case of the Mount Etna eruption on March 18, 2012". In: *Geophys. Model Dev. Discuss.*

Mailler, S. et al. (2017). "CHIMERE-2017: from urban to hemispheric chemistry-transport modeling". In: *Geoscientific Model Development* 10.6, pp. 2397–2423. DOI: 10.5194/gmd-10-2397-2017.

Van Leer, B. (1977). "Towards the ultimate conservative difference scheme. IV. A new approach to numerical convection". In: *Journal of Computational Physics* 23.3, pp. 276 –299. ISSN: 0021-9991. DOI: https://doi.org/10.1016/0021-9991(77)90095-X. URL: http://www.sciencedirect.com/science/article/pii/002199917790095X.