An Empirical Model of Electron Flux from the Seven-Year Van Allen Probe Mission

Christine Gabrielse[1], J. Lee[1], J. Roeder[1], S. Claudepierre[1], A. Runov[2], T. Paul O'Brien[1], D. L. Turner[1,3], J. Fennell[1], J. B. Blake[1], Y. Miyoshi[4], K. Keika[5], N. Higashio[6], I. Shinohara[7], S. Kurita[4], S. Imajo[4]

[1]The Aerospace Corporation
[2]Earth, Planetary, and Space Sciences Dept, UCLA
[3]Now at John Hopkins Applied Physic Laboratory
[4]Institute for Space-Earth Environmental Research, Nagoya University
[5]Graduate School of Science, The University of Tokyo
[6]Research & Development Directorate, Japan Aerospace Exploration Agency
[7]Institute of Space and Astronautical Science, Japan Aerospace Exploration Agency

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Background and Motivation

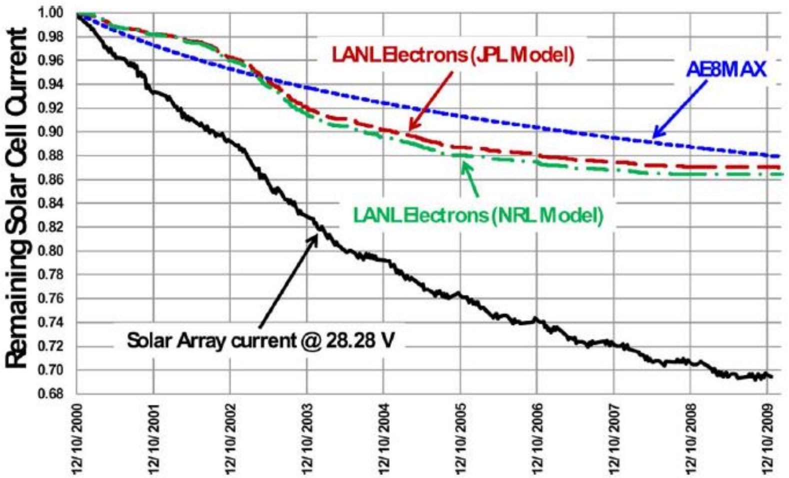
Desire data-based model on day-long timescales

- Empirical models have been designed to predict radiation environment (e.g., Roeder et al., Space Weather 2005; Chen et al., JGR, 2014), but they **may not capture actual fluxes observed a particular day**
- AE9 provides probability of occurrence (percentile levels) for flux and fluence averaged over different exposure periods—**not meant to capture daily variations**
- Effects that require shorter-term integrals of the outer radiation belt may need special attention when it comes to environmental assessments.
  - *Spacecraft charging (DeForest, 1972; Olsen, 1983; Koons et al., 2006; Fennell et al., 2008)*
- Practical example: GPS solar array current and voltage degrades faster than predicted by any model (e.g., Messenger et al., 2011)—**Are we correctly modeling the radiation environment?**

Figure: Black line is remaining factor of solar array current.

Red, green and blue lines: modeled or expected solar panel current including all known factors.

What is missing?



Messenger et al., 2011
IEEE TRANSACTIONS ON NUCLEAR SCIENCE

Background and Motivation

Use Van Allen Probe data to provide actual daily fluence estimates, mapped to secondary satellite

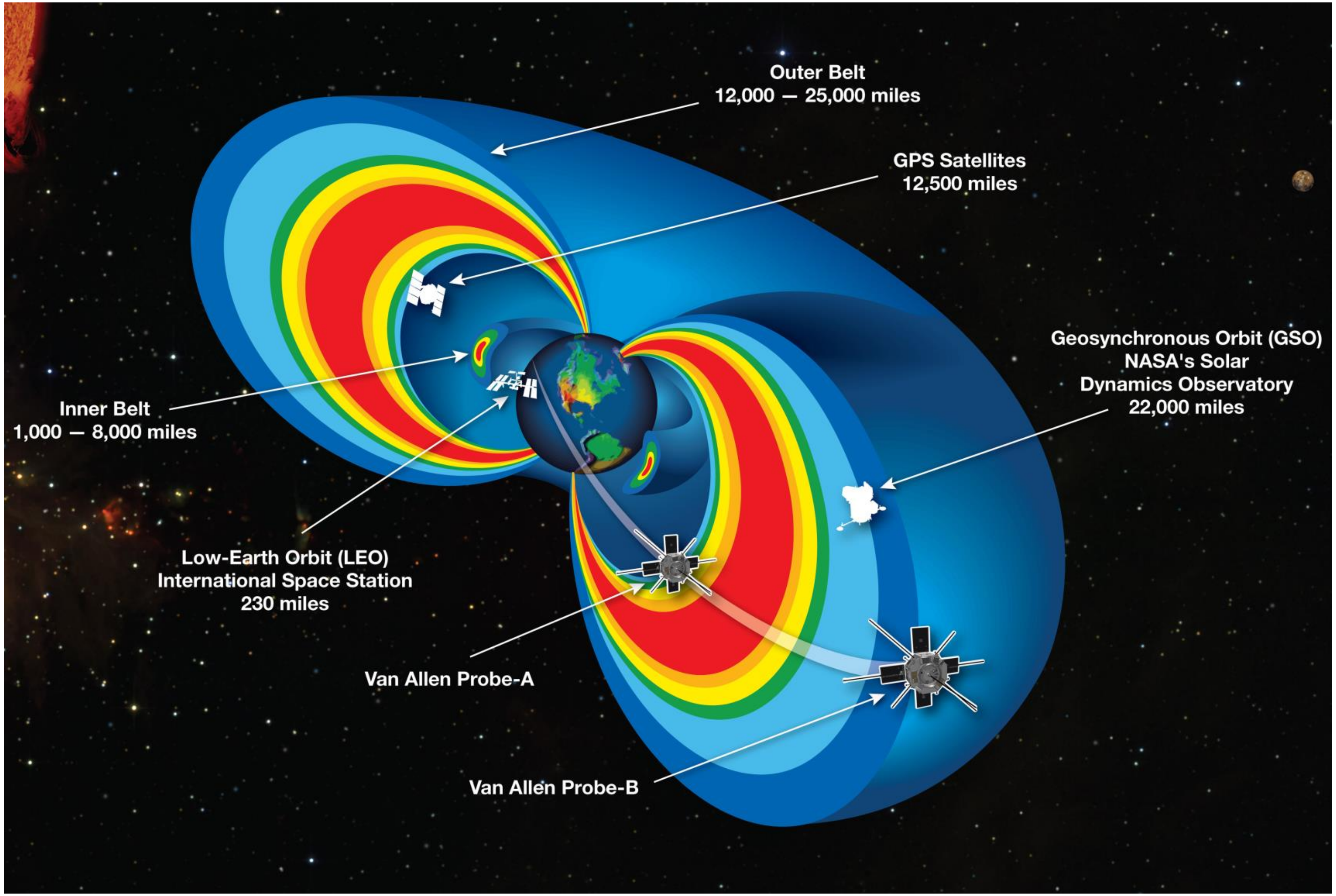


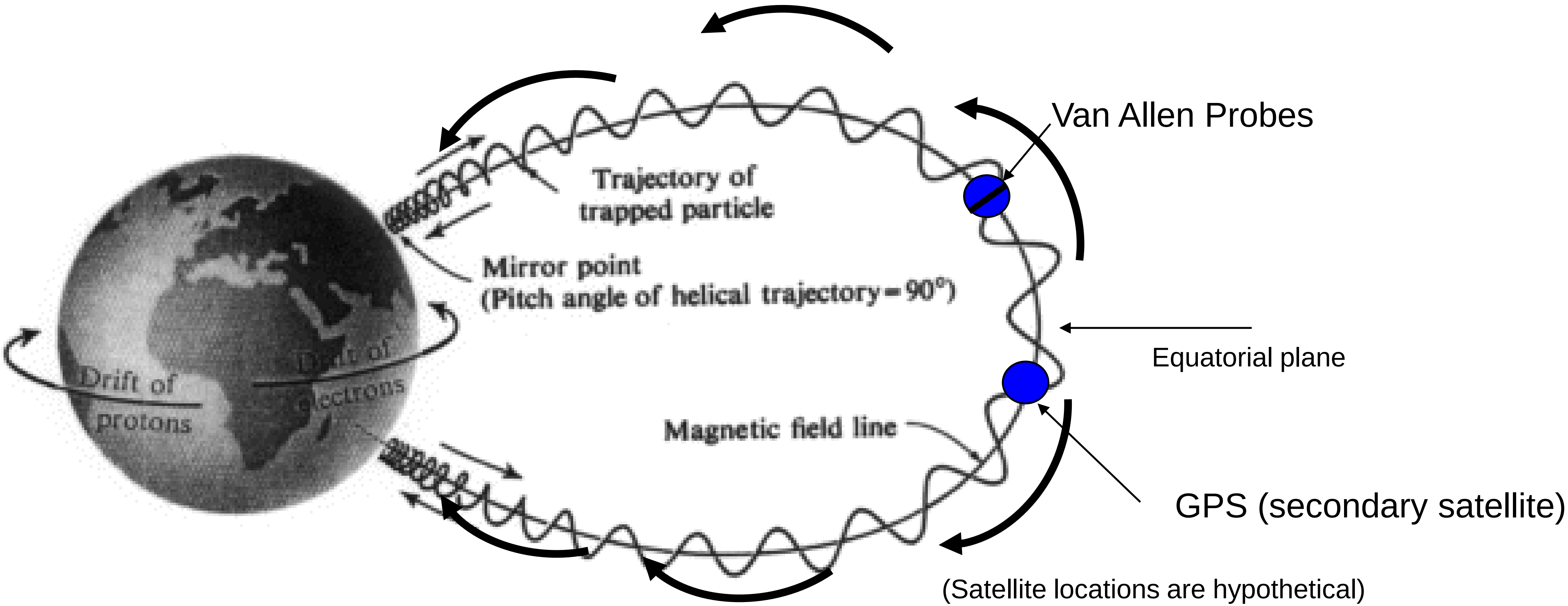
Image Credit: NASA

Model Background

Obtaining equatorial pitch angle distributions

We want to know what electron fluxes are everywhere along the field line.

But we only know the fluxes for electrons that bounce past Van Allen Probes.



