A new Bayesian hierarchical geostatistical model based on two spatial fields with case studies with short records of annual runoff in Norway

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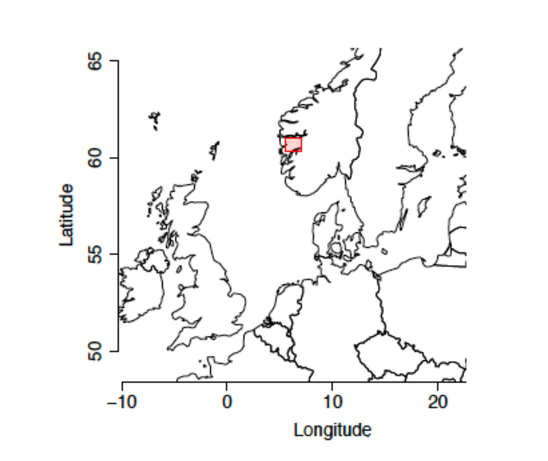
Norwegian University of Science and Technology

NVE

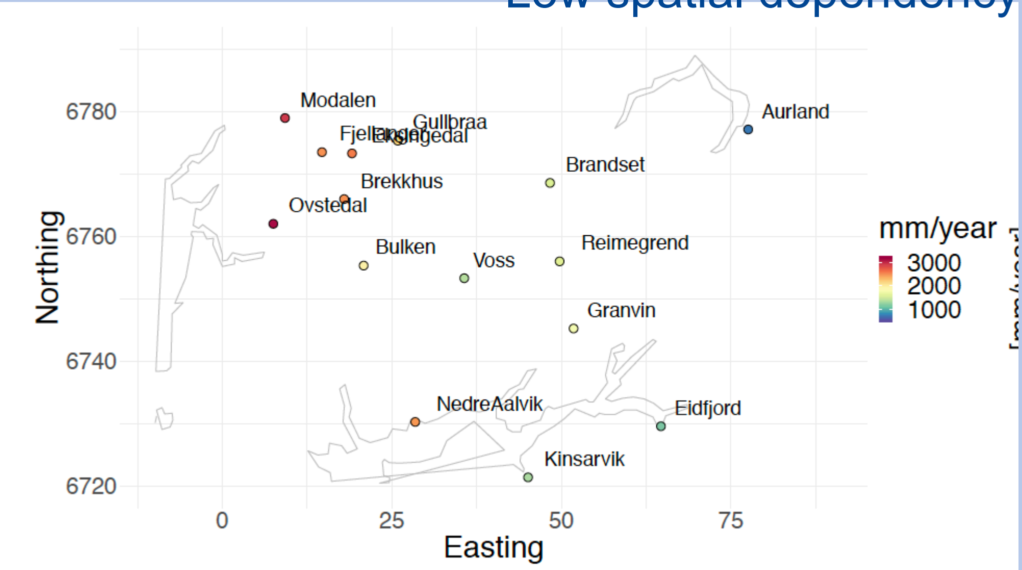
NTNU

Motivating example

Annual precipitation in Western Norway

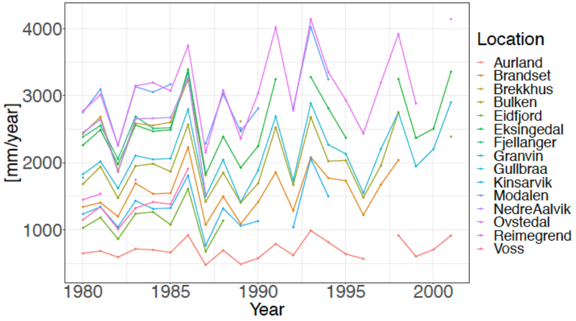


Low spatial dependency

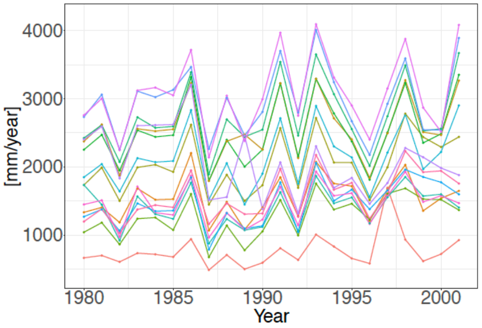


Low temporal dependency, but we find the same pattern each year

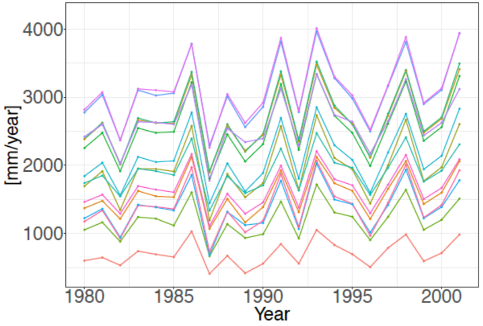
Data:



Traditional geostatistics:



Desired solution?



