The viscous and Ohmic damping of Earth's Free Core Nutation (FCN)

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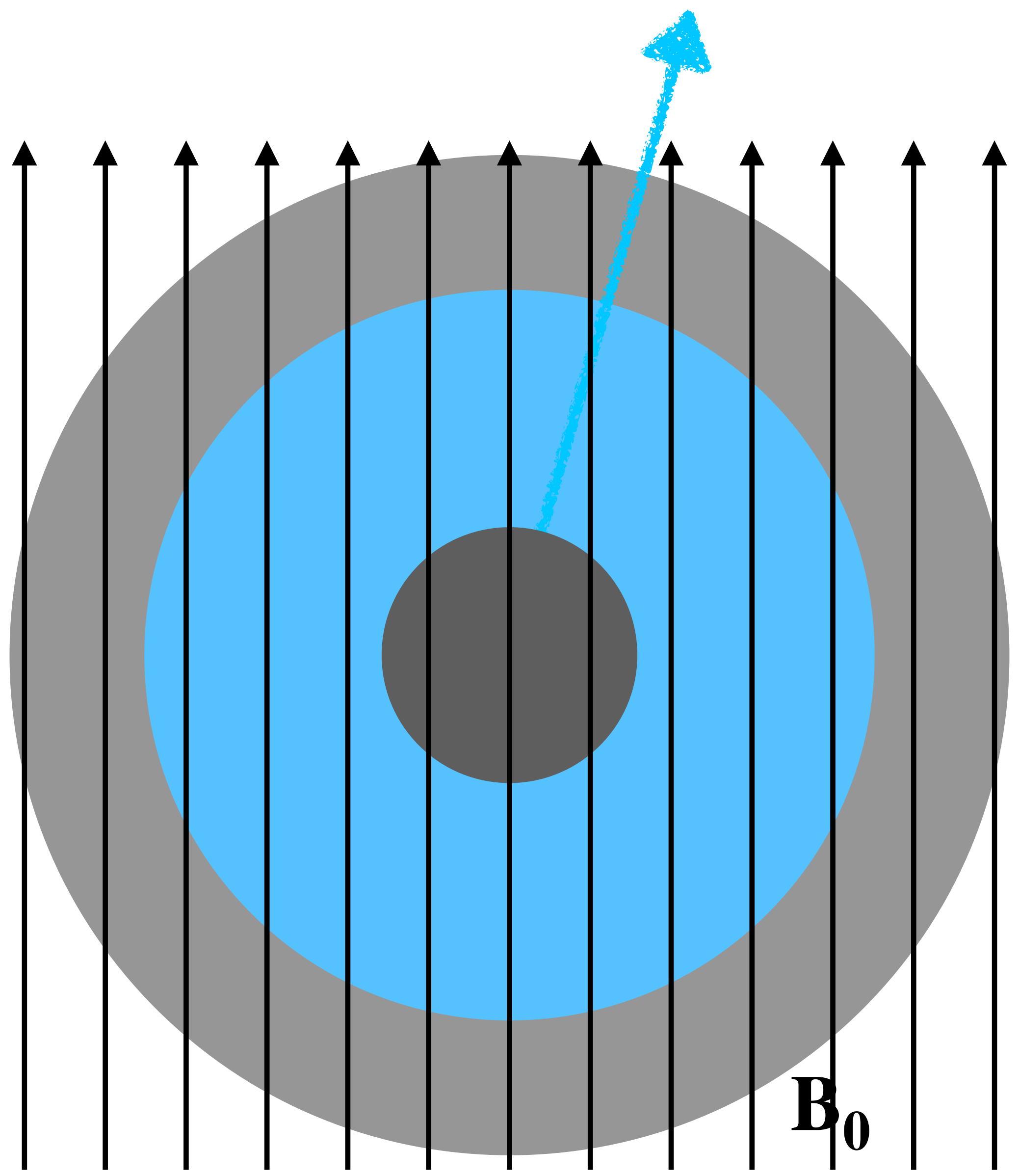
The FCN's damping can be estimated from VLBI and gravimetry observations

We build a dissipation model using the *spin-over* mode as a proxy for the FCN

We adjust the model's parameters to match the observed damping



$$\partial_t \mathbf{u} + 2\hat{z} \times \mathbf{u} = -\nabla p + E\nabla^2 \mathbf{u} + Le\,(\nabla \times \mathbf{b}) \times \mathbf{B_0}$$

$$\partial_t \mathbf{b} = \nabla \times \left(\mathbf{u} \times \mathbf{B_0}\right) + E_\eta \nabla^2 \mathbf{b}$$

Ekman $E = \dfrac{\nu}{\Omega L^2}$

Lehnert $Le = \dfrac{B_0}{\sqrt{\rho\mu_0}\,\Omega L}$ $\quad E_\eta = \dfrac{\eta}{\Omega L^2}$