(Satellite locations are hypothetical)

R.E. Mars, 2002, Lawrence Livermore National Laboratory Report UCRL-ID-151402

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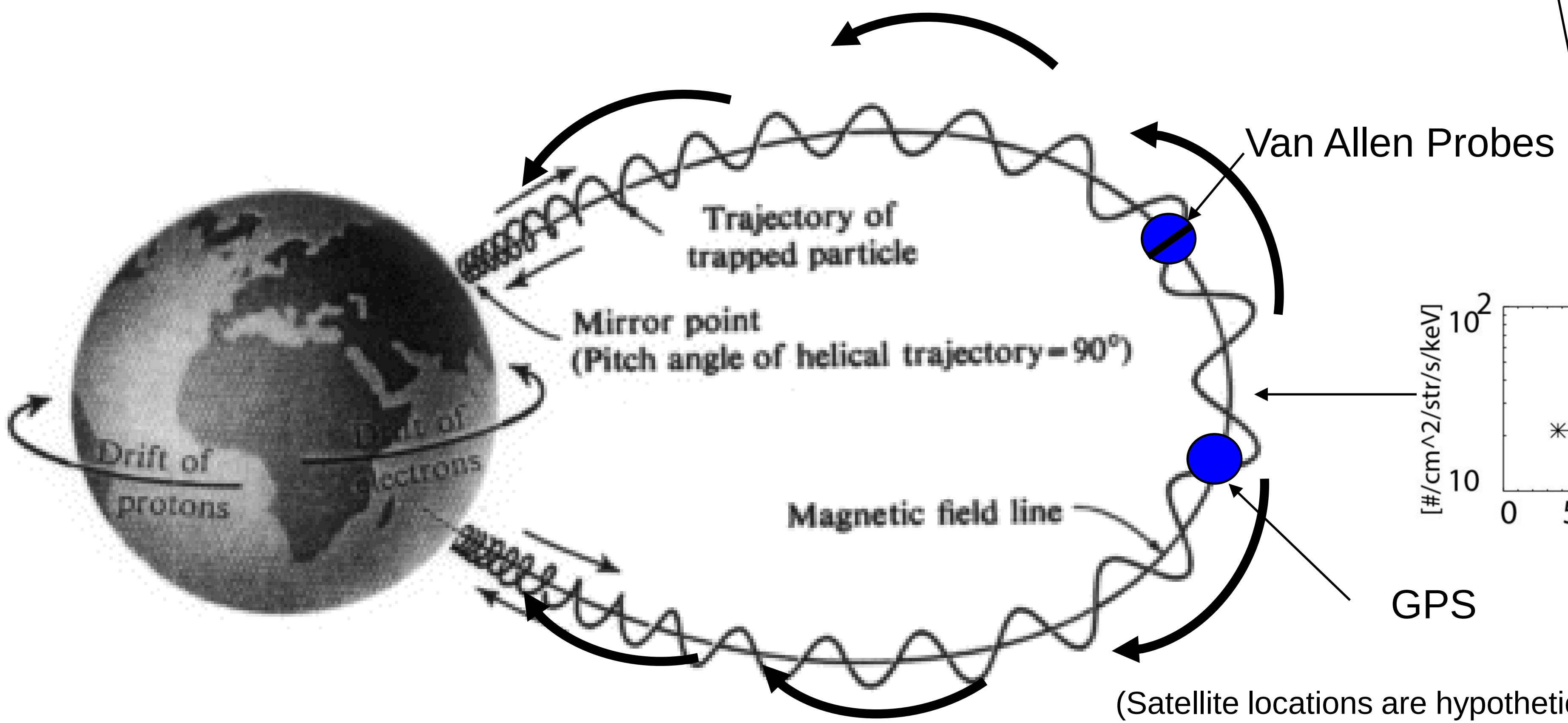

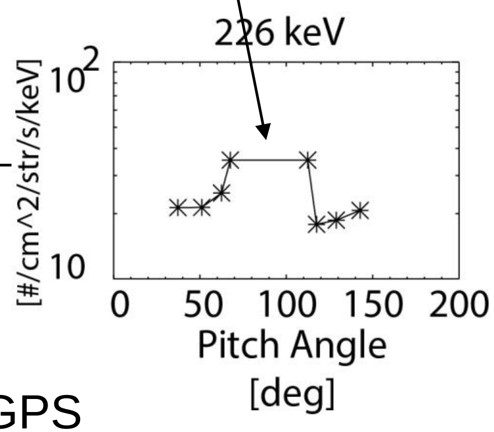

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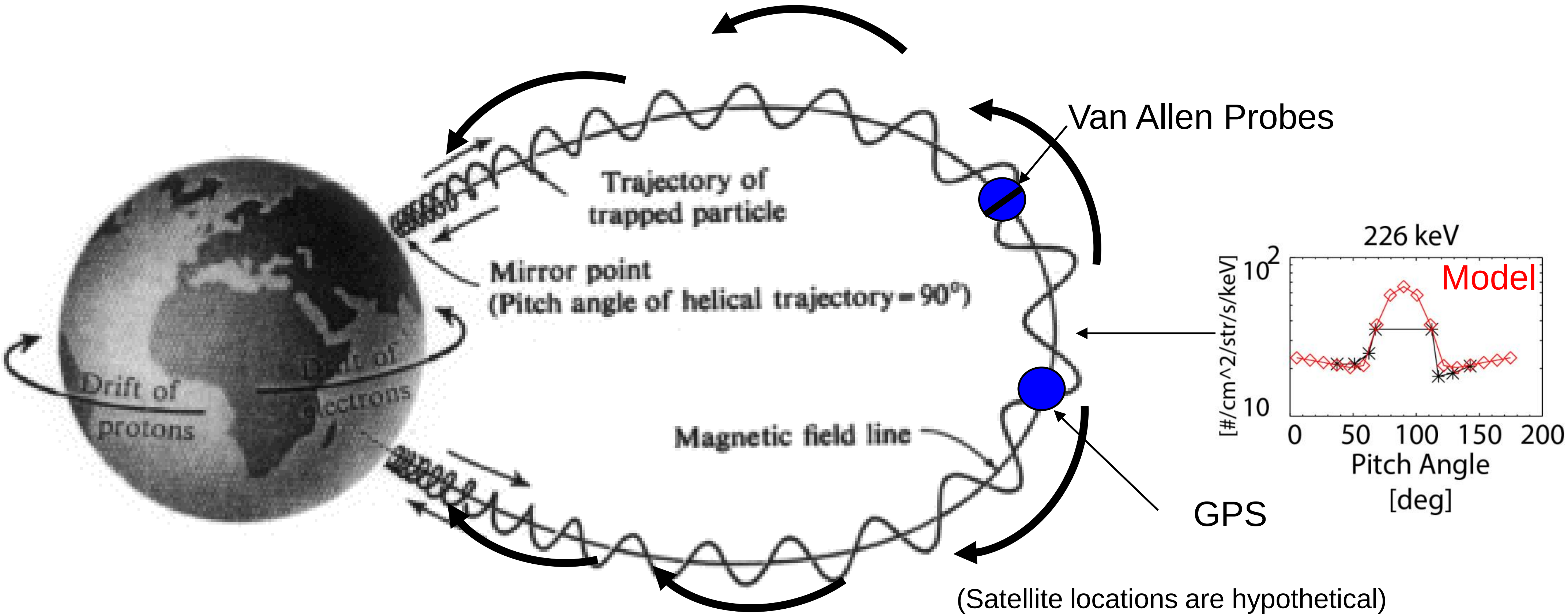

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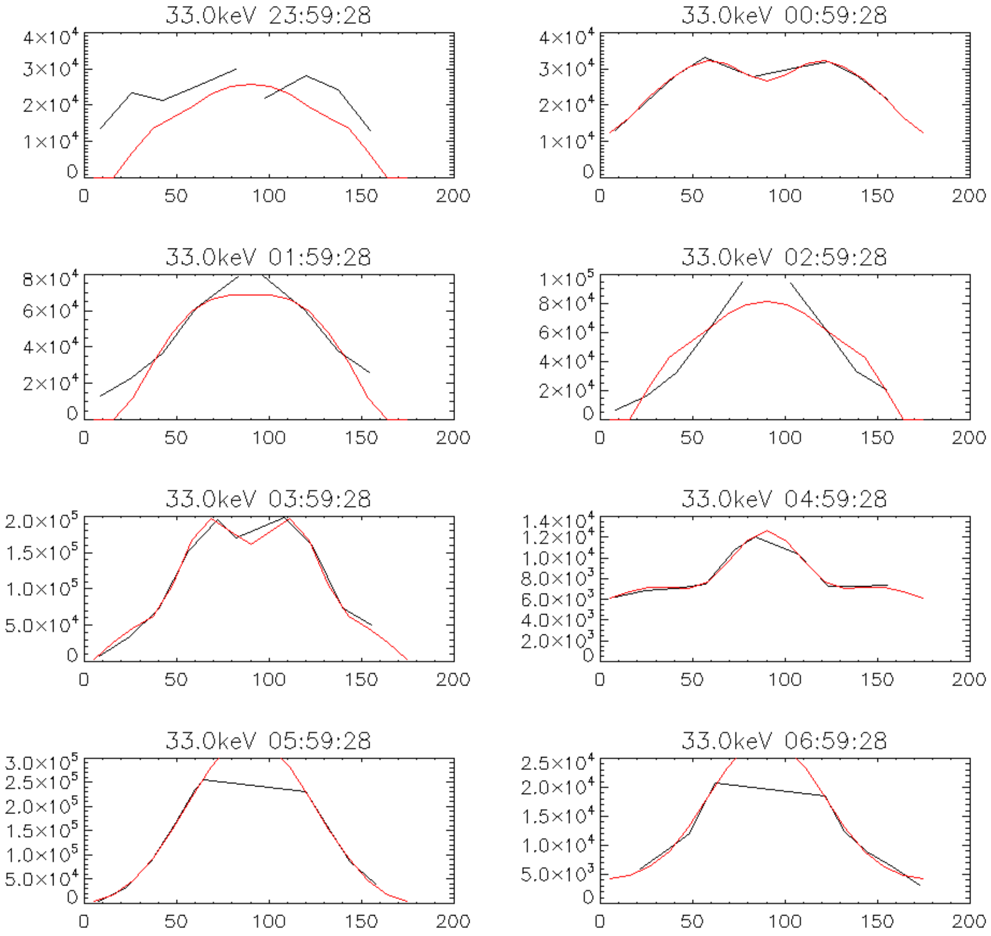
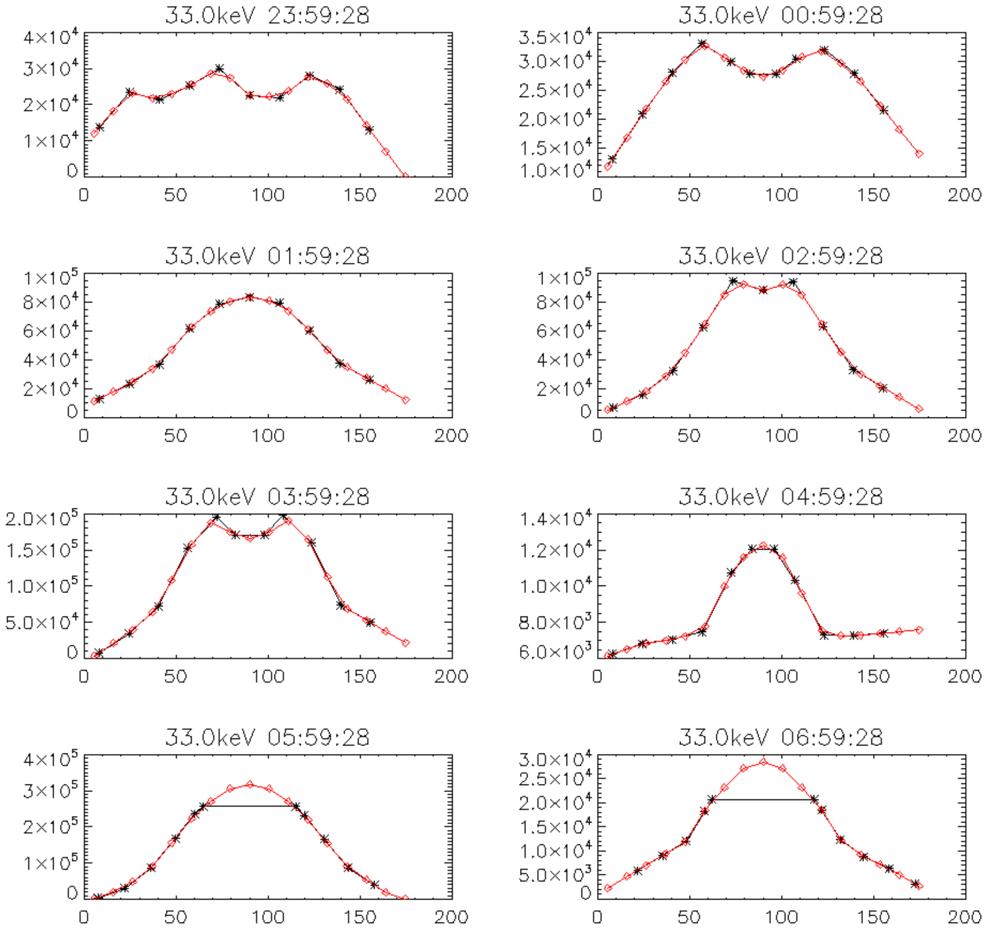
Modeling Pitch Angle Distributions