We have constructed a geostatistical method that is able to detect and capture such data patterns.

Model run-off location *u* year *j*

$$q_j(\boldsymbol{u}) = \beta_c + c(\boldsymbol{u}) + \beta_j + x_j(\boldsymbol{u}); \qquad \boldsymbol{u} \in \mathcal{R}^2; \qquad j = 1, ..r.$$

Model r years simultaneously.

Climate (long-term averages):

β_c : Intercept.

$c(\boldsymbol{u})$: Spatial effect. GRF(ρ_c,σ_c).

Annual discrepancy from climate:

β_j : Intercept. year j

$x_j(\boldsymbol{u})$: Spatial effect. GRF(ρ_x,σ_x).

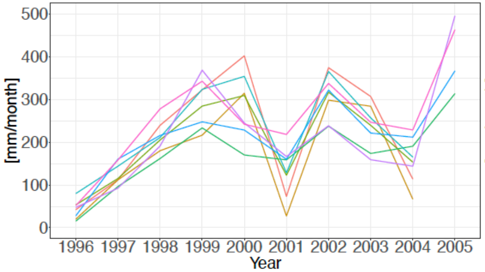
GRF: Gaussian random field. $\mathcal{N}(0, \boldsymbol{\Sigma}(\rho, \sigma^2))$.

$\boldsymbol{\Sigma}$: Covariance matrix.

ρ : Spatial range.

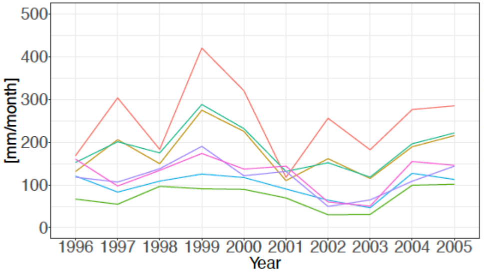
σ^2 : Spatial marginal variance.

Simulated examples from model



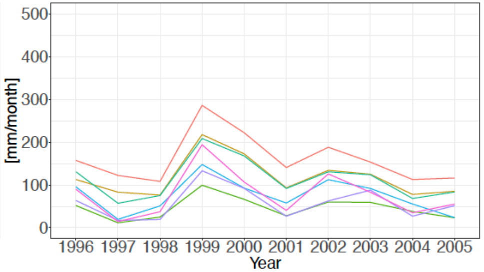
Climate < Annual

$\sigma_c < \sigma_x$



Climate ≈ Annual

$\sigma_x \approx \sigma_c$



Climate » Annual

$\sigma_c \gg \sigma_x.$

$$q_j(\boldsymbol{u}) = \beta_c + c(\boldsymbol{u}) + \beta_j + x_j(\boldsymbol{u}); \qquad \boldsymbol{u} \in \mathcal{R}^2; \qquad j = 1, ..r.$$

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For catchment $\mathcal{A}$

Areal model:
$Q_j(\mathcal{A}) = \frac{1}{|\mathcal{A}|} \int_{\boldsymbol{u} \in \mathcal{A}} q_j(\boldsymbol{u}) d\boldsymbol{u}.$

Mass-conservation.

Centroid model:
$Q_j(\mathcal{A}) = q_j(\boldsymbol{u}_{\mathcal{A}})$
$\boldsymbol{u}_{\mathcal{A}}$ is the catchment centroid.

No mass conservation, but quicker.

With the model framework we can:

- Model overlapping catchments consistently
- Combining areal (runoff) and point (precipitation) observations.
- Utilize short records
- Use informative priors in a Bayesian setting
- Fast inference using INLA (Integrated nested Laplace Approximations)