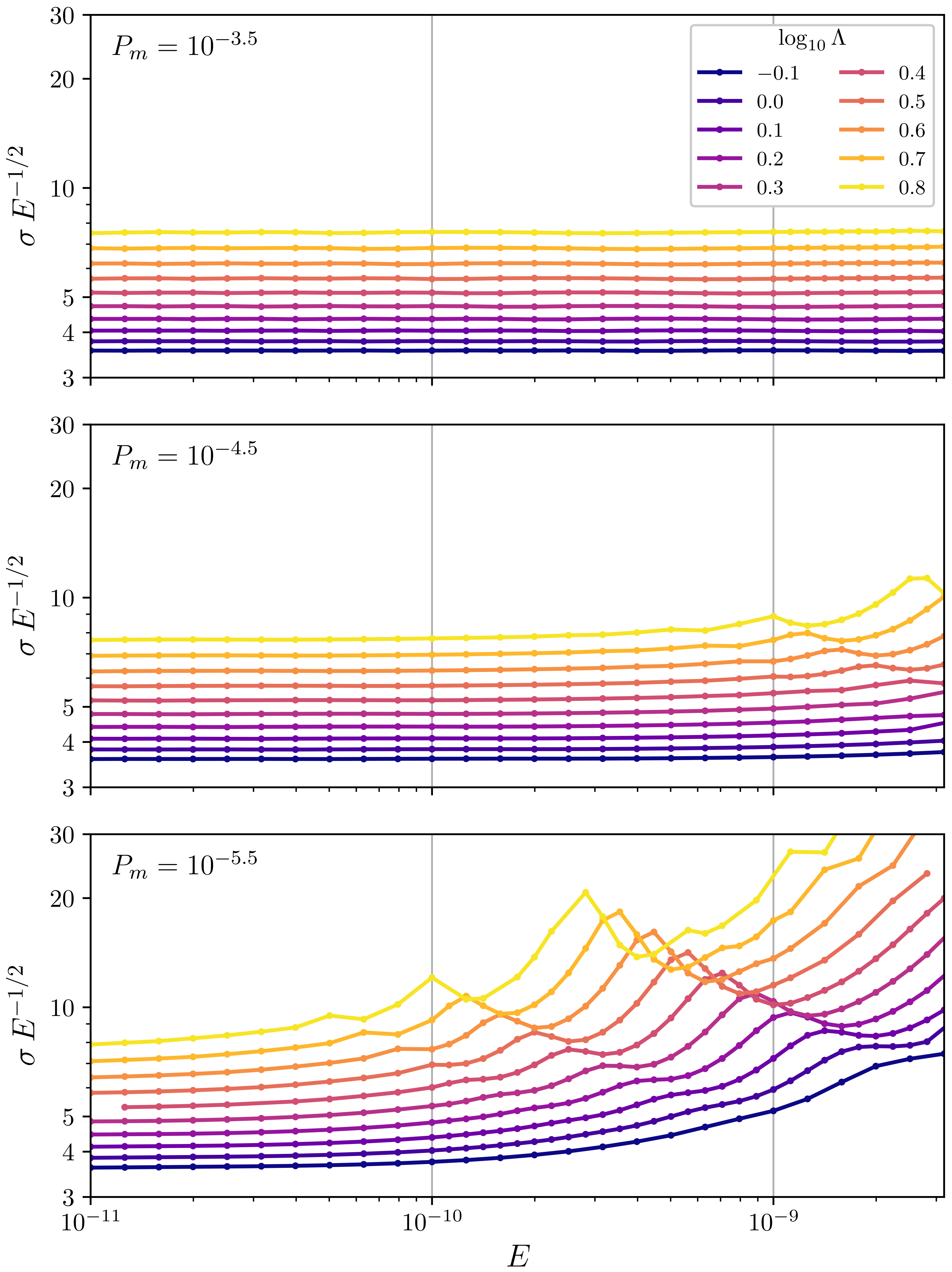
Elsasser $\Lambda = \dfrac{B_0^2\,\sigma}{\rho\,\Omega} = \dfrac{B_0^2}{\rho\mu_0\eta\,\Omega} = \dfrac{Le^2}{E_\eta}$ $\quad P_m = \dfrac{\nu}{\eta}$