Legendre Polynomial Fitting vs. Filling the Gap with a Sine Function

Legendre Polynomial fits



Data interpolates + Sine Function



Legendre Polynomial fitting may be good for statistics (e.g. Chen et al., 2014), but we want actual daily fluences

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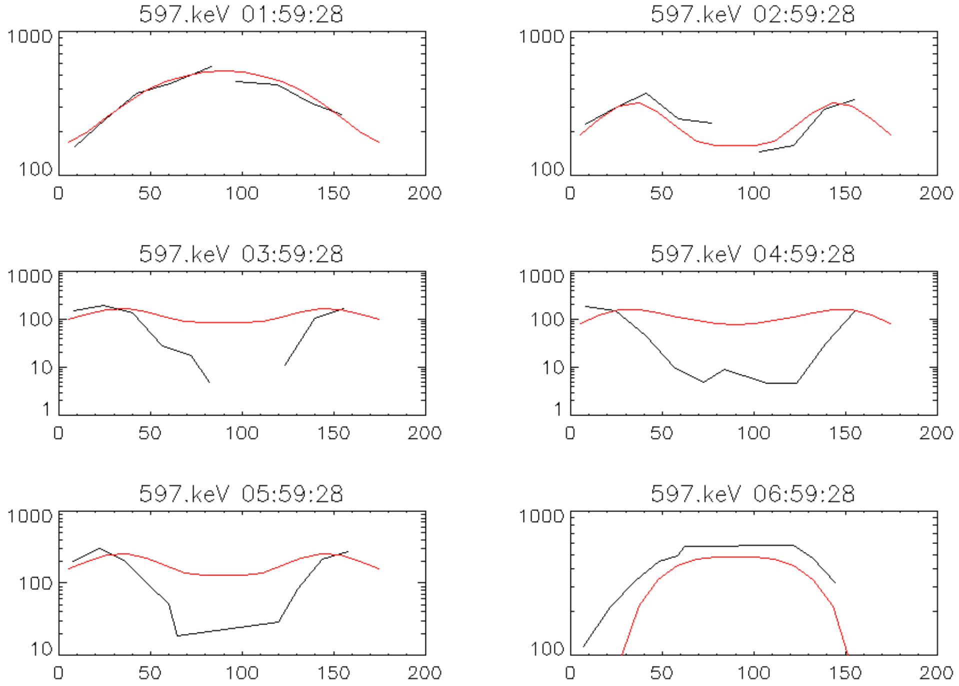

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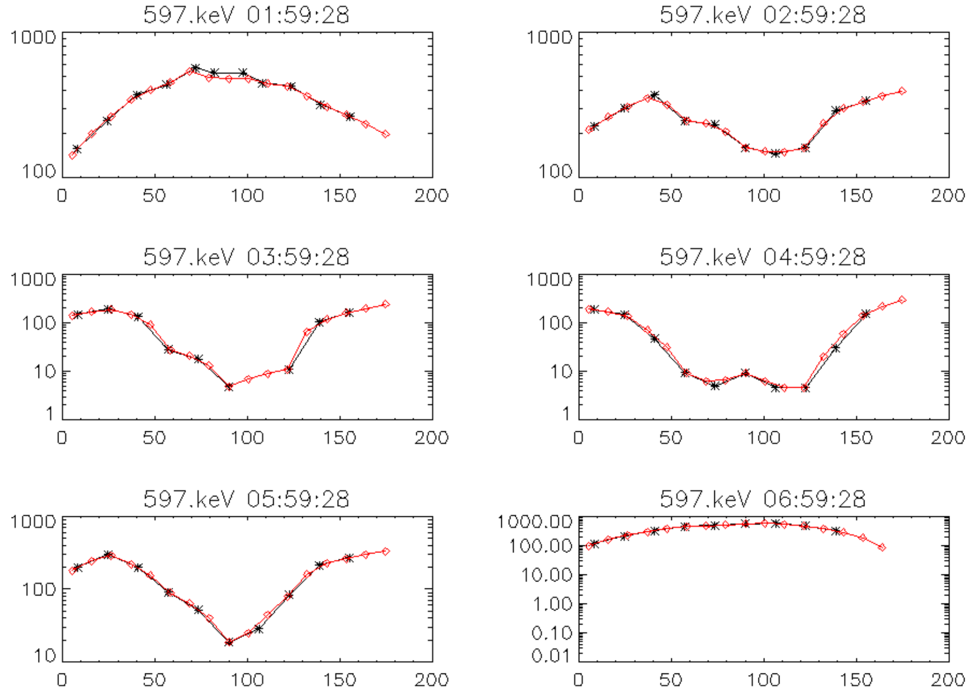

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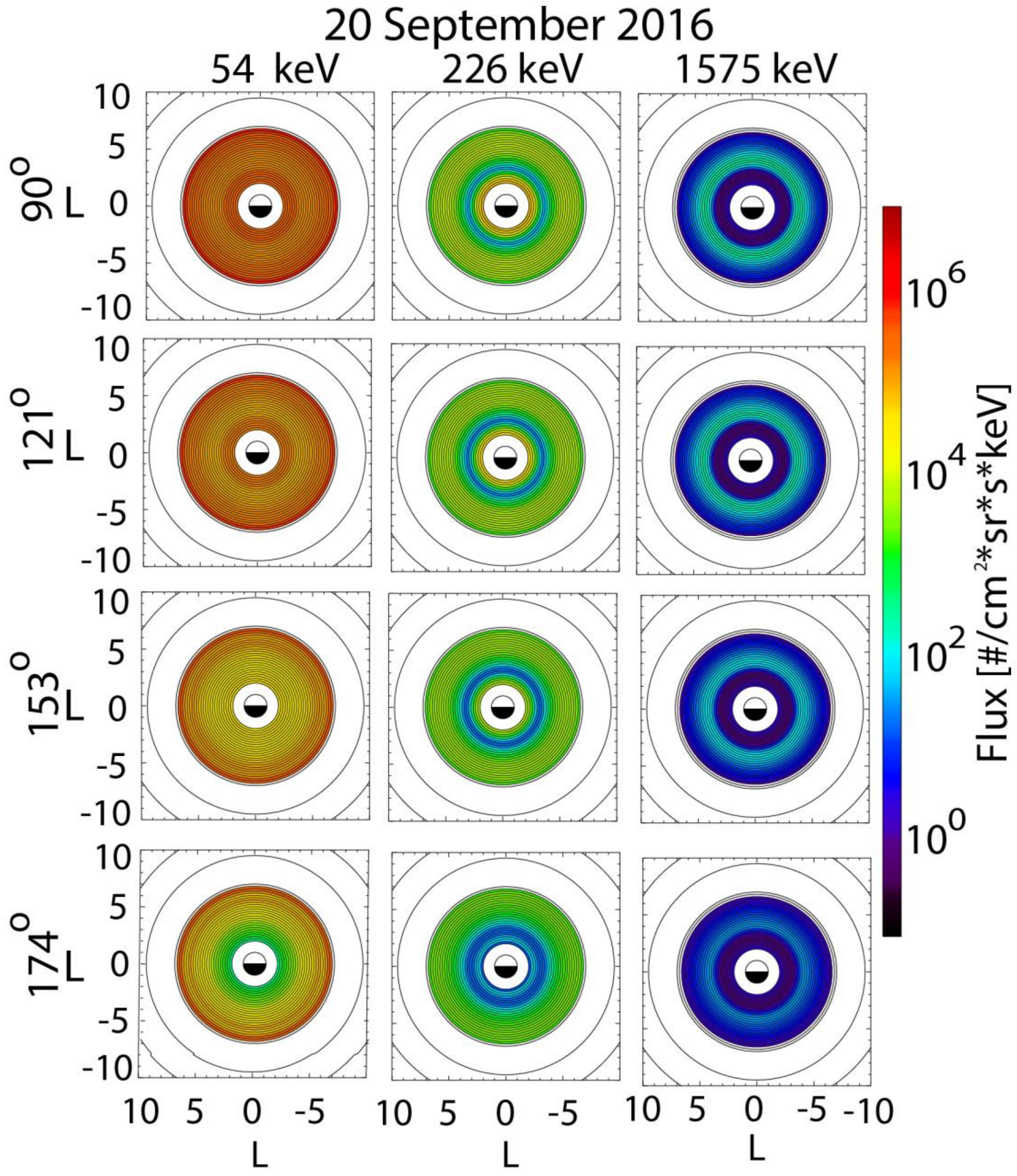
Radiation Belt Daily Electron Flux Model: RB-Daily-E

Matrix: 0.2 R_E L shell (from 2-7 R_E) x 17 pitch angle

- 1440 minutes per day x 17 pitch angles x 25 energy bins are saved.

- Bin fluxes into 0.2 R_E L shell bins by summing total fluxes and normalizing by time spent in each bin that day.
    - Allows us to change L shell bin width without reprocessing.

