Towards coupling fluid flow and rate-and-state friction in compacting visco-poro-elasto-plastic reservoirs

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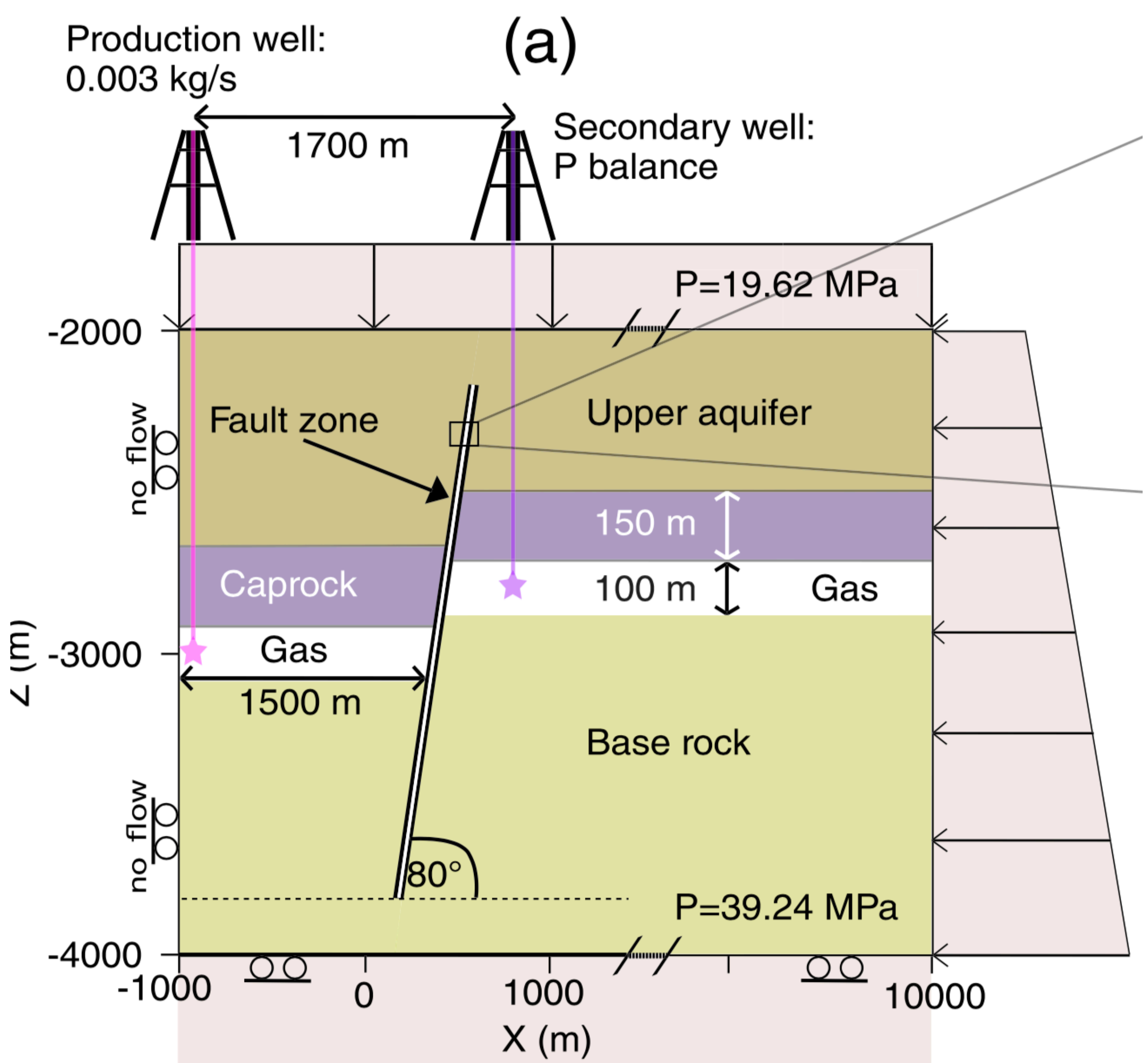
Physics

- Induced seismicity due to the production of gas from Groningen fields
- Multiphase fluid flow involved in natural gas extraction activities should be included
- Solid deformation, water and gas flow under non-isothermal conditions
- Viscous rheology and large deformations
- Porous domain with natural discontinuities (discrete faults)