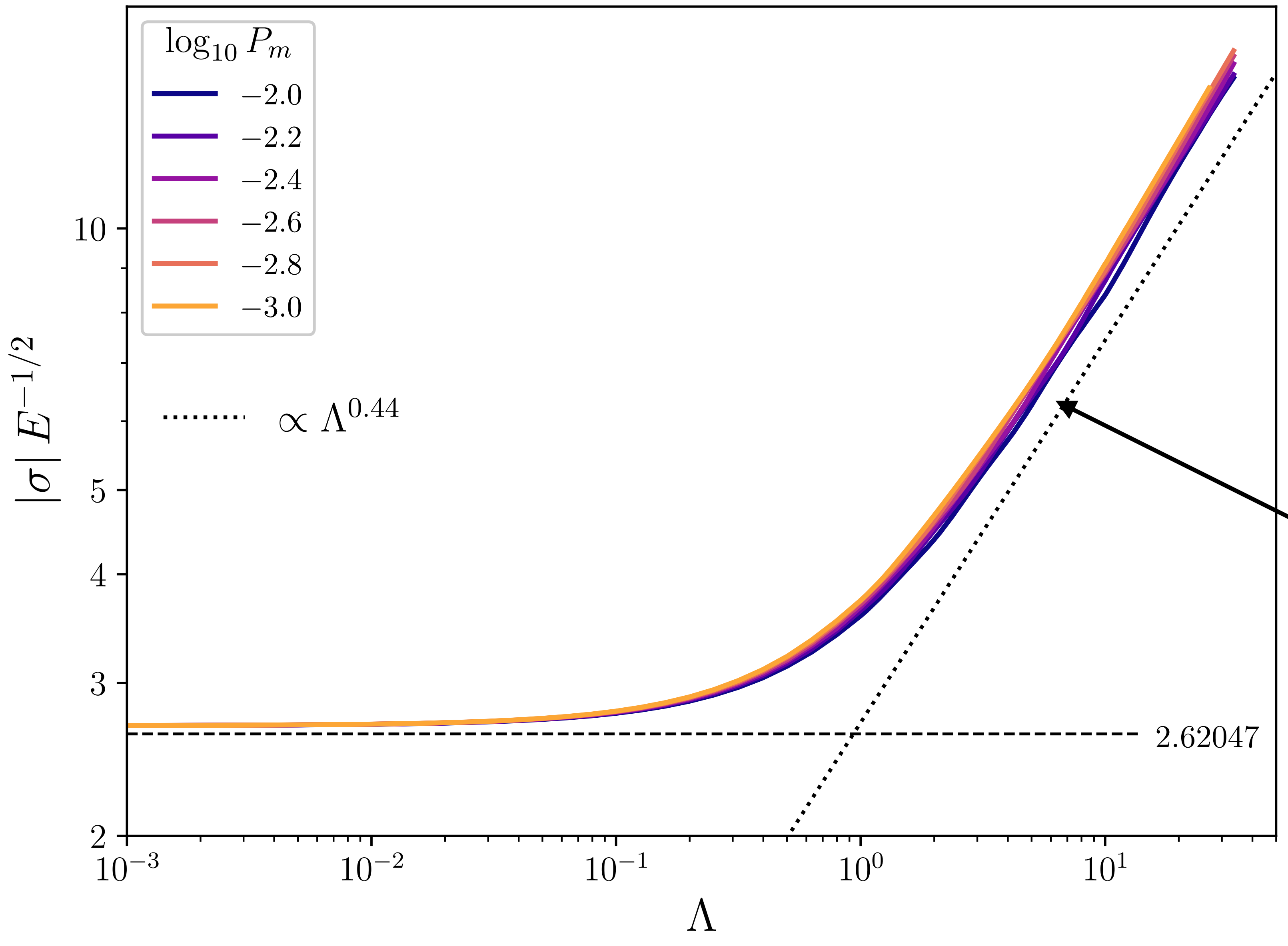
No slip, insulating mantle

$$\mathbf{u} \propto \mathrm{e}^{(\sigma + \mathbf{i}\omega)\mathbf{t}}$$

- Solve and find the proper eigenmode
- Compute its Ohmic and viscous dissipation

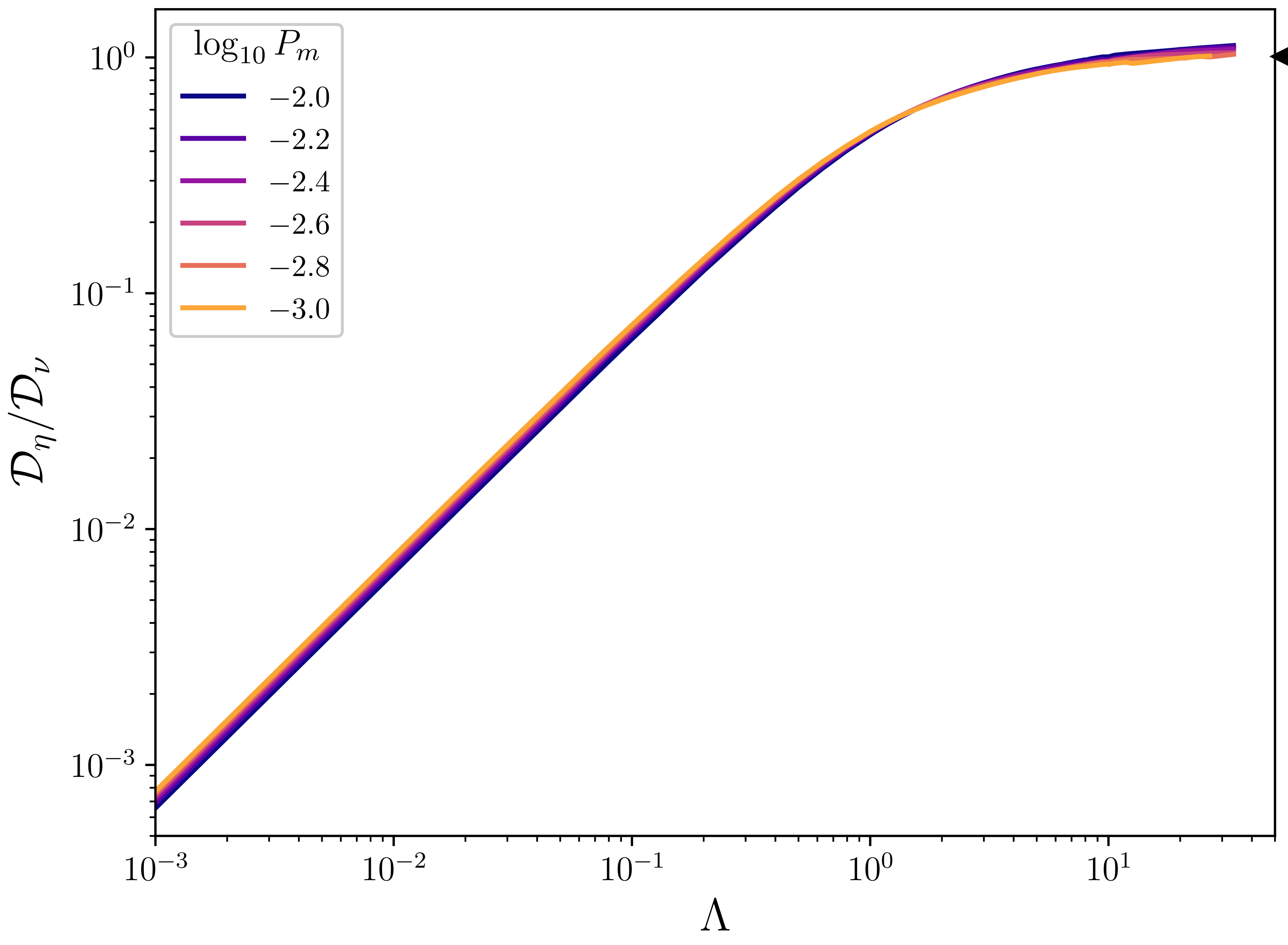
An asymptotic regime where $\sigma \propto E^{1/2}$ (flat curves below) is reached at different E, depending on Λ and P_m





