Constrained Surface-wave Dispersion Inversion Using GPR Reflection Data

Shufan Hu¹, Yonghui Zhao¹*, Wenda Bi¹, Ruiqing Shen¹, Bo Li¹, and Shuangcheng Ge²

¹School of Ocean & Earth Sciences, Tongji University, Shanghai, China
²Zhejiang University of Water Resources and Electric Power, Zhejiang, China
*Email: zhaoyh@tongji.edu.cn
OUTLINE

I  Context and motivation

II  Multiwindow-based inversion of surface-wave

III  Surface-wave inversion with GPR constraint

IV  Field data example

V  Summary
Context and motivation

- Ground penetrating radar (GPR) and Seismic Surface Wave methods (SWMs) are two nondestructive testing (NDT) methods commonly used in near-surface site investigations. GPR has a good resolvability to characterize the layered structure, while, the geometric dispersion of surface waves can be used to retrieve the variation of S-wave velocity (Vs) with depth.

- Limitations of SWMs:
  - 1D model assumption.
  - Non-uniqueness of the solution.

- Aims of this study:
  - Improving the lateral resolution of SWMs.
  - Achieving an appropriate representation of the subsurface.
Multiwindow-based inversion of surface-wave

In a dispersive medium, the cross-power spectrum of two signals observed at two stations after amplitude normalization is presented as

\[ R^w_{mn}(\omega) = \frac{S_m(\omega) \cdot S^*_n(\omega)}{|S_m(\omega)| \cdot |S_n(\omega)|} = \exp[-jk_{mn}(\omega)(x_m - x_n)] \]

where \( k_{mn}(\omega) \) is the spatial wavenumber that controls wave propagation from the \( m \)th receiver to the \( n \)th receiver.

For multichannel seismic records, surface wave dispersion curves can be estimated using the plane-wave frequency-wavenumber method given by

\[ P(\omega, k) = e^H(k)WR(\omega)W^He(k) \]

where \( H \) denotes the Hermitian transpose; \( \omega \) is the angular frequency (\( \text{rad/s} \)); \( k \) is a trial wavenumber (\( \text{rad/m} \)); \( W \) is a diagonal matrix containing shading weights for each receiver; \( R(\omega) \) is the spatio-spectral correlation matrix which is assembled by the cross-power spectra between all receiver pair combinations; \( e(k) \) is the plane-wave steering vector.

We can apply the reciprocal of amplitude as the shading weights for each receiver, the output power becomes

\[ P(\omega, k) = e^H(k)R^w(\omega)e(k) = \sum_{m=1}^{K} \sum_{n=1}^{K} \exp[-j(x_m - x_n)(k_{mn}(\omega) - k)] \]
Multiwindow-based inversion of surface-wave

Similarly, we can use the same theory to calculate the theoretical dispersion curve of multichannel seismic data, as shown below:

- Haskell-Thomson approach
- Computing spatial wavenumber between all receiver pair combinations
- Local maximum search

![Diagram showing multiwindow-based inversion of surface-wave](image-url)
Multiwindow-based inversion of surface-wave

The data vector consisting of surface-wave dispersion data $d^{obs}(\omega_i, W_j)$, gathered along the applied spatial window $W_j$ and analyzed frequency $\omega_i$, is represented as:

$$d^{obs} = \log\left(d^{obs}(\omega_i, W_j)\right), i = 1, 2, ..., I; j = 1, 2, ... J^T$$

The model vector is a collection of the shear-wave velocity $V^s$ in each cell:

$$m = \log(V^s_{pq}), p = 1, 2, ..., P; q = 1, 2, ... Q^T$$

We define the objective function $\phi(m)$ based on L1-norm regularization as:

$$\phi(m) = \frac{1}{2} \left\| W_d \left( F[m] - d^{obs} \right) \right\|_2^2 + \frac{1}{2} \lambda \left\| Lm \right\|_1$$

where $W_d$ is the data weighting matrix; $\lambda$ is the regularization parameter; $L$ is the roughness matrix, which is a combination of the first-order differential operator in the x and y directions:

$$L = \begin{bmatrix} a_x L_x \\ a_y L_y \end{bmatrix},$$

where $a_*$ can be applied as scalars or diagonal matrices with varying weight for each parameter.
Multiwindow-based inversion of surface-wave