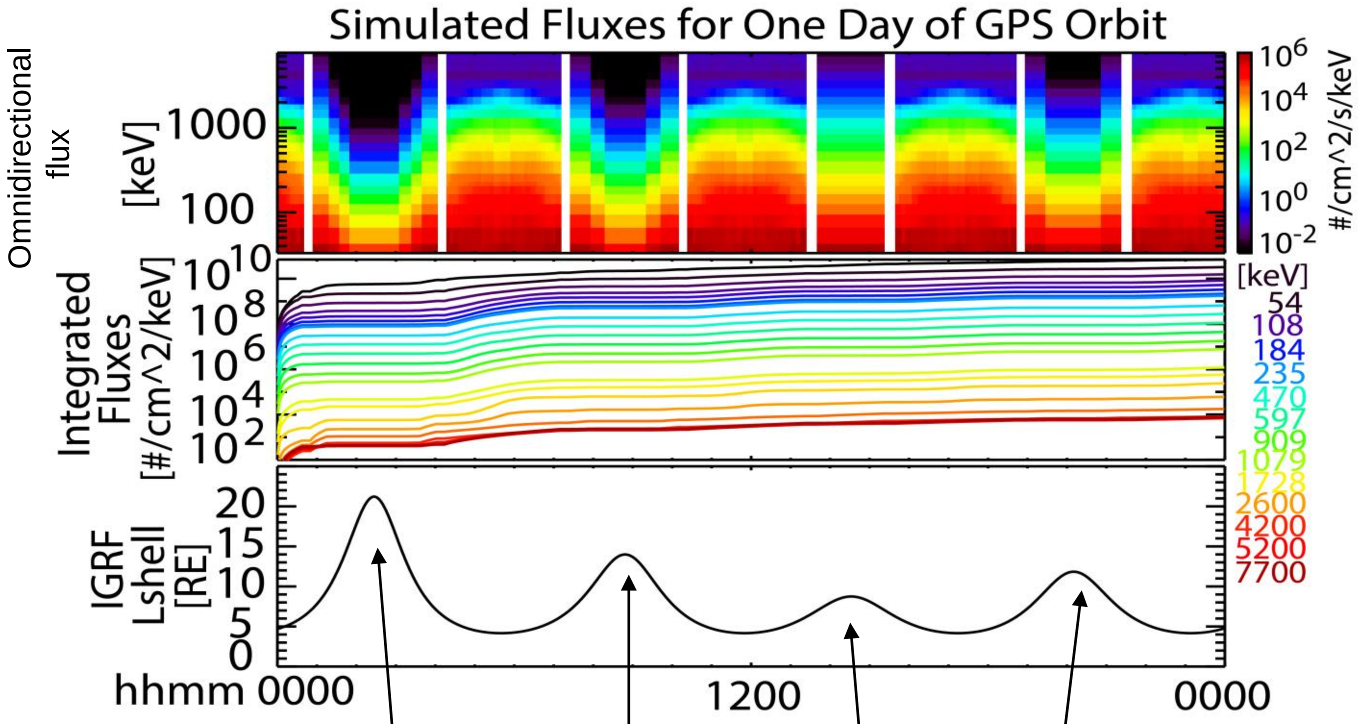


Right: Example of 54, 226, and 1574 keV fluxes at 90, 121, 153, and 174 deg equatorial pitch angles. Must assume MLT symmetry.

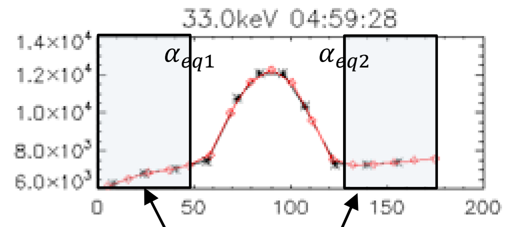
GPS Fluxes from RB-Daily-E

Fly hypothetical GPS through model each minute by:

1) Mapping GPS location to equator (IGRF) to get L bin.
2) Integrate fluxes within pitch angle range of particles that reach GPS to get omnidirectional flux. (e.g. Local 0-180 at GPS may map to equatorial pitch angles of 0-60 and 120-180: see insert.)
3) Integrate omnidirectional flux over time to obtain a daily fluence as input to degradation models.

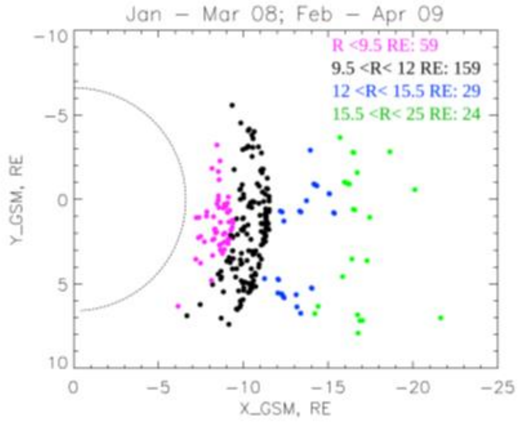


When GPS maps to L shells > 7 (Van Allen equatorial apogee), use THEMIS statistics (4 R bins spanning <9.5-25 R_E). *Runov et al.*, AGU JGR, 2015.



Hypothetical example of fluxes GPS actually sees (blue).

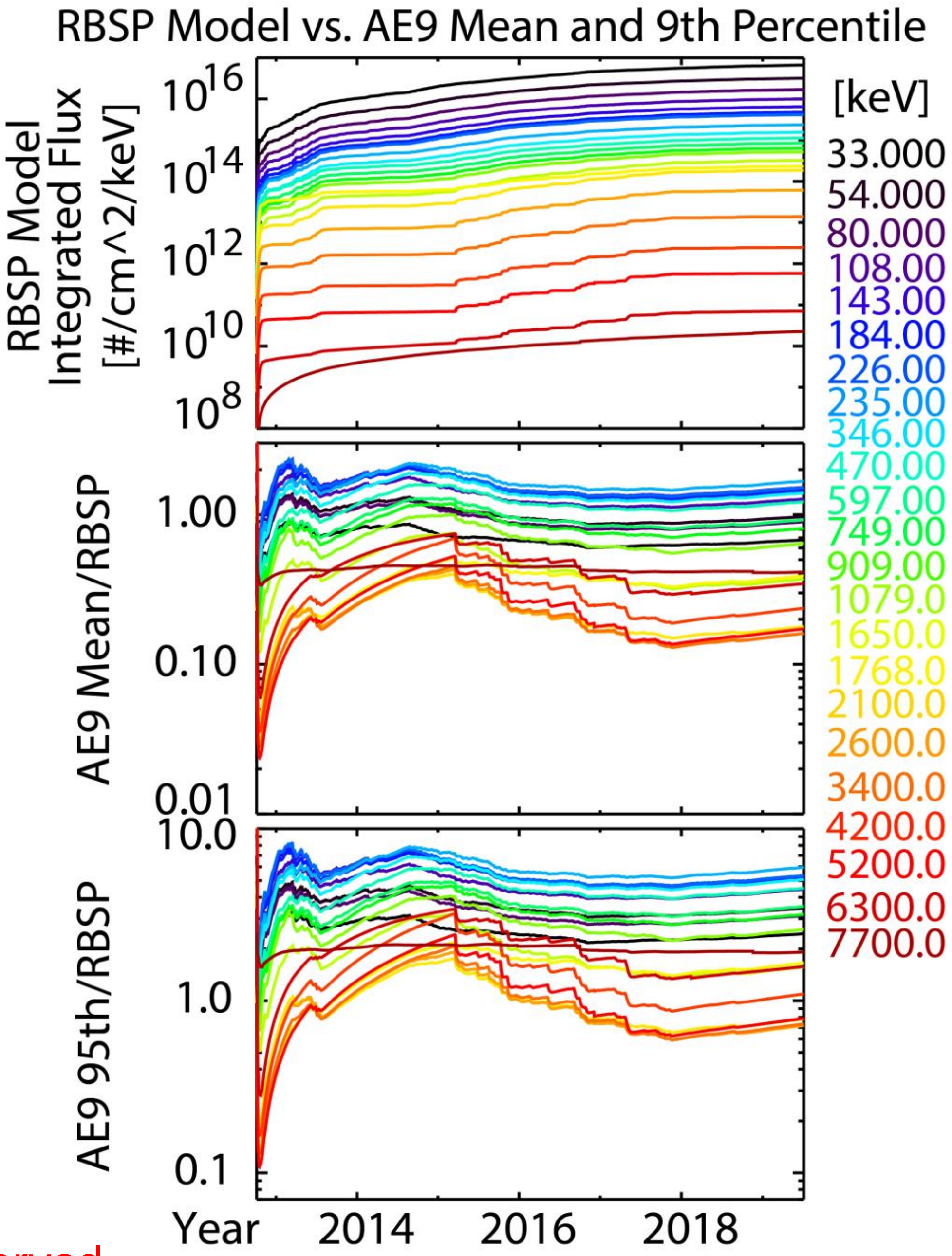
Interpolate the equatorial fluxes from RBSP to get the exact flux at α_{eq1} and α_{eq2}.



GPS Fluxes from RB-Daily-E

Integrated Fluxes over Mission Lifetime

- Plot fluences (time integrated fluxes)
  - These are inputs to solar cell degradation models
- Compare to AE9
  - AE9 Mean fluences (middle)
  - AE9 95th percentile (bottom)
- <1 means AE9 underestimates RBSP Model