When $\Lambda \gtrsim \mathcal{O}(1)$, as in the Earth's core, the total damping (Ohmic+viscous) follows a well defined scaling law

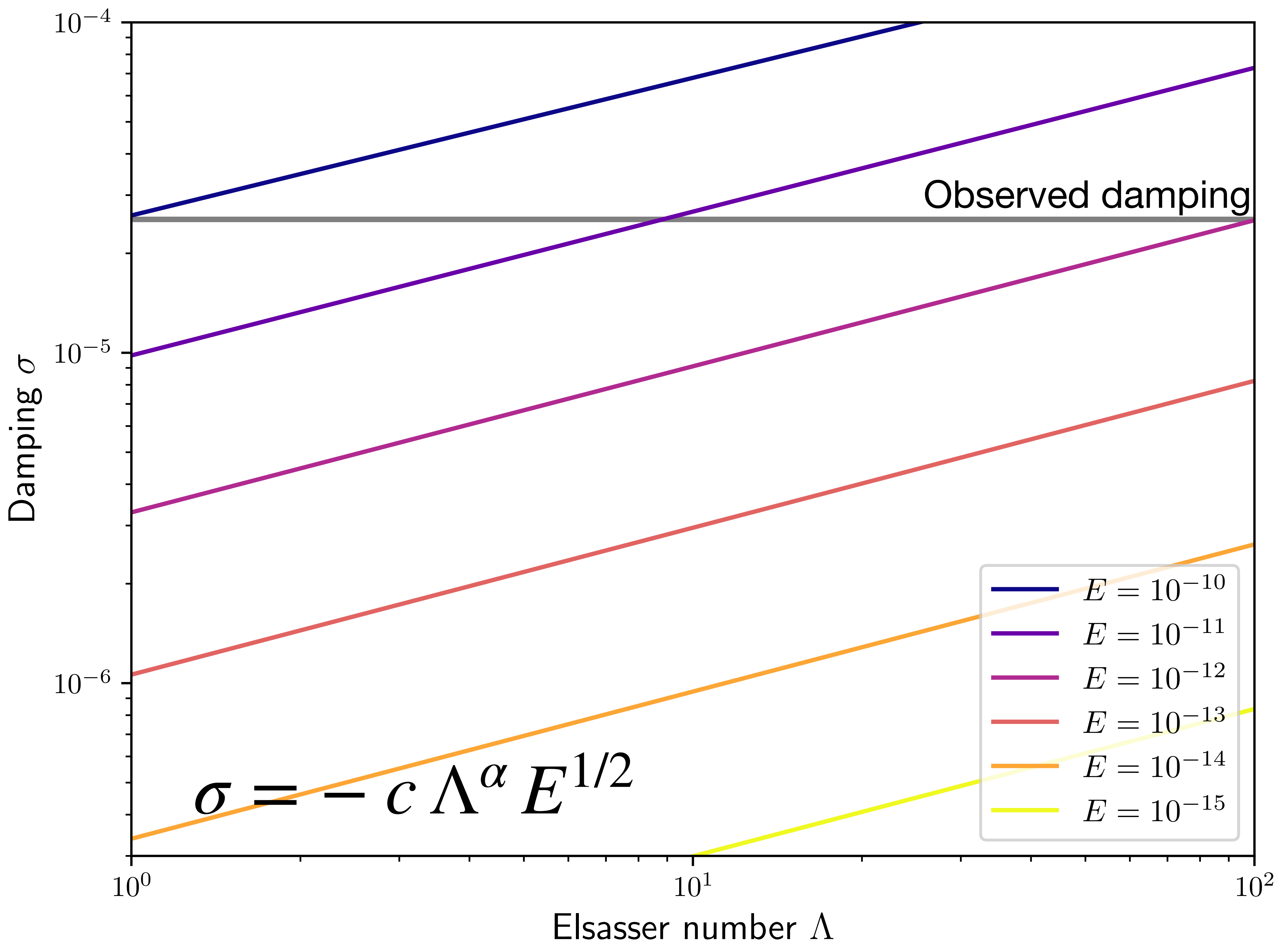
$$\sigma = -c\, \Lambda^{\alpha}\, E^{1/2}$$



The ratio between Ohmic and viscous dissipation becomes $\mathcal{D}_\eta / \mathcal{D}_\nu \approx 1$ regardless of Λ

Almost all the dissipation, whether Ohmic or viscous, takes place within the boundary layers.