Porous media formulation

Momentum balance equation (solid) (Lagrangian form):

$$\frac{\partial \sigma'_{ij}}{x_j} - \frac{\partial p_t}{x_i} + \rho_t g_i = (1-\phi)\rho_s \frac{D^s v_i^s}{Dt} + \phi \rho_f \frac{D^f v_i^f}{Dt}$$

Momentum balance equations (fluid) (Lagrangian form):

$$v_i^D = -\frac{k}{\eta_f}(\nabla p_f - \rho_f g_i)$$

Porous media formulation

Mass conservation equations

Solid

$$\frac{D^s((1-\phi)\rho_s)}{Dt} + \rho_s(1-\phi)\,\mathbf{div}(\bar{\mathbf{v}}^s) = 0$$

Fluid

$$\frac{D^f(\phi\rho_f)}{Dt} + (\phi\rho_f)\,\mathbf{div}(\bar{\mathbf{v}}^f) = 0$$

introducing equations of state and viscous effects
[Gerya, 2019; Yarushina and Podladchikov, 2015]

Solid

$$\nabla . v^s = -\frac{1}{K_d}\left(\frac{D^s p_t}{Dt} - \alpha\frac{D^f p_f}{Dt}\right) - \frac{p_t - p_f}{\eta_\phi(1-\phi)}$$

Fluid

$$\nabla . v^D = \frac{\alpha}{K_d}\left(\frac{D^s p_t}{Dt} - \frac{1}{B}\frac{D^f p_f}{Dt}\right) + \frac{p_t - p_f}{\eta_\phi(1-\phi)}$$

$$\frac{1}{\rho^s}\frac{D^s\rho^s}{Dt} = -\frac{1}{V_s}\frac{D^s V_s}{Dt} = \frac{1}{K_s}\frac{D^s p^s}{Dt} - \beta_s\frac{D^s T}{Dt} - \frac{1}{3(n-1)K_s}\frac{D^s(\mathrm{tr}\,\boldsymbol{\sigma}')}{Dt}$$

Porous media formulation

$$\frac{\partial \sigma'_{ij}}{x_j} - \frac{\partial p_t}{x_i} + \rho_t g_i = (1-\phi)\rho_s \frac{D^s v_i^s}{Dt} + \phi \rho_f \frac{D^f v_i^f}{Dt}$$

$$v_i^D = -\frac{k}{\eta_f}(\nabla p_f - \rho_f g_i)$$

$p_t,\ p_f$

v_x^s, v_x^D

v_y^s, v_y^D

$$\nabla \,.\, v^s = -\frac{1}{K_d}(\frac{D^s p_t}{Dt} - \alpha \frac{D^f p_f}{Dt}) - \frac{p_t - p_f}{\eta_\phi (1-\phi)}$$

$$\nabla \,.\, v^D = \frac{\alpha}{K_d}(\frac{D^s p_t}{Dt} - \frac{1}{\beta}\frac{D^f p_f}{Dt}) + \frac{p_t - p_f}{\eta_\phi (1-\phi)}$$

$$\eta_\varphi = \frac{2m}{(m+1)}\frac{\eta_s}{\varphi} \qquad \alpha = 1 - \frac{K_d}{K_s} \qquad K_d = (1-\varphi)\left(\frac{1}{K_\varphi} + \frac{1}{K_s}\right)^{-1} \qquad \beta = \frac{\frac{1}{K_d} - \frac{1}{K_s}}{\frac{1}{K_d} - \frac{1}{K_s} + \varphi\left(\frac{1}{K_f} - \frac{1}{K_s}\right)}$$

$$K_\varphi = \frac{2m}{(m+1)}\frac{G}{\varphi} \qquad k = k_0\left(\frac{\varphi}{\varphi_0}\right)^m \qquad G = G_0\left(1 - \frac{\varphi}{\varphi_{crit}}\right)$$

Constitutive relations:

Maxwell visco-elasto-plastic material

$$\dot{\epsilon}_{total} = \dot{\epsilon}_{elastic} + \dot{\epsilon}_{viscous} + \dot{\epsilon}_{plastic}$$

$$\dot{\epsilon}_{ij(elastic)} = \frac{1}{2G}\frac{D^s\sigma'_{ij}}{Dt} \qquad \dot{\epsilon}_{ij(viscous)} = \frac{1}{2\eta}\sigma'_{ij} \qquad \dot{\epsilon}'_{ij(plastic)} = \dot{\lambda}\frac{\partial Q}{\partial \sigma'} \quad , \quad \sigma'_{II} = \sigma_{yield}$$

