Earthquake Recurrence Intervals in Complex Seismogenetic Systems

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For detailed information:

- Extended abstract available via this link
- Pre-print available via this link
**Standard definition of Recurrence Interval**

**Recurrence Interval:** Statistical estimate of the likelihood of an earthquake to occur

\[
\Delta t (\geq M_{th}) = \frac{\text{Number of years on record} + 1}{\text{Number of events} \geq M_{th}}
\]

- Assumes Poissonian processes – events of similar size are mutually independent and have a stationary probability of occurrence (Boltzmann-Gibbs thermodynamics).
- Says *nothing* about the dynamic state of fault networks.
- **Gross approximation of the long-term average** (expectation) of RI.
- Might lead to **misestimation** if the dynamics of a seismogenetic system is *not* Poissonian.

\[
\text{Number of years on record} \geq \text{Number of events}
\]
Interevent times

- A parameter obviously associated with the RI and the dynamic state is the *interevent time* (IT),
  - IT: Lapse between consecutive earthquakes over a given area and above a magnitude threshold.

- IT has generally not been used in estimation of earthquake recurrence intervals.
- In the context of Poissonian processes, Frequency – Interevent Time (F-T) distributions should be exponential whereas they are generally not.
- Empirical F-T distributions usually are power laws that cannot possibly fit into the Poissonian (Boltzmann-Gibbs) context.
  - Attempts to resolve contradiction produced *ad hoc* theories that are generally well formulated and elegant, but unavoidably multi-parametric, unnecessarily complicated and possibly defying the principle of maximum parsimony.
Seismicity expresses a fault network (system) that evolves in a fractal-like spacetime and may be sustainably non-equilibrating (Complex), sustainably equilibrating (Poissonian), or may transition between equilibrating and non-equilibrating (Complex) states.

Complex States require a significant proportion of successive earthquakes to be dependent through short and long range interaction (correlation) introducing delayed feedback: confers memory manifested by power-law distributions.

The statistical properties of Complex States can be studied with Non-Extensive Statistical Physics (NESP) → direct generalization of Boltzmann-Gibbs thermodynamics to non-equilibrating systems.

In NESP, for real dynamic variables $x \in [0, \infty)$ the CDF is

$$P(> x) = \exp_q \left( -x \cdot x_0^{-1} \right) = \left[ 1 - (1 - q) \left( -x \cdot x_0^{-1} \right) \right] (1 - q)^{-1}$$

- $\exp_q$ is the $q$-exponential function
- $x_0$: $q$-relaxation constant;
- $q$: entropic index (level of correlation)
- For $q_T \neq 1$ $\exp_q$ is a Zipf-Mandelbrot power law
- For $q_T = 1$ $\exp_q(-x/x_0) = \exp(-x/x_0)$, i.e. exponential distribution – Poissonian process.
Earthquake magnitudes and interevent times are related.

- The larger the magnitude scale, the longer the recurrence interval and interevent time.

Joint evaluation of Frequency – Magnitude – Interevent Time distributions ensures observance of this important details.

Frequency distribution of Interevent Time should be evaluated conditionally on the frequency distribution of magnitudes.
NESP-COMPATIBLE STATISTICAL MODEL

Statistical properties of seismogenetic space-times represented with generalized bivariate q-exponential Frequency-Magnitude distributions of Gutenberg-Richter type:

\[
\frac{N(\{M \geq M_{th}, \Delta t : M \geq M_{th}\})}{N_0} = \left(1 - \frac{1 - q_M}{2 - q_M} \cdot \frac{10^M}{\alpha^{2/3}}\right)^{\frac{2-q_M}{1-q_M}} \cdot \left(1 - (1 - q_T) \cdot \frac{\Delta t}{\Delta t_0}\right)^{\frac{1}{1-q_T}}
\]

\[
P(M \geq M_{th}, \Delta t : M \geq M_{th})
\]

\[
P(M \geq M_{th})
\]

\[
P(\Delta t : M \geq M_{th})
\]

or

\[
\log N(\{M \geq M_{th}, \Delta t : M \geq M_{th}\}) = a + \left(\frac{2-q_M}{1-q_M}\right) \cdot \log \left(1 - \frac{1 - q_M}{2 - q_M} \cdot \frac{10^M}{\alpha^{2/3}}\right) + \frac{1}{1-q_T} \log \left(1 - (1 - q_T) \cdot \frac{\Delta t}{\Delta t_0}\right)
\]

Assumes: Magnitudes and interevent times are statistically independent

\(M_{th}\): Threshold magnitude

\(\alpha\): Energy scaling constant

\(q_M\): Magnitude Entropic Index: indicates level of correlation in size-space

\(q_T\): Temporal Entropic Index: indicates level of correlation in time-space

\(\Delta t\): Interevent Time

\(\Delta t_0\): q-relaxation interval \(\equiv\) characteristic recurrence of \(M \geq M_{th}\)
ESTIMATION PROCEDURE