Conclusion



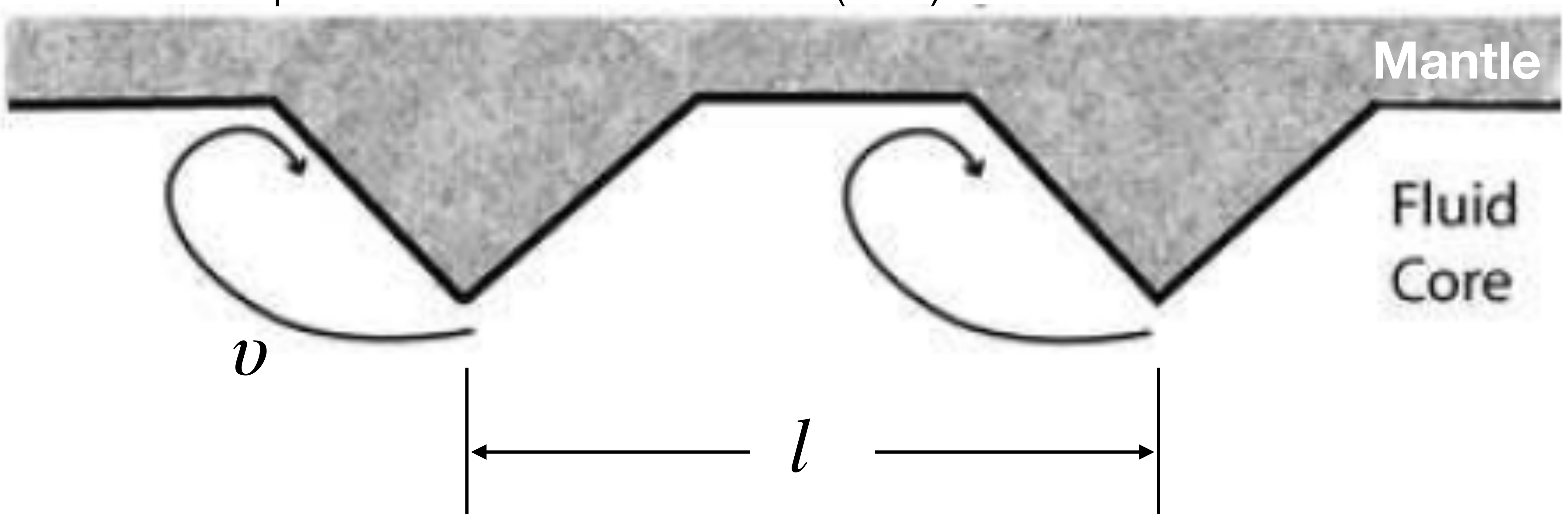
Code available here:
https://bitbucket.org/repepo/kore/src/master/README.md

kore

- The magnetic field required to match the observed damping is two orders of magnitude higher than other estimates (e.g. Gillet+2010), if we use a molecular diffusivity rather than an eddy viscosity.
- To keep a magnetic field strength in line with other estimates, we need an eddy diffusivity $\nu_e \approx 0.01\mathrm{m}^2/\mathrm{s}$, or an Ekman number $E \approx 10^{-11}$.
- Such an eddy viscosity is realised if the core surface velocity from Earth's precession is considered ($\sim 2\,\mathrm{mm/s}$), and choose an integral length scale of $\sim 5\,\mathrm{m}$.
- **Thus, given the observed FCN damping, our results are a strong indication that the boundary layers, at least at the CMB, are the seat of significant amounts of turbulence, and that the associated integral length scale can be used to constrain the characteristic length scale of the CMB roughness.**

CMB roughness induces turbulence

Schematic adapted from Buffett & Christensen (2007)