The Drucker-Prager yielding condition

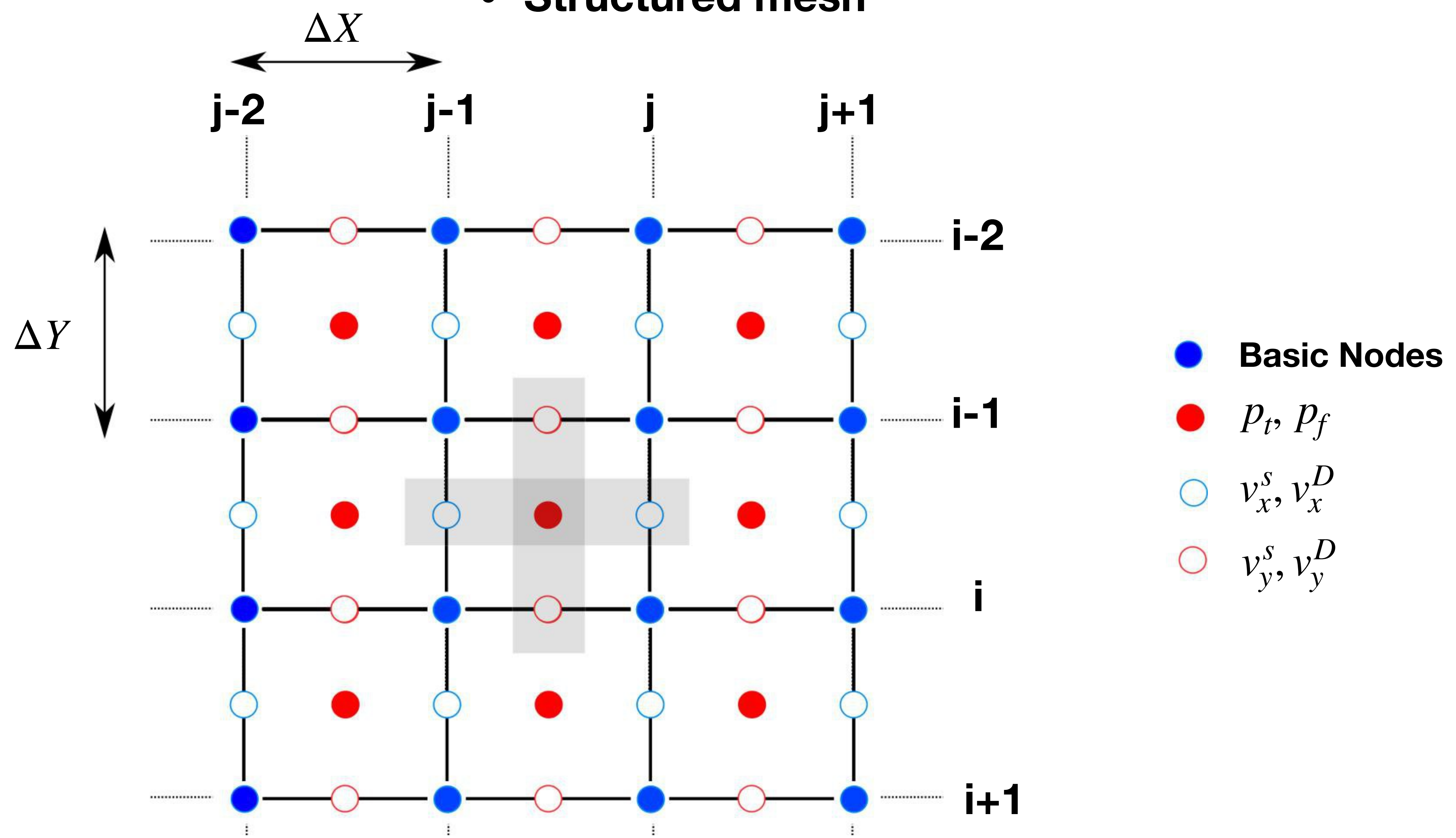
$$\sigma_{yield} = \sigma_c + \mu(p_t - p_f)$$