- All parameters positive (bounded from below)
- Entropic indices bounded from above ($\leq 2$)
- Problem solved with trust-region reflective algorithm + with least absolute residual (LAR) minimization to suppress outliers.
Example 1: Crustal (schizospheric) systems in Transformational Plate Margins: California, USA

Data Source: North California Earthquake Data Centre @ http://www.ncedc.org.
- $M_L \geq 3.0$; Period 1968-2017.5

Data Source: South California Earthquake Data Centre @(http://www.data.scec.org).
- $M_L \geq 2.5$; Period 1968-2017.5
San Andreas Fault – Northern Segment

- **Left column:** *Prior* to the Loma Prieta event: 
  
a) The temporal entropic index is *significant* (complexity/ moderate correlation) and *increases with magnitude*;  
b) the *q-relaxation* interval resembles the “standard” and increases quasi-exponentially.

- **Right column:** *After* the Loma Prieta event: 
  
c) The temporal entropic index is $<1.2$ and indicates *weak correlation*;  
d) the *q-relaxation* interval resembles the “standard” and increases quasi-exponentially.

- Absence of stationarity in dynamic expression of the system is evidence against Self-Organized Criticality.
Both systems: In (a) and (c) the temporal entropic index is very significant (very high correlation) and increases with magnitude; in (b) and (d) $q$-relaxation interval does not increase.

Both systems locked in the landward side of the primary plate boundary and experience strong long-range interaction.

Shape of the $q$-relaxation curve indicates that upon occurrence of any event systems respond promptly and in a non-hierarchical manner; this is a hallmark of Self-Organized Criticality.
Example 2: Crustal (schizospheric) systems in Convergent Plate Margins

Alaskan – Aleutian Arc

IZU – Bonin – Mariana Arc

Data Source: Regional earthquake database of the Alaska Earthquake Centre @ http://www.aeic.alaska.edu/html_docs/db2catalog.html

- Only seismicity shown in orange is considered

Data Source: National Research Institute for Earth Science and Disaster Resilience (NIED) of Japan, @http://www.hinet.bosai.go.jp

- Only the seismicity shown in blue is considered (PSP-C)
Both systems are marginally correlated: $q_T \leq 1.3$ indicates low level of long-range interaction and delayed feedback.

Both systems: the q-relaxation interval and the “standard” recurrence interval are comparable and increase exponentially.
Example 3: Sub-Crustal (below Moho) systems in Convergent Plate Margins

Alaskan – Aleutian Wadati-Benioff Zone

IZU – Bonin – Marianna Wadati-Benioff Zone

Data Source: Regional earthquake database of the Alaska Earthquake Centre @ http://www.aeic.alaska.edu/html_docs/db2catalog.html
- Only seismicity shown in orange is considered

Data Source: National Research Institute for Earth Science and Disaster Resilience (NIED) of Japan, @ (http://www.hinet.bosai.go.jp)
- Only the seismicity designated as PSP-D blue is considered
Alaskan – Aleutian Subduction

Izu – Bonin – Mariana Subduction

systems are uncorrelated: $q_T \leq 1.15$ indicates practically non-existent long-range interaction and delayed feedback; systems are Poissonian.

Both systems: the $q$-relaxation interval and the “standard” recurrence interval are practically identical and increase exponentially as expected of Poissonian systems.
Summary of Correlation Properties (entropic states)

- **Crustal** systems in transformational plate boundaries are generally **correlated**.
- **Crustal** systems in convergent and divergent plate margins are generally **weakly–moderately correlated**.
- **Sub-crustal**/Wadati-Benioff zone systems are definitely uncorrelated (**quasi-Poissonian**).
General Conclusions after analysis of 20 seismogenetic systems

- The q-exponential distribution is a universal descriptor of Interevent Time statistics.
- The duration of q-relaxation intervals is generally reciprocal to the level of correlation (\(q_T\)). The higher the correlation, the shorter the q-relaxation.
  - Both may change with time and across system boundaries.

- Crustal systems in transformational plate boundaries:
  - A few systems with very strong correlation and very short/ slowly increasing recurrence intervals exhibit attributes of Self-Organized Criticality.
  - Most other such systems are complex and with apparently significant long-range interaction but most probably non-critical!

- Crustal systems in convergent plate margins:
  - q-relaxation and standard recurrence intervals both increase exponentially, some at comparable and some at different rates.
  - Such fault networks exhibit moderate to strong correlation (complexity).
  - Attributes indicate that such systems are possibly non-critical.

- Sub-crustal and Wadati-Benioff zones:
  - q-relaxation and standard recurrence intervals increase exponentially with magnitude and are congruent.
  - Such systems are generally uncorrelated and appear to be Poissonian in nature.

- The blending of earthquake populations from adjacent but dynamically different systems randomizes the statistics of the mixed catalogue and over large seismogenetic provinces, reduces the apparent level of Complexity.
Additional examples, documentation, discussion and a possible interpretation of the observations can be found in a pre-print available via this link.

Possible utility/utilization of the new information remains to be specified with future work.

Thank you for your Patience.