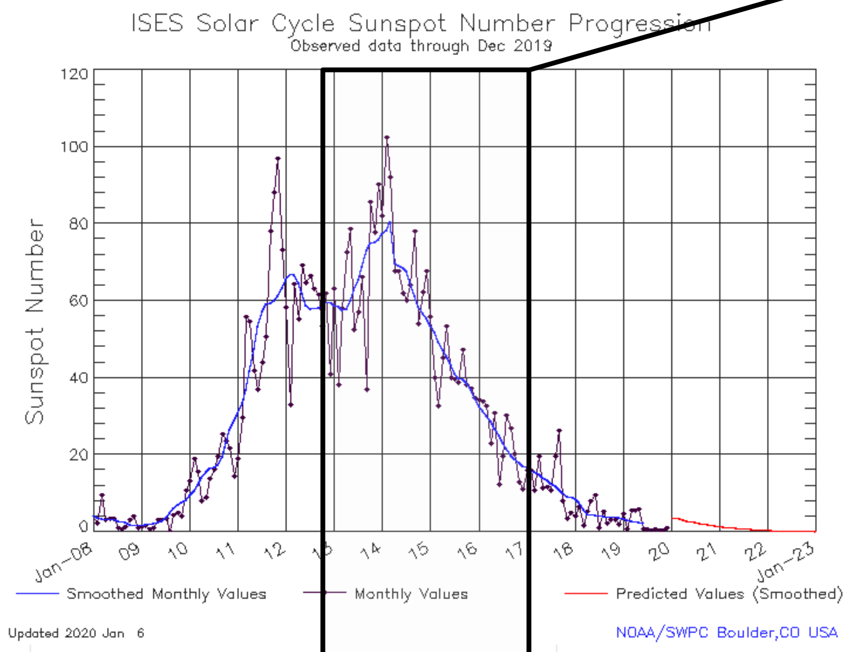


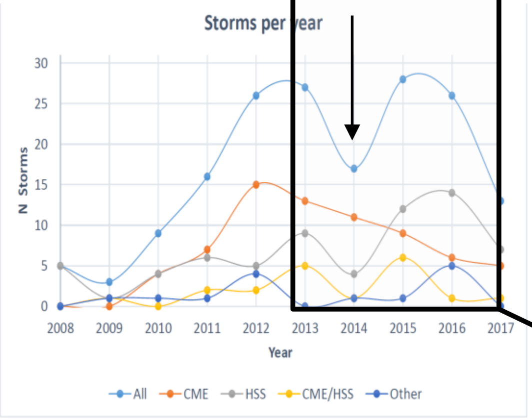
GPS Fluxes from RB-Daily-E

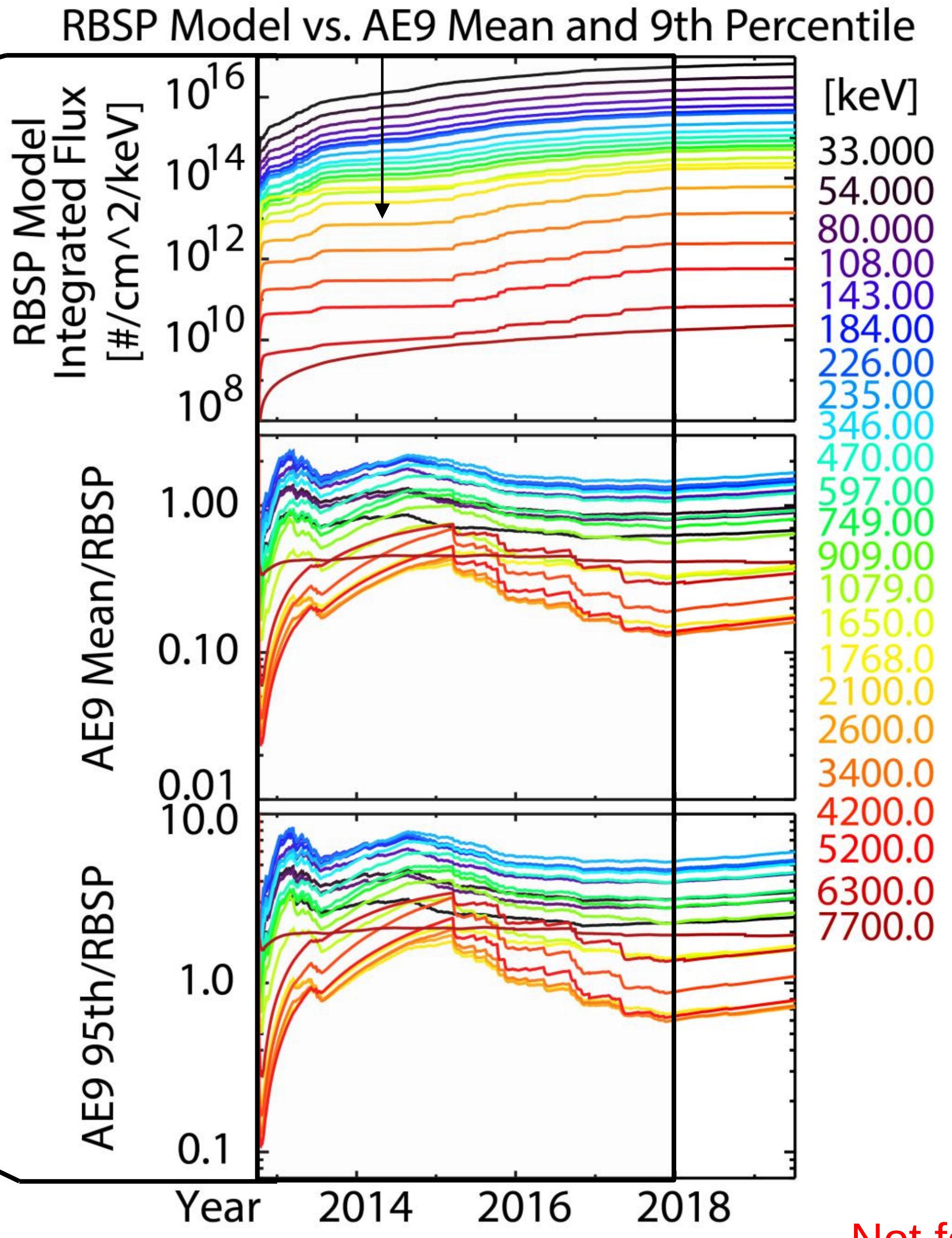
Flux Relationship to Storms

Fewer storms in 2014
→Integrated fluxes stagnate

More High Speed Stream storms occur during the declining solar cycle (*Mursula & Zieger*, 1996; *Gabrielse et al.*, 2019).
→2015 integrated fluxes sharply increase

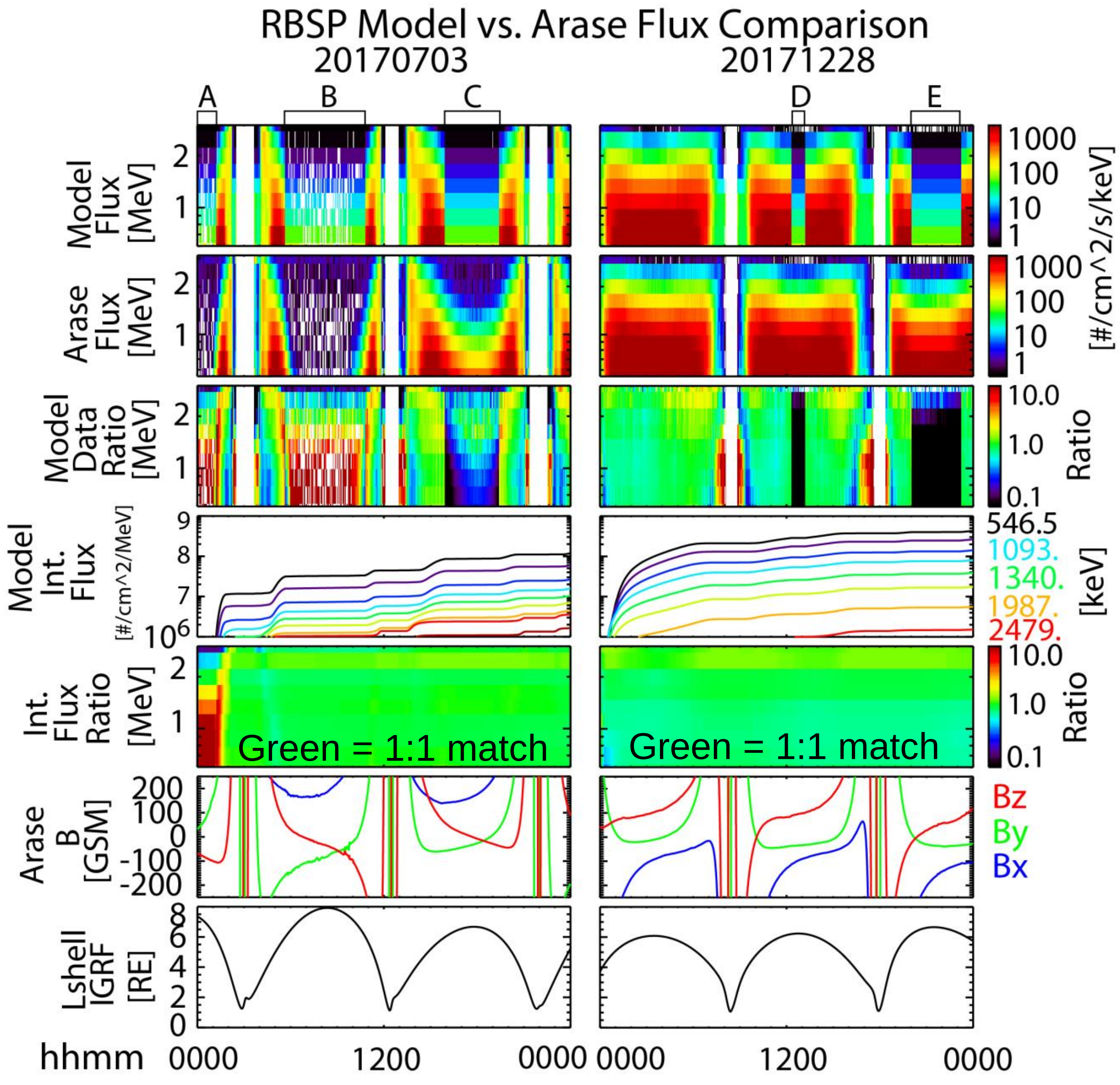






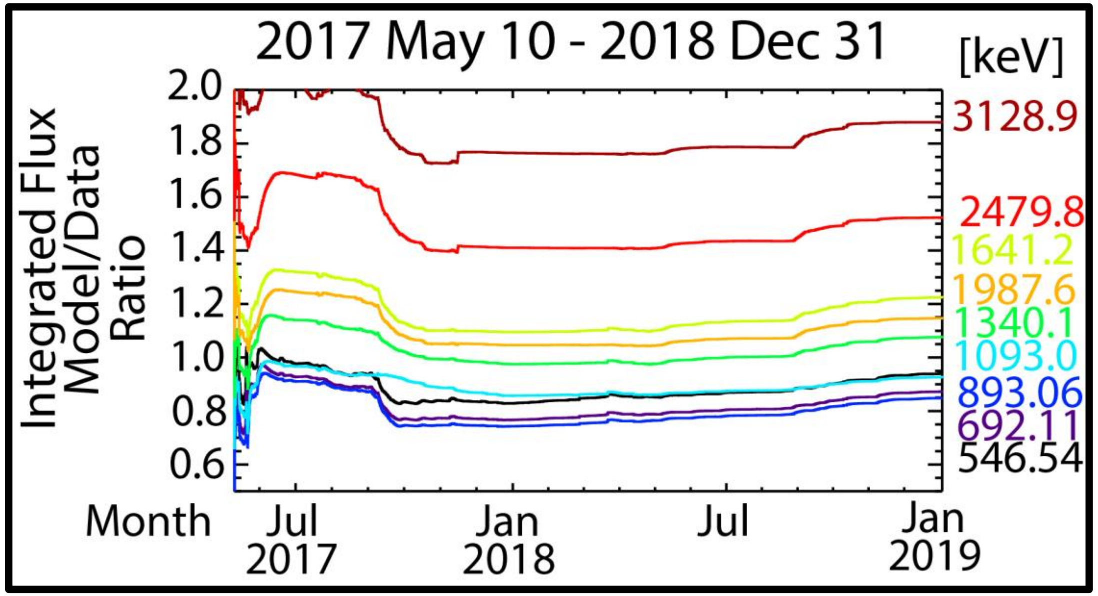
Gabrielse et al., AGU GRL, 2019

Compare RB-Daily-E with Arase XEP Data



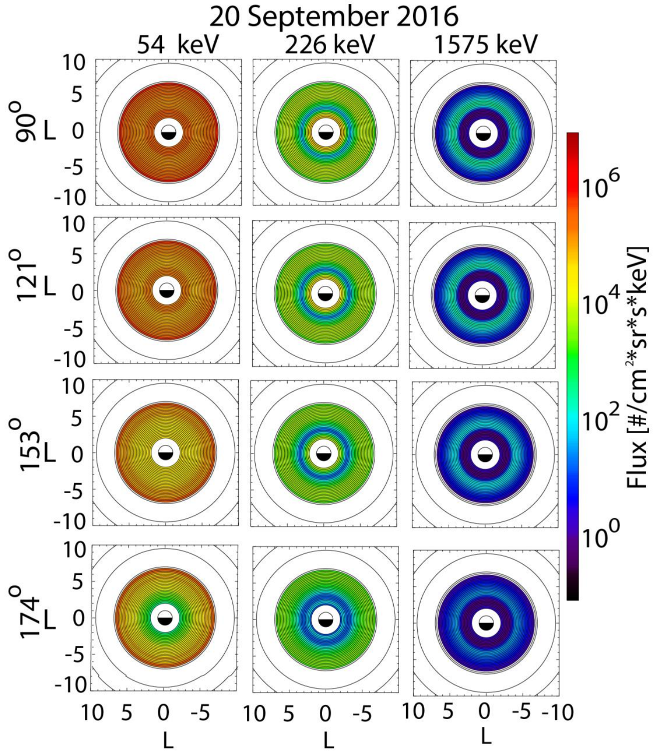
- Good comparison between RB-Daily-E and Arase XEP fluxes (*Higashio et al.*, 2018)
    - Within a factor of 2, but **for most energies within a factor of ~<1.2.**