Mesh objectivity, rate and state dependent friction

$$\mu = f(V, \theta)$$

Finite difference method on fully staggered grids

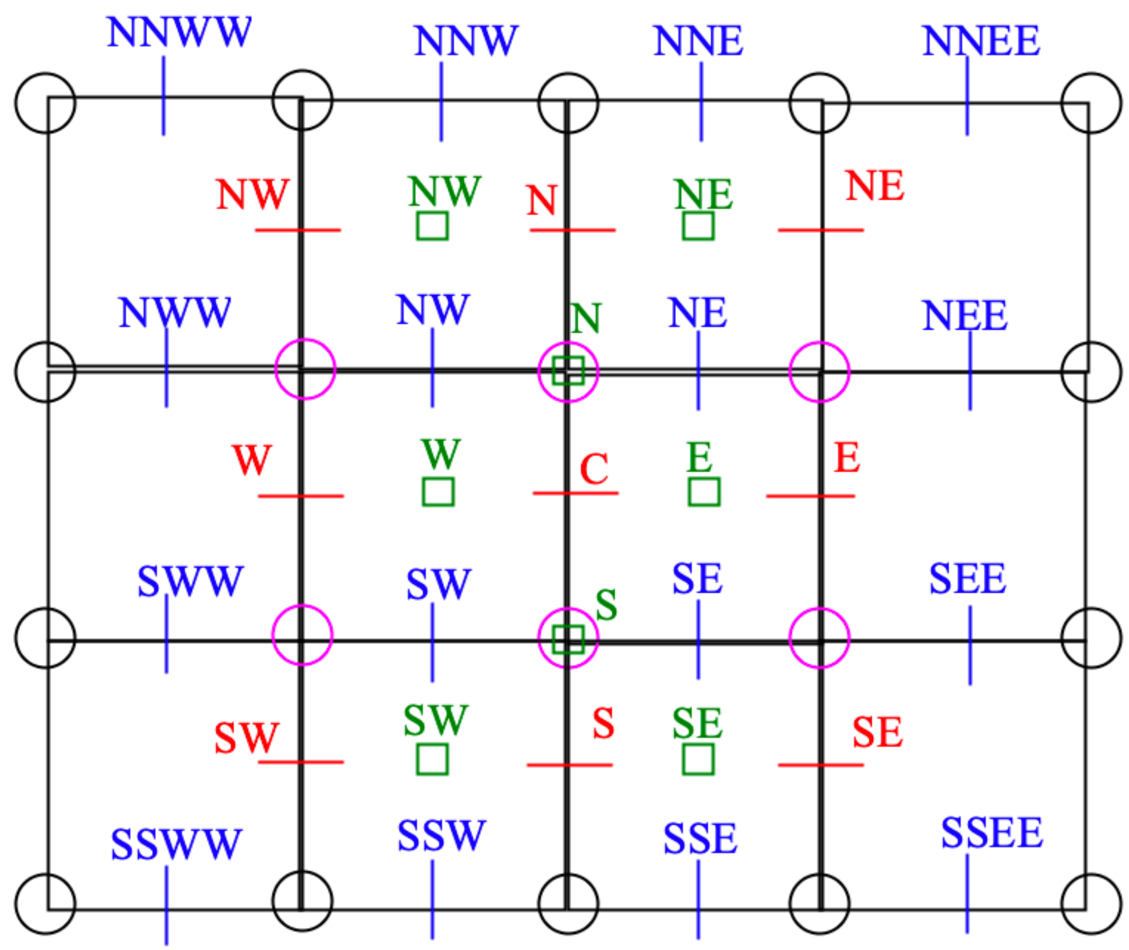
- **Structured mesh**



$$\frac{\left(v^s_{x(i,j)} - v^s_{x(i,j-1)}\right)}{dx} + \frac{\left(v^s_{y(i,j)} - v^s_{y(i-1,j)}\right)}{dy}$$

$$= -\frac{1}{K_{\varphi(i,j)}\left(1-\varphi_{(i,j)}\right)}\left(\frac{\left(p_{t(i,j)} - p_{t0(i,j)}\right)}{dt} - \frac{\left(p_{f(i,j)} - p_{f0(i,j)}\right)}{dt}\right) - \frac{p_{t(i,j)} - p_{f(i,j)}}{\eta_{\varphi(i,j)}\left(1-\varphi_{(i,j)}\right)}$$

FDM stencil for elasticity equation in x direction (point C)