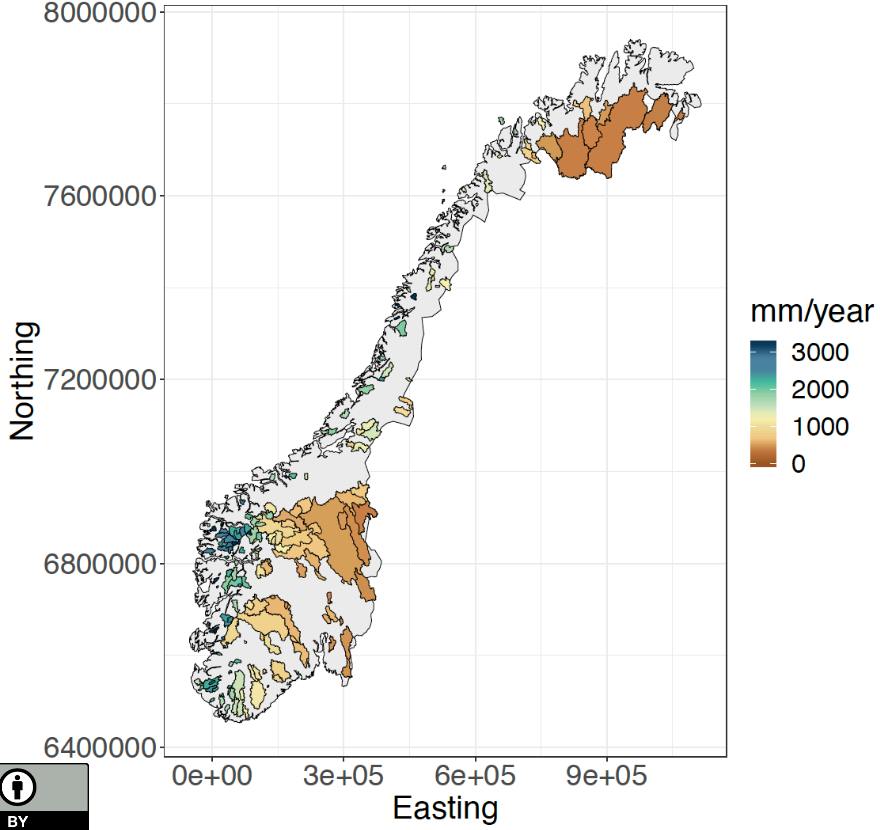
Papers available :

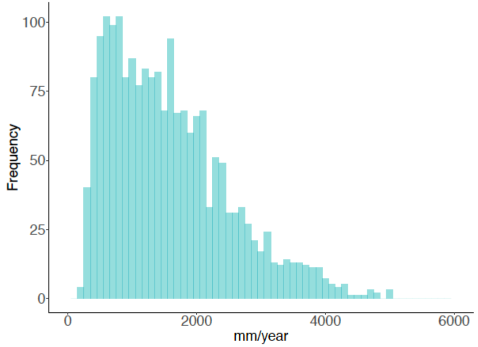
- Thea Roksvåg, Ingelin Steinsland, Kolbjørn Engeland , *A geostatistical two field model that combines point observations and nested areal observations, and quantifies long-term spatial variability -- A case study of annual runoff predictions in the Voss area* Under revision, available at arXiv:1904.02519

- Roksvåg, T., Steinsland, I., and Engeland, K.: *A geostatistical framework for estimating flow indices by exploiting short records and long-term spatial averages – Application to annual and monthly runoff*, Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2019-415, in review, 2019.

Example using short records

Test the areal and centroid model on Norwegian annual runoff data.





200 catchments.
Data from 1996-2005.

Divide the catchments into 20 groups/folds.

For each cross-validation fold we:

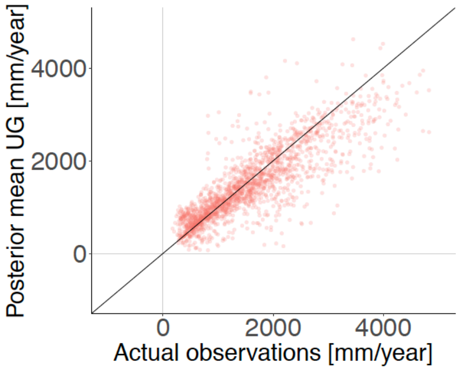
- Remove data.
- Predict the missing values for individual years $\in \{1996, 2005\}$.
- Evaluate the predictive performance of the areal and centroid model, compare to Top-Kriging.

Ungauged catchments (UG): Treat the target catchments as ungauged.

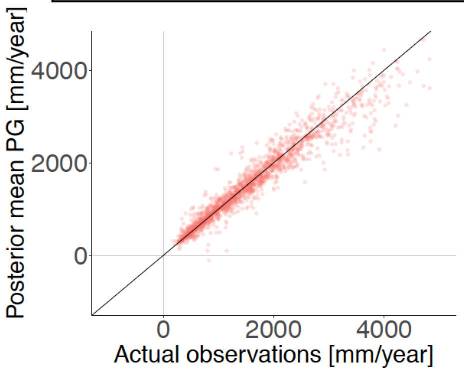
Partially gauged catchments (PG): Include one random observation from the target catchment from one of the years $\in \{1996, 2005\}$. Assess the value of short records.