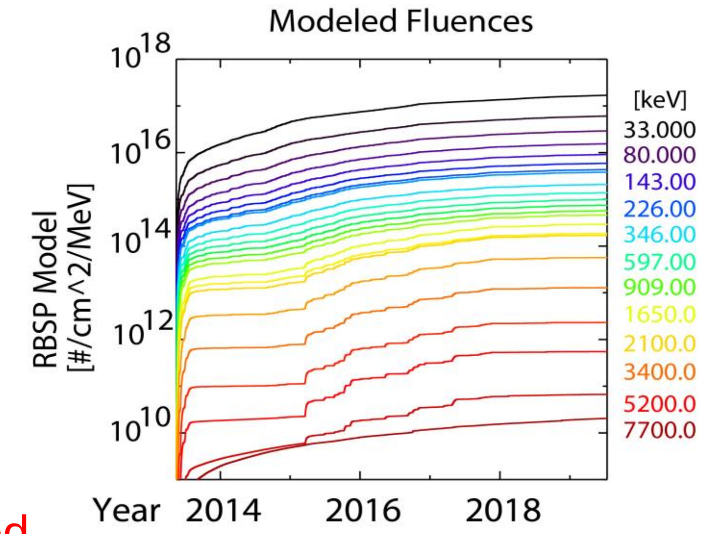
- A-E: Times we relied on THEMIS statistics. THEMIS typically underestimated what Arase observed at these high energies.
    - THEMIS was only fit up to ~293 keV (Arase XEP lower energy is ~600 keV).



Summary

- We built a daily average electron flux model based on 7 years of Van Allen Probe (RBSP) data.
  - *L shell x Pitch angle x Energy*
  - *2 RE to 7 RE, > 7RE supplement with THEMIS statistics*
  - *33 keV to 7.7 MeV*
- RB-Daily-E provides daily average fluences for a given satellite providing ephemeris and date range as input.
- RB-Daily-E accurately predicted Arase fluences within a factor of ~1.2 for energies 600-1987 keV, and within a factor of ~2 for higher energies.
- RB-Daily-E performs more precisely than AE9 Mean or 95th percentile.
- Practical application: RB-Daily-E outputs can be used in solar cell degradation models.
  - *We have had exciting results in this arena that we hope to share in the future.*



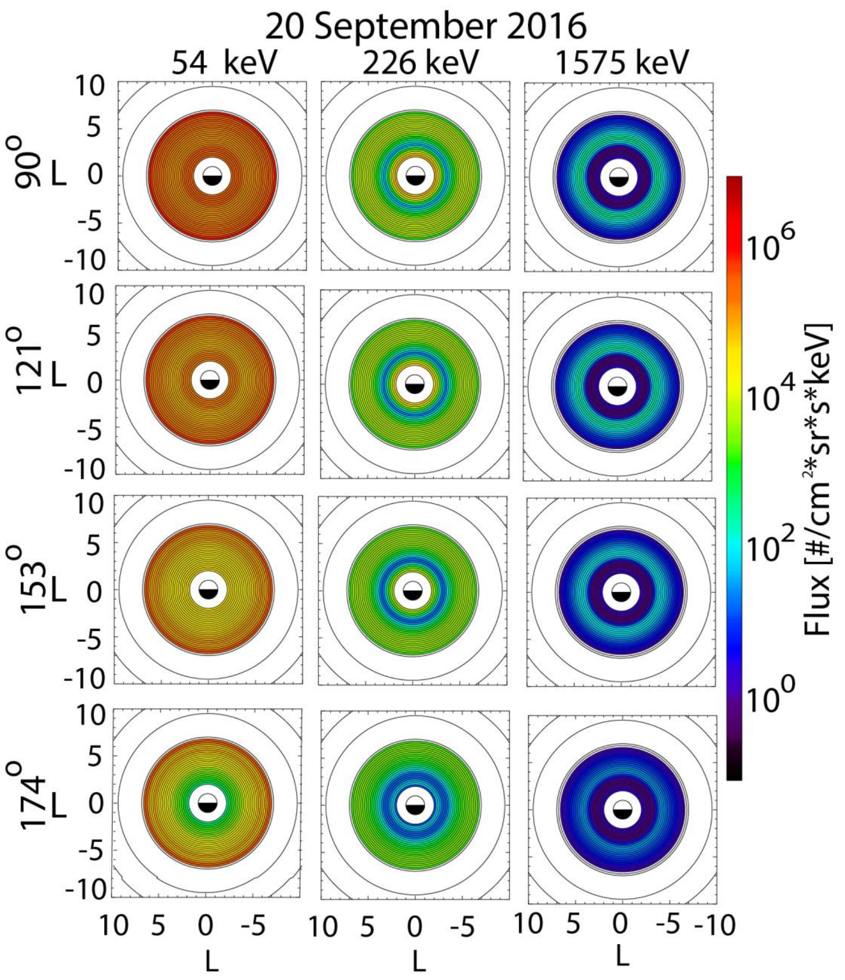


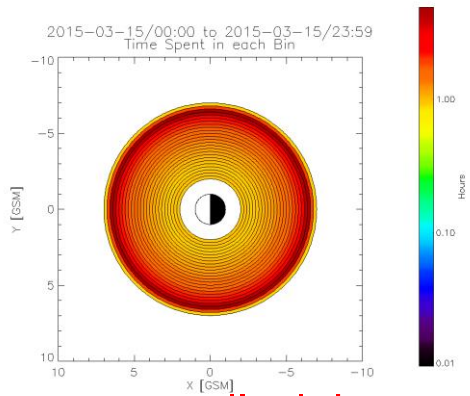
Backup

Methodology Overview

Pitch Angle Distributions at the Equator

- Obtain equatorial pitch angle distributions (PADs) from Van Allen Probe data
  - Map Van Allen Probe satellites to equator
  - Determine equatorial PAD from local PAD
- Create daily flux averages for each L shell bin (2-7 RE with 0.2 RE increments), energy bin, and pitch angle bin.
- Fly GPS through the model to obtain daily fluences
  - Use GPS ephemeris to map GPS to equator
  - Determine which portion of the equatorial PAD will reach GPS
  - Integrate to obtain omnidirectional flux at GPS
  - Collect flux estimate every minute, and integrate over the day
- Use daily integrated fluxes as input to degradation models

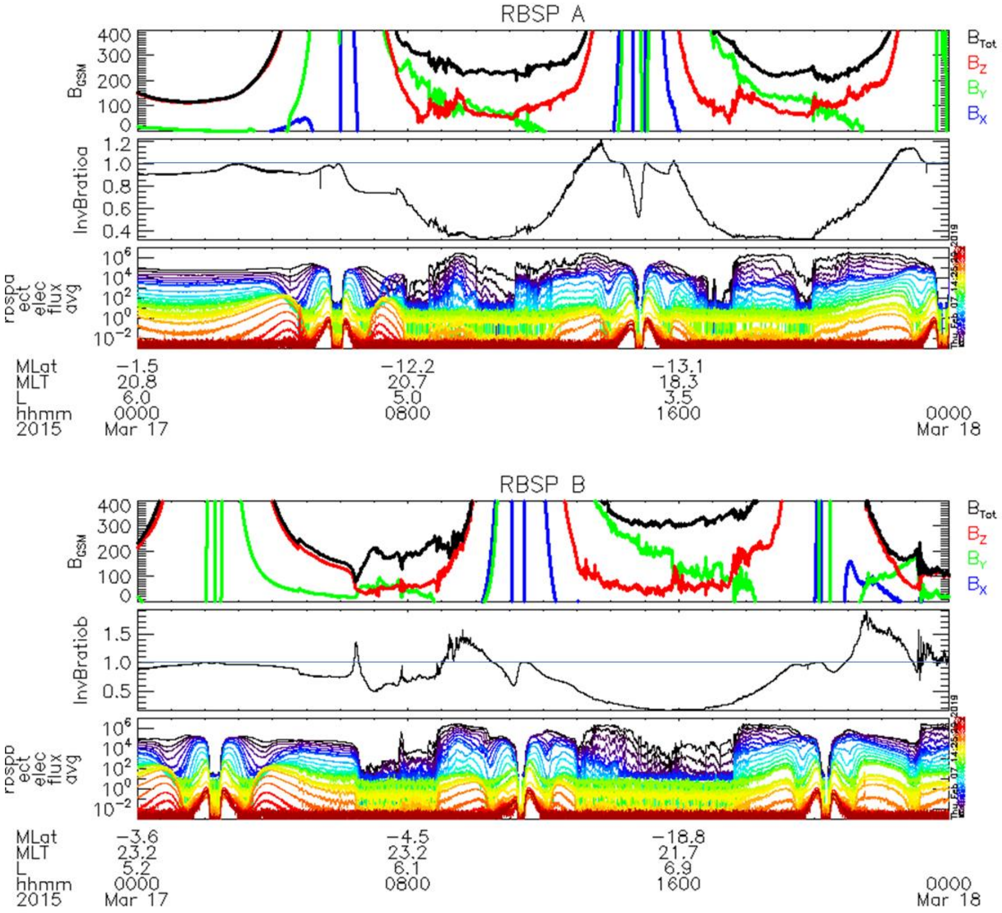




Methodology

Mapping various pitch angles

- Because GPS could be at the equator when Van Allen Probes is far from the equator, we need to fill in the populations that Van Allen Probes did not see.
- Step 1: Convert local pitch angles to equatorial pitch angles.
    - Use the L shell and Beq data provided with the MagEIS product. Use REPT
    - Use MagEIS data without background correction.



$$\alpha_{eq} = sin^{-1}\sqrt{\frac{B_{eq}}{B_{sc}} sin^2(\alpha_{sc})}$$

InvBratio= $\frac{B_{eq}}{B_{sc}}$

There are some points when Beq > Bsc, so InvBratio >1.

This doesn't really mean anything, it must result from the data not following the model. (For example, dipolarization could mean observed B is larger than modeled B at the equator. Or a compression of the Magnetosphere could increase observed B.)

Because asin(1.5)=NaN, the mapped pitch angles are stored as NaNs when Beq>Bsc. They are interpolated over.

Treated α_{sc} as α_{eq}.

