Spectral fabric model coupling grain orientation, grain volume, and lattice strain-energy distribution functions

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Grain **number** distribution (ODF)

$$\dot n = -\dot n_{\mathcal{R}} + \dot n_{\mathcal{G}} + \dot n_{\mathcal{M}} + \dot n_{\mathcal{S}}$$

Volume distribution

$$\dot v = -\dot v_{\mathcal{R}} + \dot v_{\mathcal{G}} + \dot v_{\mathcal{M}} + \dot v_{\mathcal{S}}$$

Strain-energy distribution

$$\dot w = -\dot w_{\mathcal{R}} + \dot w_{\mathcal{G}} + \dot w_{\mathcal{M}} + \dot w_{\mathcal{S}} + \dot w_{C}$$

Lattice rotation | Normal growth | Migration recryst. | Nucleation | Cold work

Ensemble total **number of grains (N), volume (V), and stored strain-energy (W)**:

$$N = \int_{S^2} n \,\mathrm{d}\Omega\,, \quad V = \int_{S^2} v \,\mathrm{d}\Omega\,, \quad W = \int_{S^2} w \,\mathrm{d}\Omega$$

Ensemble-average grain diameter:

$$\langle d \rangle = \frac{1}{4\pi} \int_{S^2} \left(\frac{6}{\pi} \frac{v}{n} \right)^{1/3} \mathrm{d}\Omega$$

Model assumes spherical grains

Distributions expanded in term of spherical harmonics, e.g.:

$$n = |\psi|^2$$

$$\psi(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_l^m Y_l^m(\theta,\phi)$$

Conforms with the parabolic growth law:

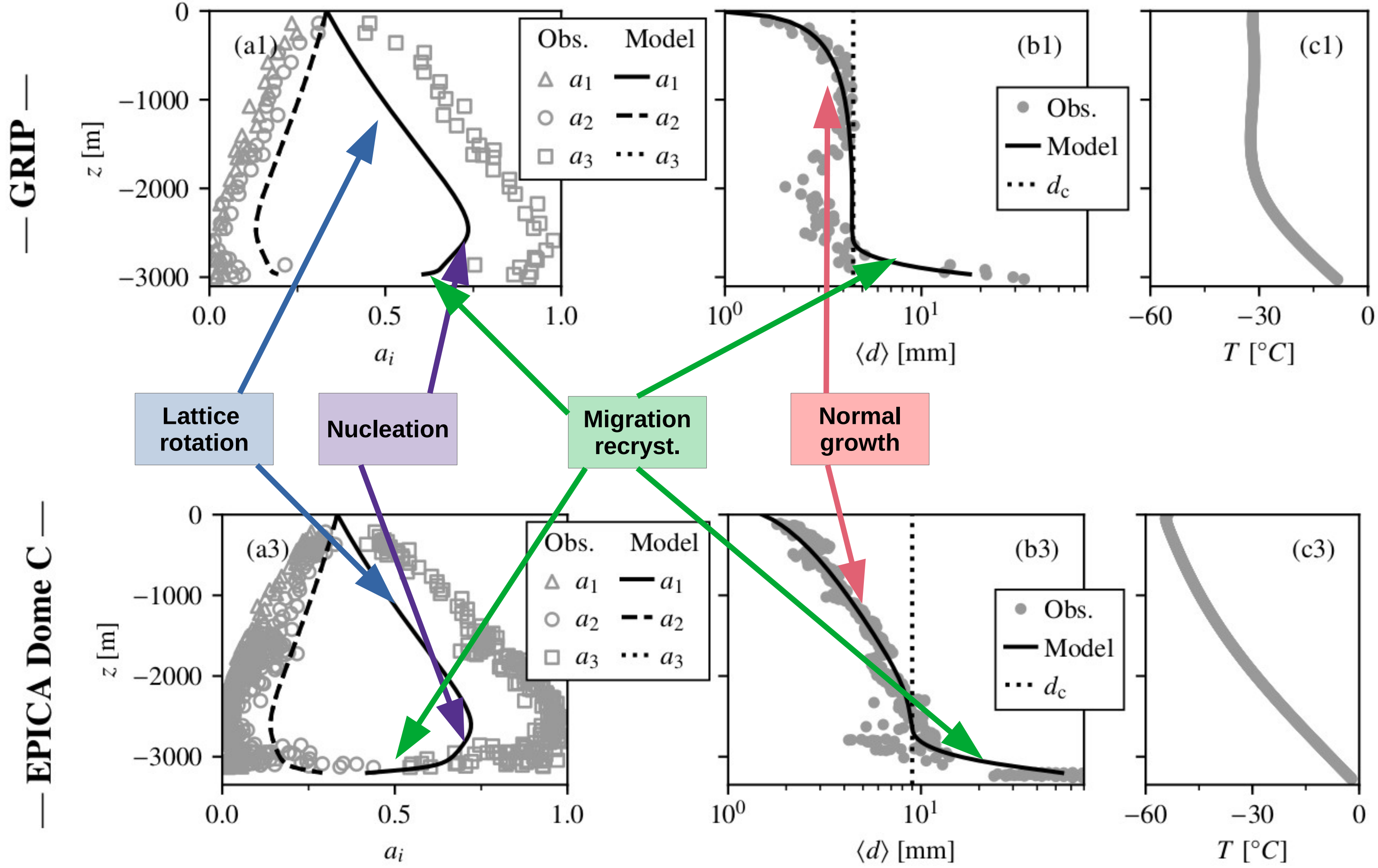
$$\frac{\langle \dot d \rangle}{\langle d \rangle} = \gamma\Lambda \left(\frac{1}{\langle d \rangle^2} - \frac{1}{\langle d_c \rangle \langle d \rangle} \right)$$

A volume-consuming process driven by a power-law dependency on the strain-energy density

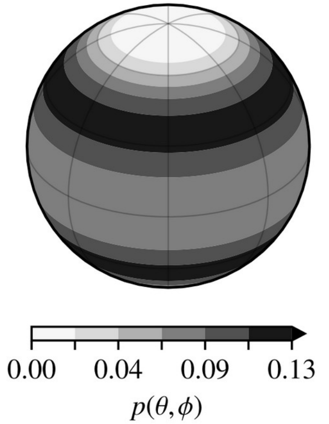
Driven by strain-energy decay and assumes classical nucleation theory

All processes are required to conserve be volume conserving

Tentative model simulations of GRIP and Dome C ice-cores



Nucleation density function used for unconfined uniaxial vertical compression:



Eigenvalues (a_i) may straight forward be calculated from ODF moments: $\left\langle \hat{\mathbf{r}}^k \right\rangle = \frac{1}{N} \int_{S^2} \hat{\mathbf{r}}^k n \, \mathrm{d}\Omega$