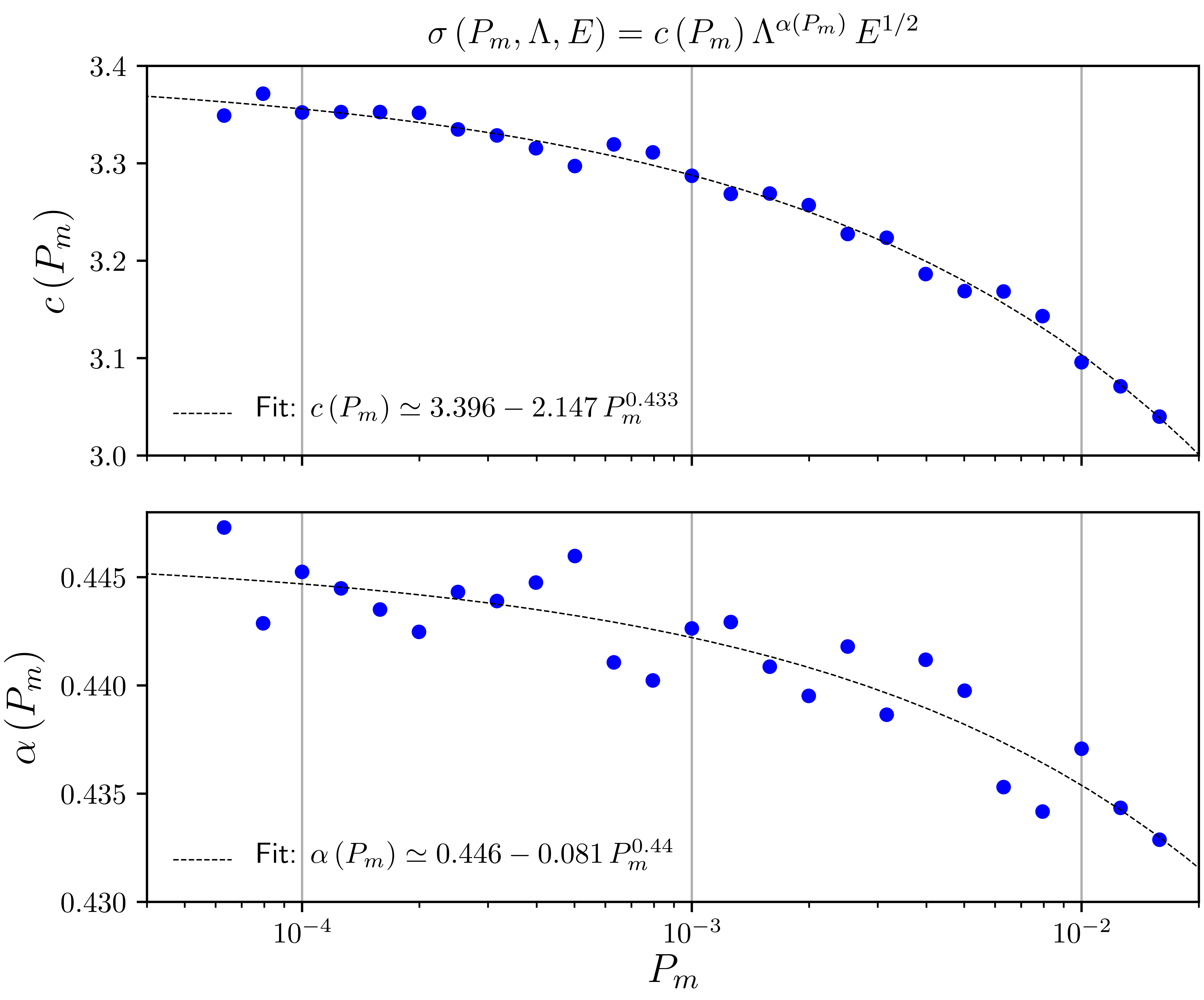
Inferred core surface velocity v

- From downward extrapolation of the secular variation of the magnetic field (observed at the Earth's surface)
  $v \approx 0.1\,\mathrm{mm/s}$
- From FCN's differential motion
  $v \approx 0.7\,\mathrm{mm/s}$
- From Earth's precession $v \approx 2\,\mathrm{mm/s}$