$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{yy} \end{bmatrix} = \begin{bmatrix} D_{11} D_{12} D_{13} \\ D_{21} D_{22} D_{23} \\ D_{31} D_{32} D_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{xy} \\ \epsilon_{yy} \end{bmatrix}$$

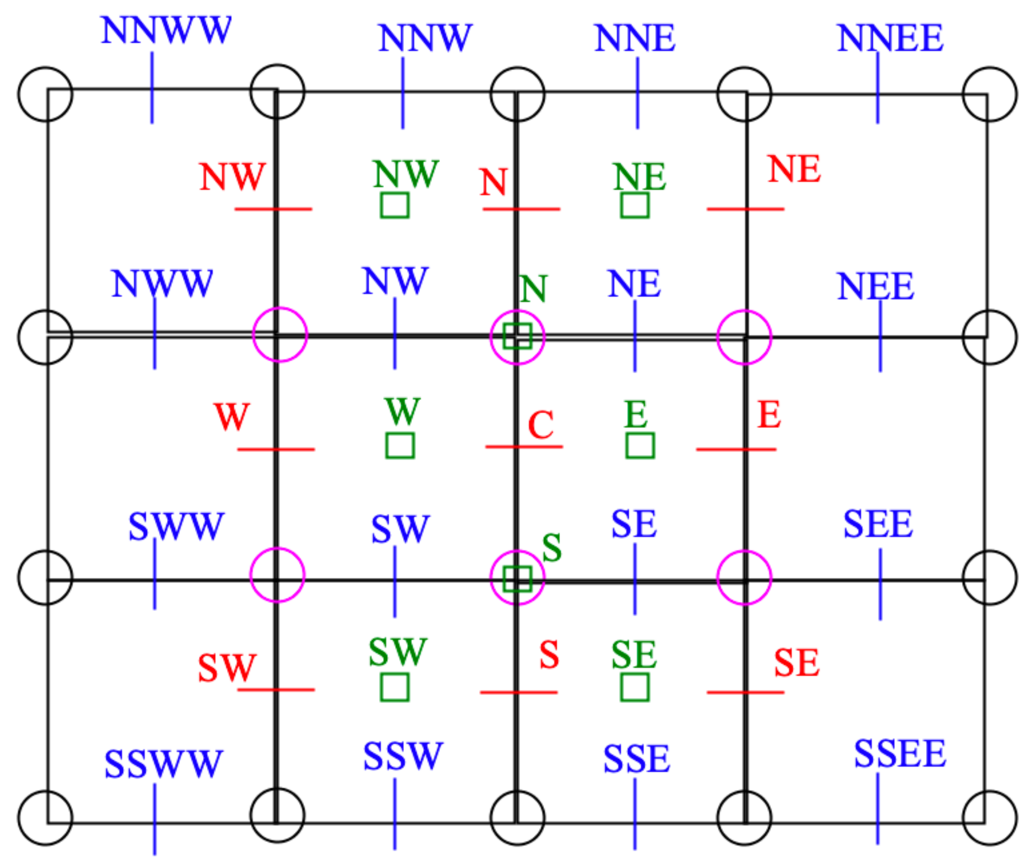
—: ux

| : uy

FDM stencil for elasticity equation in x direction (point C)

$$\epsilon_{xx} = \begin{bmatrix} \frac{\mathrm{UxN} - \mathrm{UxNW}}{\mathrm{dx}} & \frac{\mathrm{UxNE} - \mathrm{UxN}}{\mathrm{dx}} \\ \frac{\mathrm{UxC} - \mathrm{UxW}}{\mathrm{dx}} & \frac{\mathrm{UxE} - \mathrm{UxC}}{\mathrm{dx}} \\ \frac{\mathrm{UxS} - \mathrm{UxSW}}{\mathrm{dx}} & \frac{\mathrm{UxSE} - \mathrm{UxS}}{\mathrm{dx}} \end{bmatrix}$$

$$\epsilon_{yy} = \begin{bmatrix} \frac{\mathrm{UyNNW} - \mathrm{UyNW}}{\mathrm{dy}} & \frac{\mathrm{UyNNE} - \mathrm{UyNE}}{\mathrm{dy}} \\ \frac{\mathrm{UyNW} - \mathrm{UySW}}{\mathrm{dy}} & \frac{\mathrm{UyNE} - \mathrm{UySE}}{\mathrm{dy}} \\ \frac{\mathrm{UySW} - \mathrm{UySSW}}{\mathrm{dy}} & \frac{\mathrm{UySE} - \mathrm{UySSE}}{\mathrm{dy}} \end{bmatrix}$$



