Spectral fabric model coupling grain orientation, grain volume, and lattice strain-energy distribution functions

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Grain number distribution (ODF)
\[ \dot{n} = \dot{n}_R + \dot{n}_G + \dot{n}_M + \dot{n}_S \]

Volume distribution
\[ \dot{v} = \dot{v}_R + \dot{v}_G + \dot{v}_M + \dot{v}_S + \dot{w} \]

Strain-energy distribution
\[ \dot{w} = \dot{w}_R + \dot{w}_G + \dot{w}_M + \dot{w}_S + \dot{w}_C \]

Distributions expanded in term of spherical harmonics, e.g.:
\[ n = |\psi|^2 \]
\[ \psi(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_l^m Y_l^m(\theta, \phi) \]

Conforms with the parabolic growth law:
\[ \frac{\langle d \rangle}{\langle d_e \rangle} = \gamma \Lambda \left( \frac{1}{\langle d \rangle^2} - \frac{1}{\langle d_e \rangle \langle d \rangle} \right) \]

A volume-consuming process driven by a power-law dependency on the strain-energy density

Driven by strain-energy decay and assumes classical nucleation theory

All processes are required to conserve be volume conserving

Ensemble total number of grains (N), volume (V), and stored strain-energy (W):
\[ N = \int_{S^2} n \, d\Omega, \quad V = \int_{S^2} v \, d\Omega, \quad W = \int_{S^2} w \, d\Omega \]

Ensemble-average grain diameter:
\[ \langle d \rangle = \frac{1}{4\pi} \int_{S^2} \left( \frac{6 \, v}{\pi \, n} \right)^{1/3} \, d\Omega \]

Model assumes spherical grains
Tentative model simulations of GRIP and Dome C ice-cores

Eigenvalues ($a_i$) may straightforward be calculated from ODF moments:

$$\langle \hat{p}^k \rangle = \frac{1}{N} \int_{S^2} \hat{p}^k n \, d\Omega$$