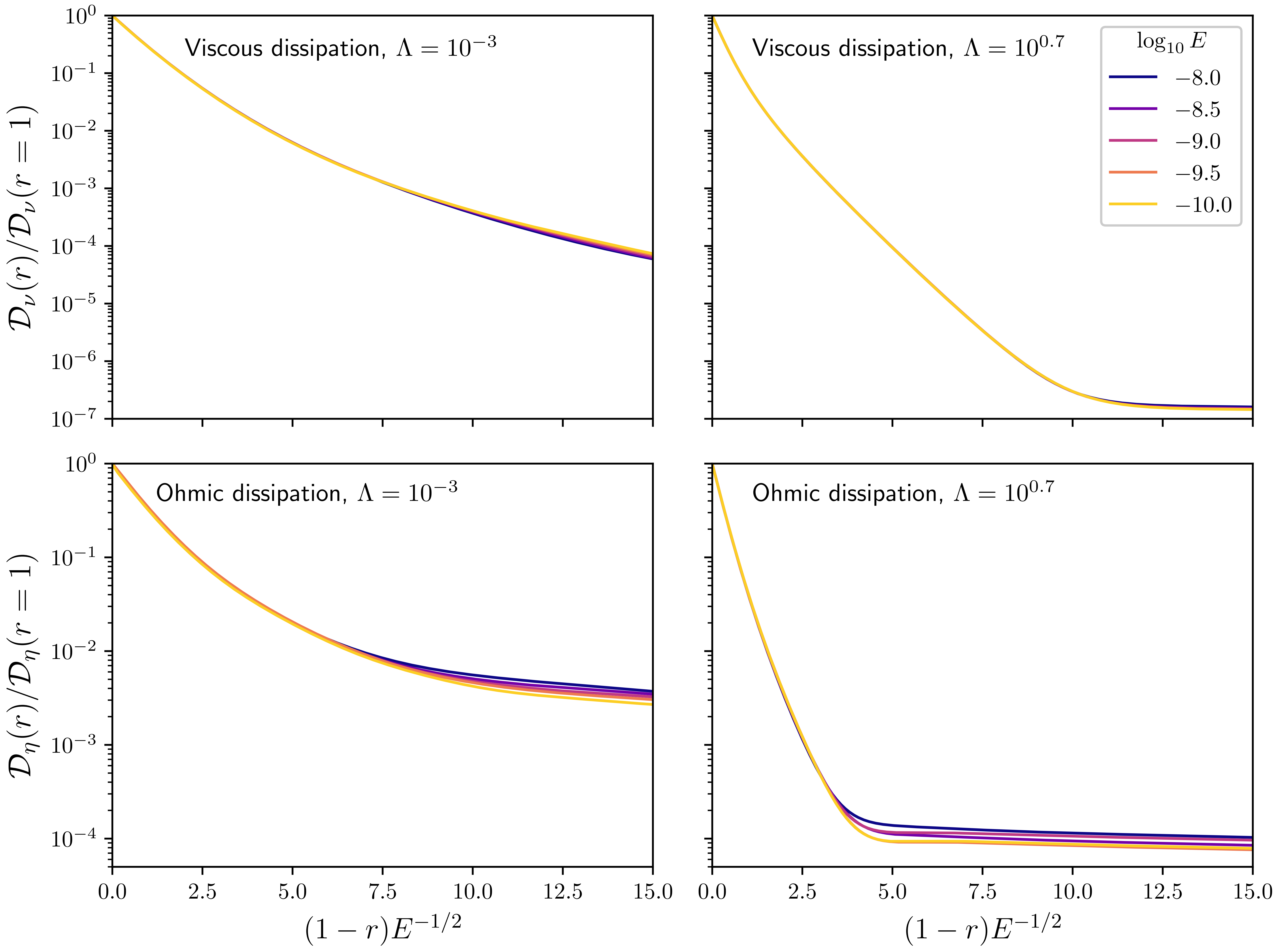
Taking $v \approx 2\,\mathrm{mm/s}$ as inferred from Earth's precession, and $\nu_e \approx 0.01\mathrm{m^2/s}$ as constrained by observations, then $l \approx 5\,\mathrm{m}$