$$\epsilon_{xy} = \begin{bmatrix} \frac{(1-\mathrm{fsxN})(\mathrm{UxNW}-\mathrm{UxW})}{\mathrm{dy}} + \frac{(1-\mathrm{fsxW})(\mathrm{UyNW}-\mathrm{UyNWW})}{\mathrm{dx}} & \frac{(1-\mathrm{fsxN})(\mathrm{UxN}-\mathrm{UxC})}{\mathrm{dy}} + \frac{\mathrm{UyNE}-\mathrm{UyNW}}{\mathrm{dx}} & \frac{(1-\mathrm{fsxN})(\mathrm{UxNE}-\mathrm{UxE})}{\mathrm{dy}} + \frac{(1-\mathrm{fsxE})(\mathrm{UyNEE}-\mathrm{UyNE})}{\mathrm{dx}} \\ \frac{(1-\mathrm{fsxS})(\mathrm{UxW}-\mathrm{UxSW})}{\mathrm{dy}} + \frac{(1-\mathrm{fsxW})(\mathrm{UySW}-\mathrm{UySWW})}{\mathrm{dx}} & \frac{(1-\mathrm{fsxS})(\mathrm{UxC}-\mathrm{UxS})}{\mathrm{dy}} + \frac{\mathrm{UySE}-\mathrm{UySW}}{\mathrm{dx}} & \frac{(1-\mathrm{fsxS})(\mathrm{UxE}-\mathrm{UxSE})}{\mathrm{dy}} + \frac{(1-\mathrm{fsxE})(\mathrm{UySEE}-\mathrm{UySE})}{\mathrm{dx}} \end{bmatrix}$$

Non-linear solution algorithm

first-order truncated Taylor series

Newton-Raphson
(enhanced convergence)

$$\mathbf{r}^{k-1} = \mathbf{f}^{int}(\mathbf{a}_{n+1}^{k-1}) - \mathbf{f}_{n+1}^{ext}$$

$$\mathbf{r}^{k,n+1} = \mathbf{r}^{k-1,n+1} + \mathbf{K}\,\delta\mathbf{a}^{k,n+1} = 0$$

$$\mathbf{K}\,\delta\mathbf{a}^{k} = -\,\mathbf{r}^{k-1}$$

$$\mathbf{K} = \frac{\partial\mathbf{r}}{\partial\mathbf{a}_{n+1}}$$

$$\mathbf{a}_{n+1}^{k} = \mathbf{a}_{n+1}^{k-1} + \delta\mathbf{a}^{k}\,.$$

Accurate stress evolution: a return mapping algorithm

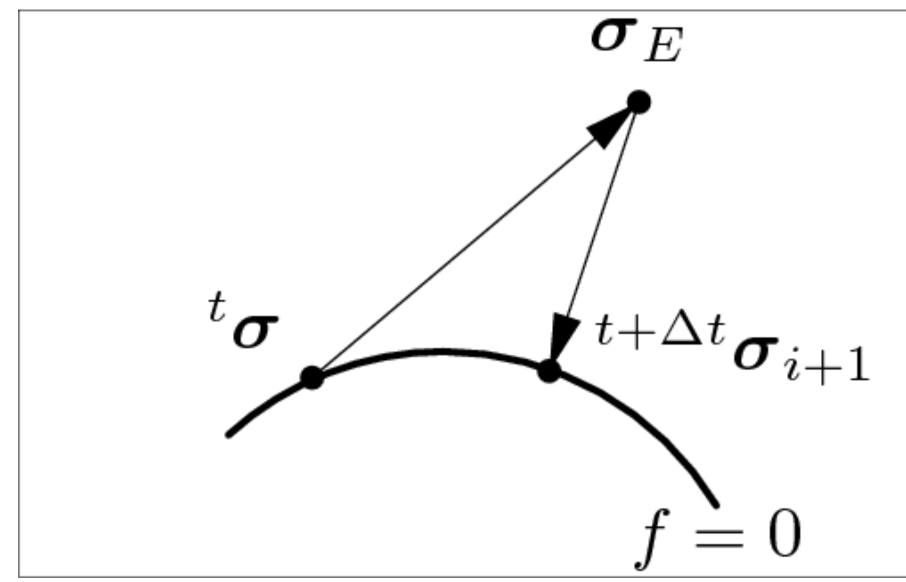
Implicit Euler backward algorithm (Newton-Raphson iterations) for non-associative plasticity