A synthetic test:

Fig.1 (a) Synthetic model. The red circles and black lines depict the locations of shots and receivers, respectively. (b) The corresponding first common-shot-gather. (c) Stacked dispersion curves extracted with a moving spatial window of 11 receivers and an overlap of 9 receivers. (d) Stacked dispersion curves extracted with a moving spatial window of 21 receivers and an overlap of 20 receivers.
Multiwindow-based inversion of surface-wave

Fig.2 (a) Initial model. (b-c) 2D Vs sections retrieved by traditional surface-wave inversion (we call it as the single window-based inversion, SWI; L1-norm regularization is also used in SWI for the sake of comparison) and multiwindow-based inversion (MWI), respectively. (d) The plot of the velocity value for the row of model cells between the depths of 3 and 3.5 m.

- The lateral resolution is improved.
- It is hard to interpret the exact depth of the interface.
- The velocity of the half-space is underestimated.
Surface-wave inversion with GPR constraint

A constant $a_x$ used in the roughness matrix $L$ can be inappropriate if the implied smoothing is applied along the direction across an interface or boundary.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\varepsilon_r$ (-)</th>
<th>$\sigma$ (mS/m)</th>
<th>$\mu_r$ (-)</th>
<th>$V_p$ (m/s)</th>
<th>$V_s$ (m/s)</th>
<th>$\rho$ (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>10</td>
<td>1</td>
<td>340</td>
<td>170</td>
<td>1.9</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>540</td>
<td>270</td>
<td>1.9</td>
</tr>
</tbody>
</table>
Surface-wave inversion with GPR constraint

Fig. 3 (a) Initial model. (b) 2D Vs sections retrieved by MWI.

- It is difficult to delineate the isolated high-velocity body along the depth direction.
Surface-wave inversion with GPR constraint

Fig. 4 (a) Synthetic radargram generated by gprMax (Warren et al., 2016). (b) Radargram after AGC, migration and time-depth conversion. The velocity is adapted by hyperbola fitting. (c) Vs model cell built with GPR constraints. (d) Inverted Vs model with GPR constraints (the velocity in the initial model is also homogenous).

The bottom depth is 3.5 m (compared to the true value of 4 m).

\[ a^* \text{ is zero along the direction across the boundary and the interface.} \]
Field data example

- GPR data is acquired by MALA radar and the equipped 50 Hz antenna.
- The seismic signal is generated by a hammer source and recorded by a Geode seismograph (Geometrics Inc.) with 24 4-Hz vertical geophones at 1.5 m interval. The common-receiver-spread recording geometry is used and 25 shot records are acquired along the receiver spread at 1.5 m interval.
- The stratum in the constrained inversion result is more clear and is similar to the drilling data near this site.

Fig.5 (a) Processed GPR profile. (b) Vs model retrieved with MWI. (c) Vs model retrieved with MWI and GPR constraints.
Summary

The proposed MWI strategy can fully exploit the data redundancy in multichannel seismic records. The dispersion data extracted from a smaller spatial window provide higher lateral resolution information regarding the shallow stratum, while the data evaluated from a wider spatial window allows for the investigation of deeper formations.

With the constraints evaluated from GPR reflection data, the non-uniqueness of the solution in the surface wave dispersion inversion is mitigated, and the retrieved Vs model presents a structure that is more consistent with the existing structural features.

A constant velocity model used in time-to-depth conversion might produce an erroneous depth constraint for surface wave inversion. The incorporation of 2D velocity analysis in GPR data is our current study topic.
Thank you for your attention!