Methodology

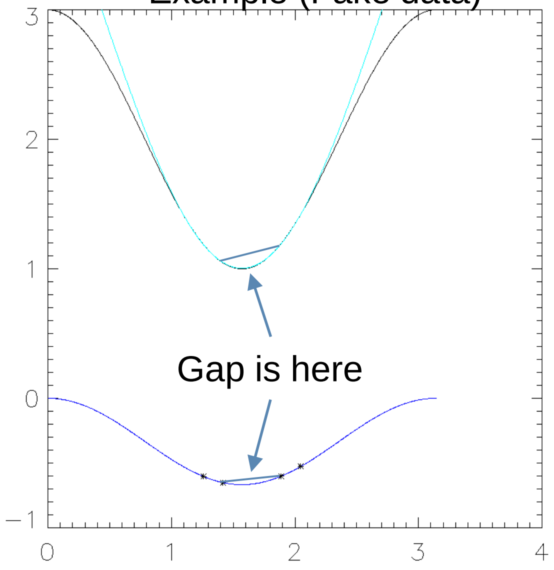
Pitch angle model

To fill the gap, I use the four data points surrounding the gap and treat them as part of a sine function of the following form:

$$flux = A * \sin(B\,\alpha + \frac{\pi}{2} - B\,\frac{\pi}{2})$$

MPFIT uses the Levenberg-Marquardt technique to solve the least-squares problem.

Example (Fake data)



Black: “the data” to fit
Navy: Normalized data
4 dots: the 4 normalized data points to fit
Cyan: Fitted sine function.

Remember: Only need to fit the gap, the wings do not matter.

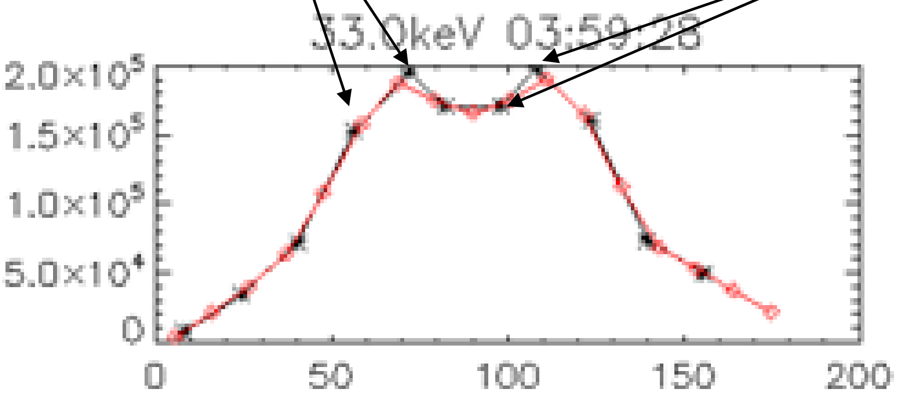
Method to fill the gap:

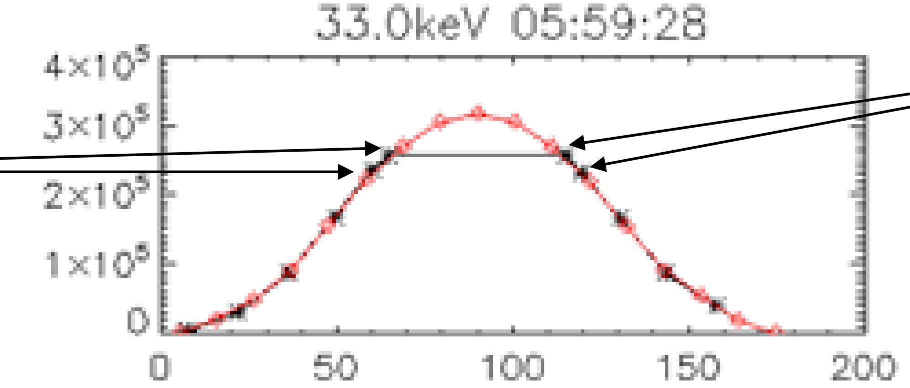
1. Normalize the PAD by the maximum flux at that time and energy channel (so flux ranges from 0 to 1).
2. Determine if there is a decrease or an increase at 90 based on the four surrounding data points.
   - If there is a decrease,
     - Subtract the PAD by 1 (see example). Always want 0 and 180 to be near 0.
     - Set initial A parameter to -0.5. (Helps the fitting routine know where to start.)
   - If there is an increase,
     - Set initial A parameter to 1. (Helps the fitting routine know where to start.)
3. Run fitting routine, which fits sine function to 4 data points near gap.
   - Outputs: A and B.
4. Use A and B in Flux eqn to calculate flux for 80, 90, and 100 deg equatorial pitch angle.
5. Multiply by the maximum flux to “un-normalize” the mapped data.
6. Use real mapped data & values calculated in step 4. Interpolate to the 17 pitch angle values in REPT to get consistent equatorial pitch angle outputs.

Methodology

Pitch angle model

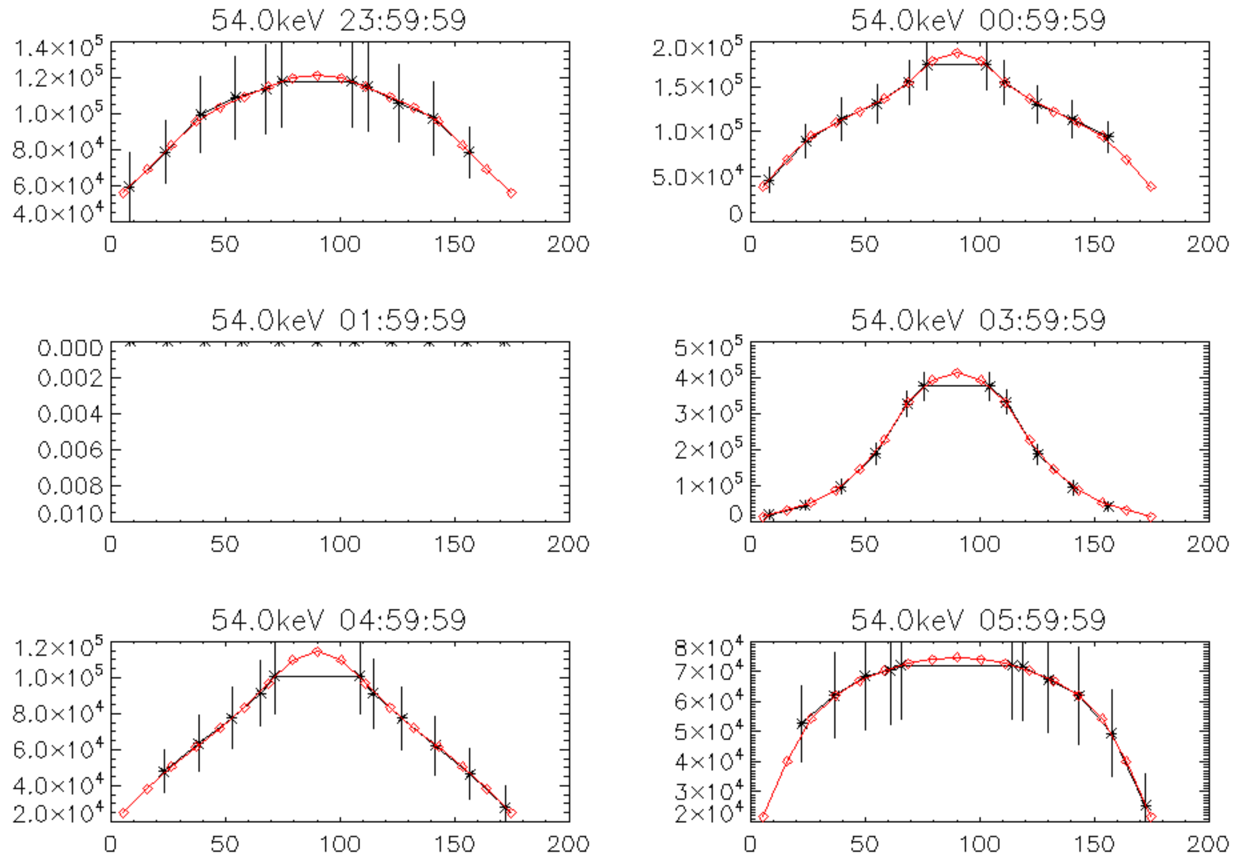
Real examples. Arrows point to the four data points used to do the sine fit.

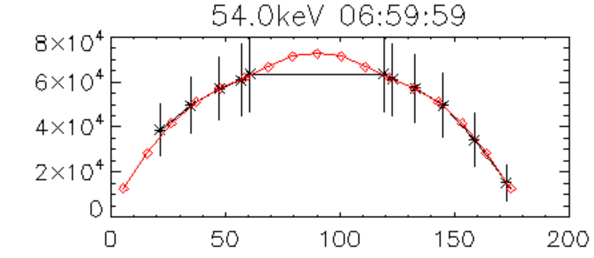




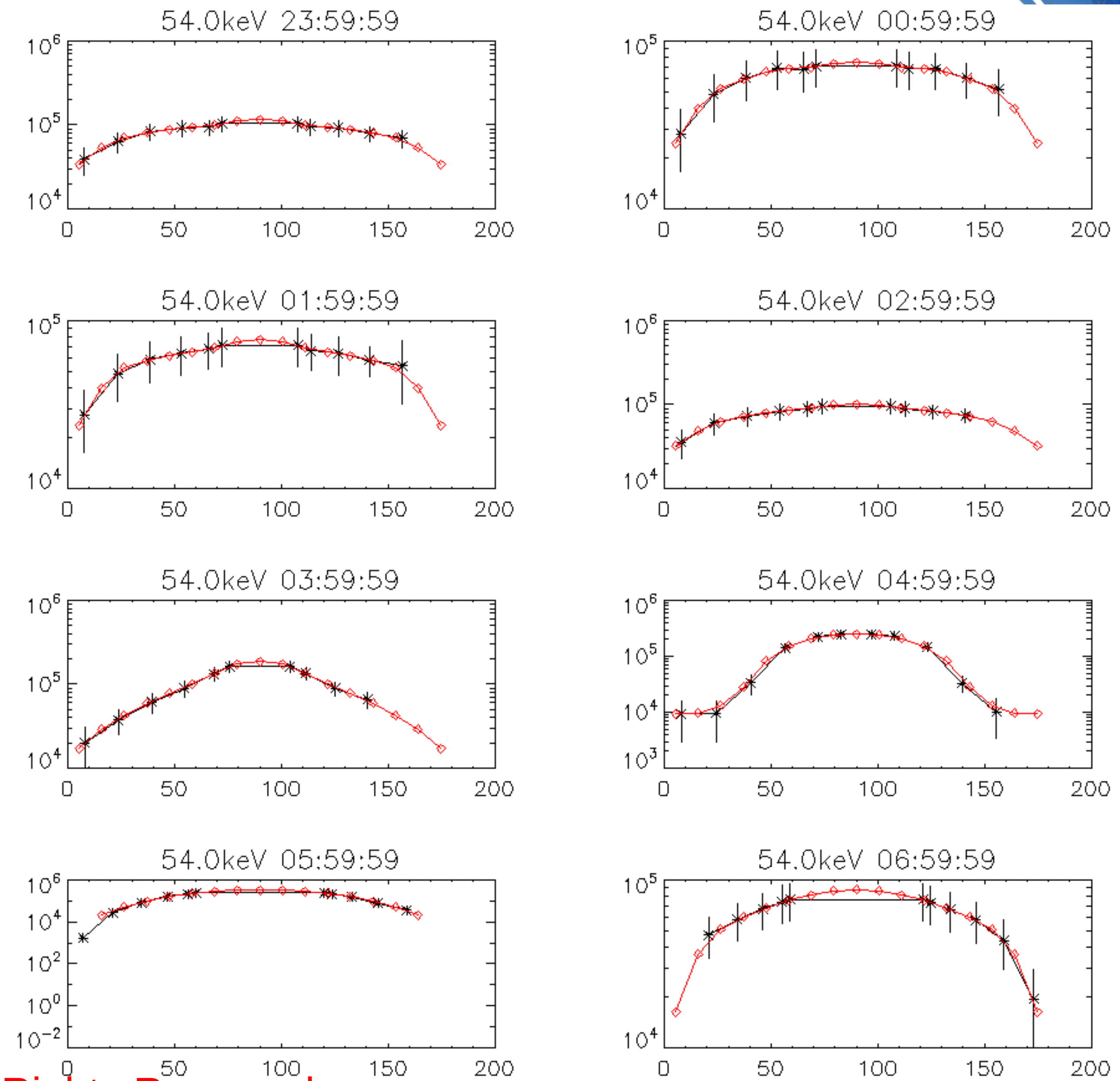
$$flux = A * \sin(B\,\alpha + \frac{\pi}{2} - \mathrm{B}\,\frac{\pi}{2})$$

54 keV fits to fluxes on 2015-03-26 for RBSP-A (left) and RBSP-B (right)





Fitting was very good.