$$\dot{\sigma} = D^e(\dot{\epsilon} - \dot{\epsilon}^p) \qquad \dot{\epsilon}^p = \dot{\lambda}\frac{\partial Q}{\partial \sigma}$$

$$f(V, p_f) = \sqrt{J_2} - \sigma_{yield} = 0$$

$$\dot{f}(V, p_f) = 0$$

Return-mapping algorithm



Finite Thickness of Shear Bands in Frictional Viscoplasticity and Implications for Lithosphere Dynamics
Duretz, de Borst, Le Pourhiet, 2019.
Kelvin type and rate dependent yield functions

Rate-dependent plasticity with rate-and-state dependent friction

$$\dot{\epsilon} = \dot{\epsilon}^{e} + \dot{\epsilon}^{vp}$$

$$\dot{\epsilon}^{vp} = \dot{\lambda}\frac{\partial Q}{\partial \sigma} \quad , \quad Q = \sqrt{J_2} - P\, sin(\psi)$$

$$F = \sqrt{J_2} - C f_1 - P f_2 - \eta^{vp}\dot{\lambda} = 0$$

$$f_2 = a\,\textbf{arcsinh}\left[\frac{V_p}{2V_0}\textbf{exp}\left(\frac{\mu_0 + b\,\textbf{ln}(\frac{\theta V_0}{L})}{a}\right)\right] \quad , \quad \frac{\textbf{d}\theta}{\textbf{d}t} = 1 - \frac{V_p\,\theta}{L}.$$

$$V_p = 2\dot{\epsilon}'_{II}(p)D. \qquad \frac{\theta^{n+1} - \theta^{n}}{\Delta t} = 1 - \frac{V_p \theta^{n+1}}{L}$$

$$F^{t+1} = \sqrt{J_{II}^{trial}} - G^{ve}\Delta\lambda - f_2(P^{trial}\Delta\lambda) - \eta^{vp}\frac{\Delta\lambda}{\Delta t} = 0$$

$$\sigma^{t+1} = (1 - \frac{G^{ve}\Delta\lambda}{\sqrt{J_{II}^{trial}}})\,\sigma^{trial} \quad , \quad F^{t+1} = 0$$

Rate-dependent plasticity with rate-and-state dependent friction

$$\dot{\epsilon}^{vp} - \frac{\Delta\lambda}{\Delta t}\frac{\partial Q}{\partial \sigma} = 0 \qquad \dot{\epsilon}^{vp} = \dot{\lambda}\frac{\partial Q}{\partial \sigma} \quad , \quad Q = \sqrt{J_2} - P\, sin(\psi)$$

$$f_2(\lambda) = a\, \textbf{arcsinh}\left[\frac{\alpha\dot{\epsilon}'_{II}(vp)}{2V_0}\textbf{exp}\left(\frac{\mu_0 + b\,\textbf{ln}(\frac{\theta^{n+1}V_0}{L})}{a}\right)\right] \quad , \quad \frac{\theta^{n+1} - \theta^n}{\Delta t} = 1 - \frac{V_p^{n+1}\theta^{n+1}}{L}$$

$$r_F = \sqrt{J_{II}^{trial}} - G^{ve}\Delta\lambda - f_2(P^{trial}\Delta\lambda) - \eta^{vp}\frac{\Delta\lambda}{\Delta t} = 0 \qquad V_p = \alpha\dot{\epsilon}'_{II}(p)$$

$$r_\sigma = \sigma^{t+1} - \sigma^t - D^{ve}\Delta\epsilon^{t+1} + \Delta\lambda D^{ve}\frac{\partial Q}{\partial \sigma} = 0$$

$$\frac{\partial r_f}{\partial \sigma}d\sigma + \frac{\partial r_f}{\partial \lambda}d\lambda = -\, r_F \qquad \frac{\partial r_\sigma}{\partial \sigma}d\sigma + \frac{\partial r_\sigma}{\partial \lambda}d\lambda = -\, r_\sigma$$

Ongoing…

- Rate-dependent plasticity with rate-and-state dependent friction
- Porous formulation, semi-saturated situation with/without third-phase, non-isothermal, compressibility, dilatancy
- Large scale problems in 3-D