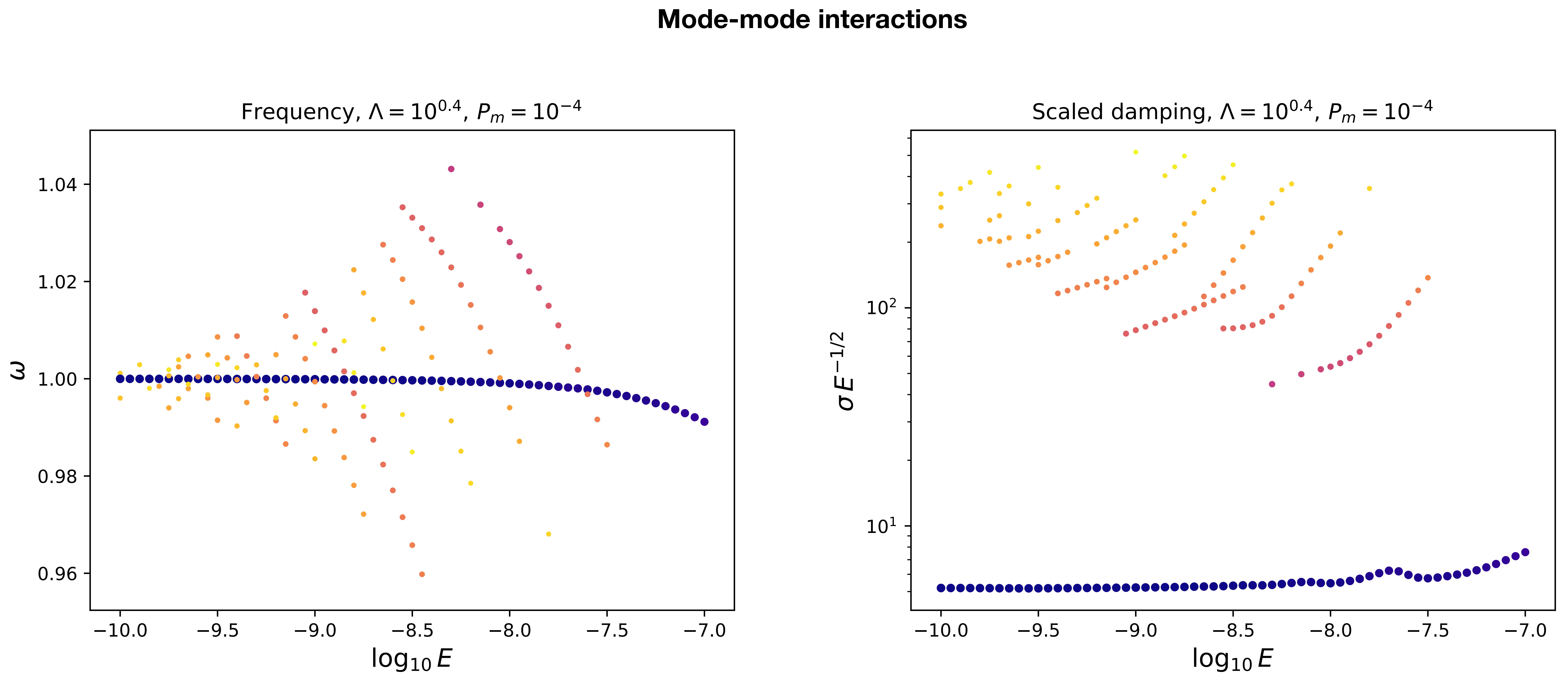
Eddy viscosity is $\nu_e \approx l\,v$

The damping in the $\Lambda \gtrsim \mathcal{O}(1)$ regime

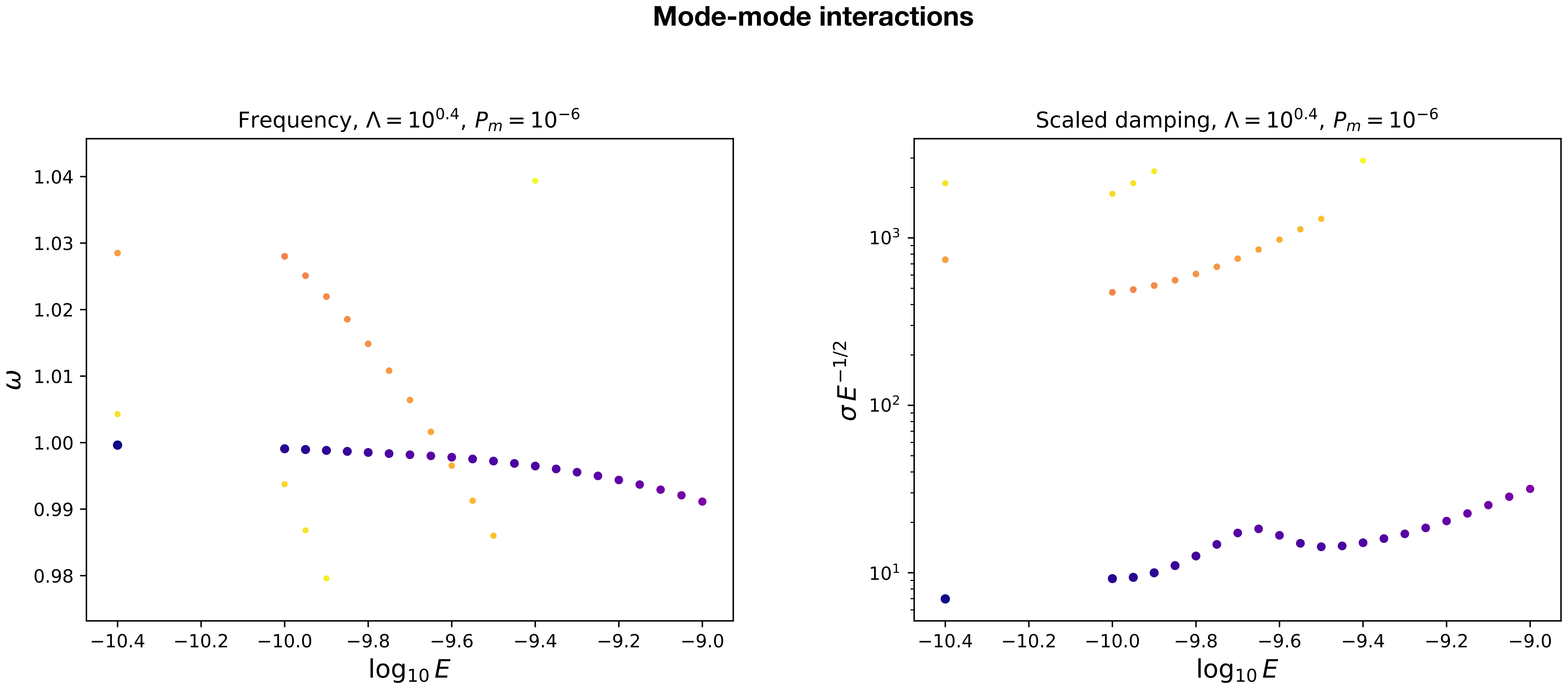


Almost all the dissipation, whether Ohmic or viscous, takes place within the boundary layers.





Mode-mode interactions
Frequency, $\Lambda = 10^{0.4}$, $P_m = 10^{-4}$
ω
1.04
1.02
1.00
0.98
0.96
−10.0
−9.5
−9.0
−8.5
−8.0
−7.5
−7.0
$\log_{10} E$
Scaled damping, $\Lambda = 10^{0.4}$, $P_m = 10^{-4}$
$\sigma E^{-1/2}$
10^2
10^1
−10.0
−9.5
−9.0
−8.5
−8.0
−7.5
−7.0
$\log_{10} E$


Mode-mode interactions
Frequency, $\Lambda = 10^{0.4}$, $P_m = 10^{-6}$
ω
1.04
1.03
1.02
1.01
1.00
0.99
0.98
−10.4
−10.2
−10.0
−9.8
−9.6
−9.4
−9.2
−9.0
$\log_{10} E$
Scaled damping, $\Lambda = 10^{0.4}$, $P_m = 10^{-6}$
$\sigma E^{-1/2}$
10^3
10^2
10^1
−10.4
−10.2
−10.0
−9.8
−9.6
−9.4
−9.2
−9.0
$\log_{10} E$