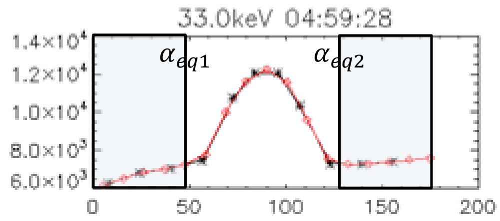
Applied Product (Results)

GPS daily flux counts over time, separated by energy

Knowing the equatorial fluxes, we fly a virtual GPS through the flux map to determine what fluences GPS observed.

1. Map GPS to equator to obtain its L shell and Beq. Initially use IGRF. (Now using T89)
2. Using Beq and Bsc, calculate maximum equatorial pitch angle that GPS observed.

$$\alpha_{eq1} = sin^{-1}\sqrt{\frac{B_{eq}}{B_{sc}} sin^2(90°)} \qquad \alpha_{eq2} = 180 - \alpha_{eq1}$$



Example of fluxes GPS actually sees (blue).

3. Integrate fluxes for all observed pitch angles:

$$j_{omni}(E) = \frac{\int_0^{\pi} j(\alpha, E)\sin\alpha\, d\alpha}{\int_0^{\pi} \sin\alpha\, d\alpha}$$

I interpolated the equatorial fluxes from RBSP to get the exact flux at α_{eq1} and α_{eq2}.

$$\text{J(omni)=} \frac{\left(\int_0^{\alpha_{eq1}} flux(t,W,\alpha)\sin(\alpha)d\alpha + \int_{\alpha_{eq2}}^{180} flux(t,W,\alpha)\sin(\alpha)d\alpha\right) \quad \text{[\#/cm\^{}2/s/str/keV]}}{\left(\int_0^{\alpha_{eq1}} \sin(\alpha)d\alpha + \int_{\alpha_{eq2}}^{180} \sin(\alpha)d\alpha\right)}$$

Where flux is a function of time (t), energy (W), and pitch angle (α).

Applied Product (Results)

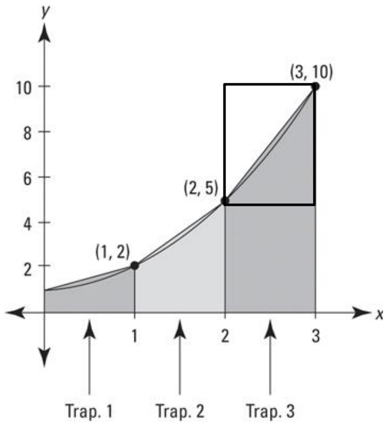
GPS daily flux counts over time, separated by energy

Using the trapezoid rule, we can numerically integrate over the equatorial pitch angle distribution to get the omnidirectional flux at GPS.

$$j_{omni} = \frac{\sum_{n=0}^{m_1}(j_{n+1} + j_n)\big(-\cos(\alpha_{n+1}) + \cos(\alpha_n)\big) + \sum_{n=m_2}^{m_3}(j_{n+1} + j_n)\big(-\cos(\alpha_{n+1}) + \cos(\alpha_n)\big)}{(\cos(0) - \cos(\alpha_{m1})) + (\cos(\alpha_{m2}) - \cos(\pi))}\left(\frac{1}{2}\right)$$

Where m1 corresponds to the pitch angle bin α_{eq1}, the largest pitch angle below 90 that reaches GPS, m2 corresponds to the pitch angle bin α_{eq2}, the lowest pitch angle bin above 90 that reaches GPS, and m3 corresponds to the 180 (or π) pitch angle bin. We divide by 2 for trapezoid rule.

We multiply the above by 4π to integrate over steradians, and by 60 because GPS has a one-min cadence. Multiplying by 60 integrates over time. Units are #/cm^2-keV. (Easily converted to /MeV.)



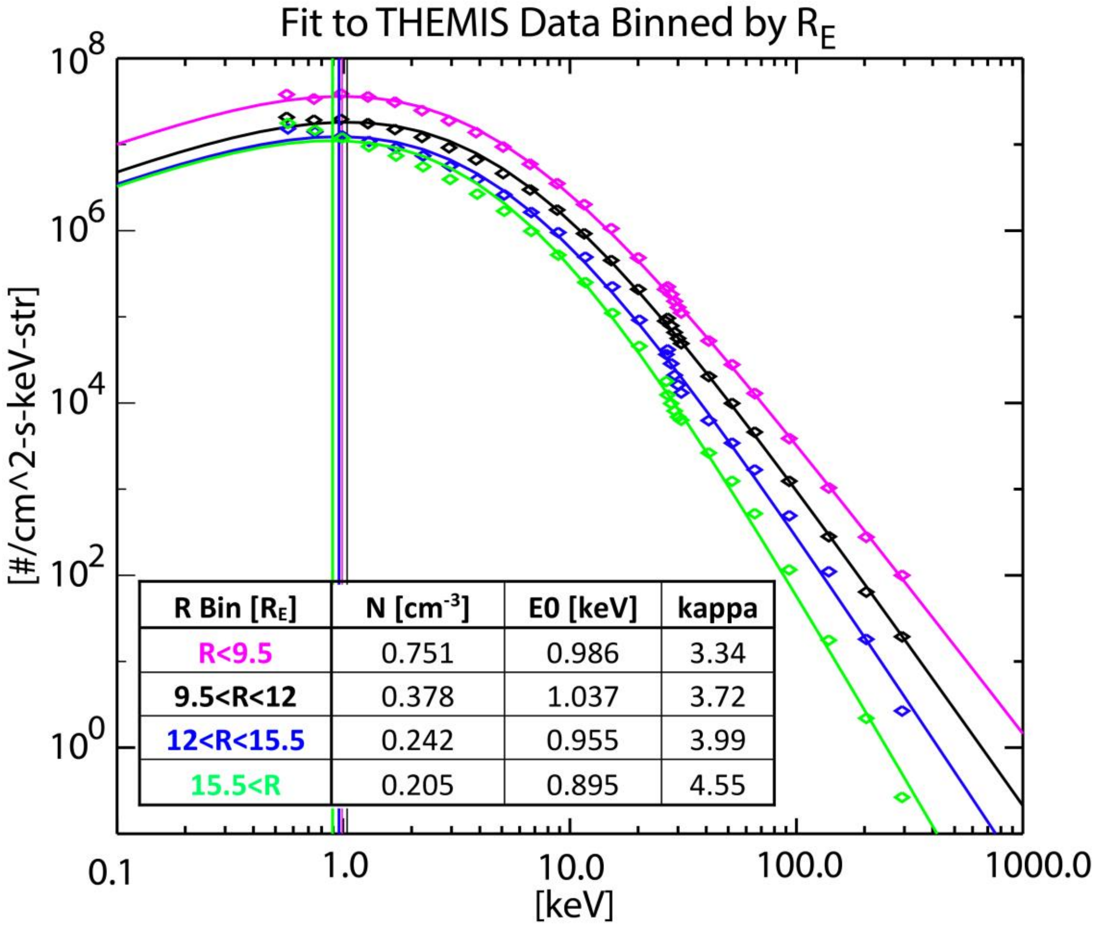
Trapezoid rule: find the area in the rectangle formed by (X2,Y2) and (X3, Y2). Add this area to half of the area of the rectangle formed by (X2,Y2) and (X3, Y3).

J2*(α3- α2)+(J3-J2) *(α3- α2)/2
-> (α3- α2)*(J2+(J3-J2)/2)
->(α3- α2)(1/2) * (2J2+J3-J2)
->(α3- α2)(1/2) * (J2+J3)

Figure from “Dummies.com”

Runov et al. 2015 Results

Four R-bins from THEMIS statistics in the plasma sheet extends our flux model out to 25 R_E.



R Bin [R_E]	N [cm^{-3}]	E0 [keV]	kappa
R<9.5	0.751	0.986	3.34
9.5<R<12	0.378	1.037	3.72
12<R<15.5	0.242	0.955	3.99
15.5<R	0.205	0.895	4.55

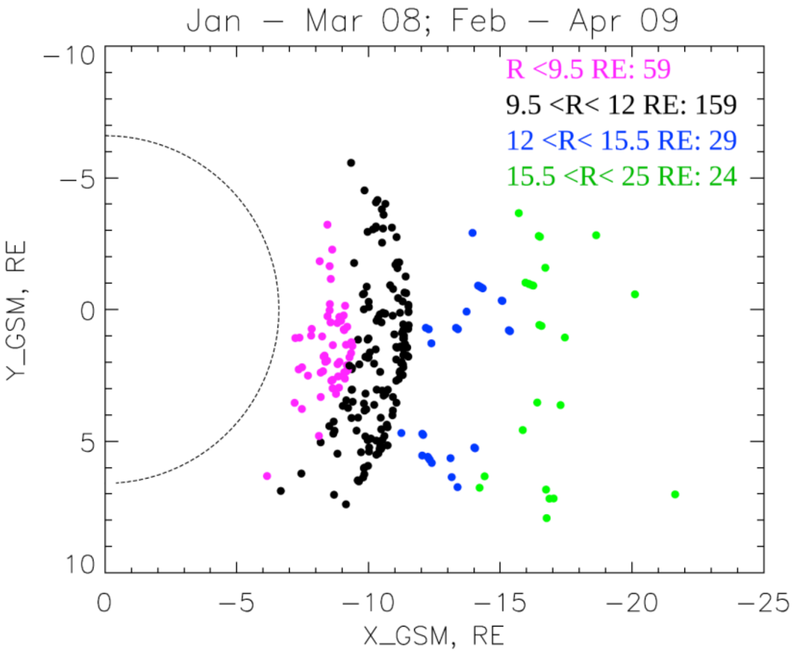


Figure 1. XY_{GSM} distribution of DFB/bubble events detected during the 2008 and 2009 THEMIS tail seasons. Colors indicate geocentric distance (magenta: $R < 9.5\ R_E$, black: $9.5 \geq R < 12\ R_E$, blue: $12 \geq R < 15.5\ R_E$, and green: $15.5 \geq R < 25\ R_E$).

Runov et al., AGU JGR, 2015

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