Results: Partially gauged catchments (PG)

$\sigma_c = 902$ mm/year. $\sigma_c^2/(\sigma_c^2 + \sigma_x^2) = 90\%$.
$\sigma_x = 270$ mm/year. The climatic effects are dominating.

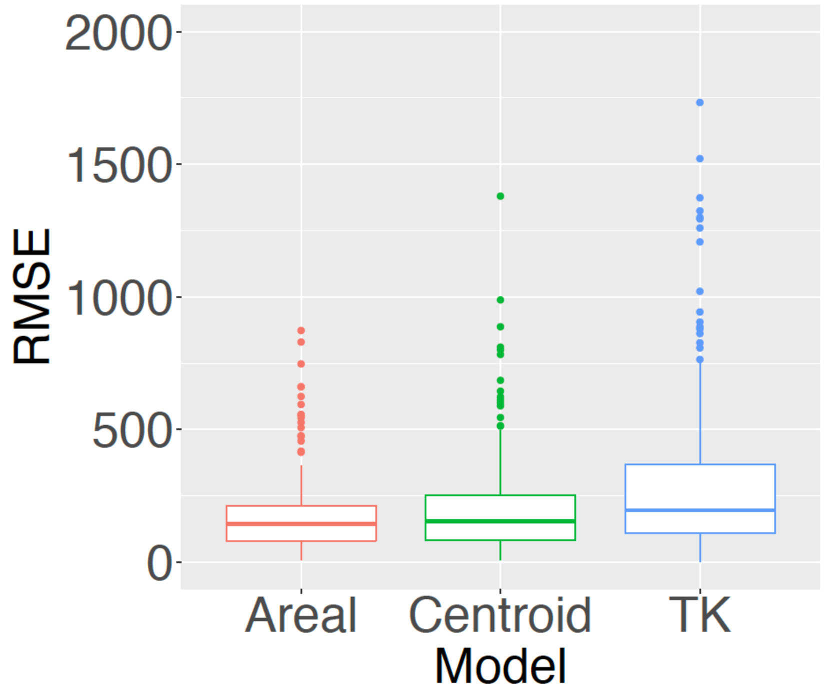


UG for the areal model.
RMSE=363 mm/year.



PG for the areal model.
RMSE= 184 mm/year.

The average reduction in RMSE is around 50 % when adding a short-record of length 1.



The areal and centroid models outperform Top-Kriging for partially gauged catchments.

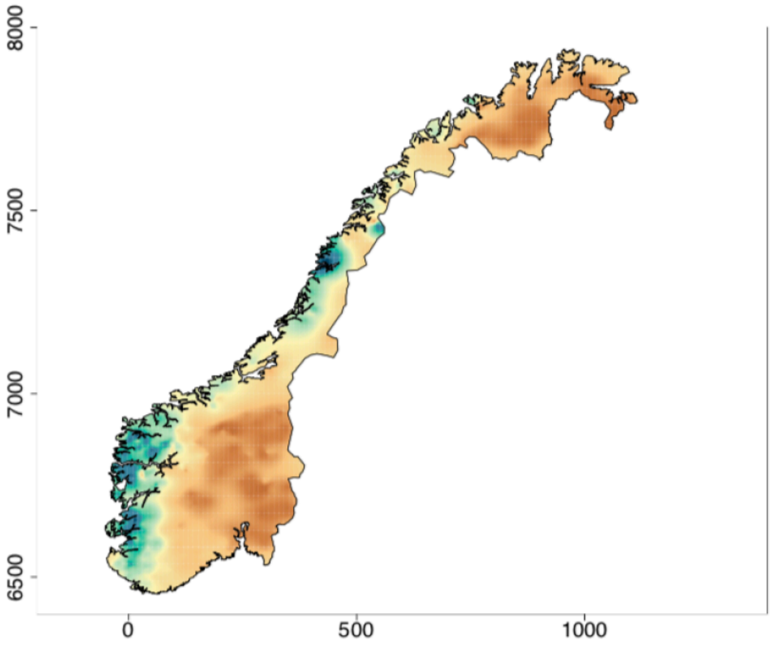
Top-Kriging treats each year individually, while we model 10 years simultaneously.

Conclusions short records

The spatial variability is stable in Norway over years ($\sigma_c >> \sigma_x$).

→ There is a lot of information stored in the short runoff records in Norway.

→ Comparing UG and PG, the reduction in RMSE was on average 50% when adding a short-record of length 1.



Thank you!

Want to read more?

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