Causality and information transfer in systems with extreme events

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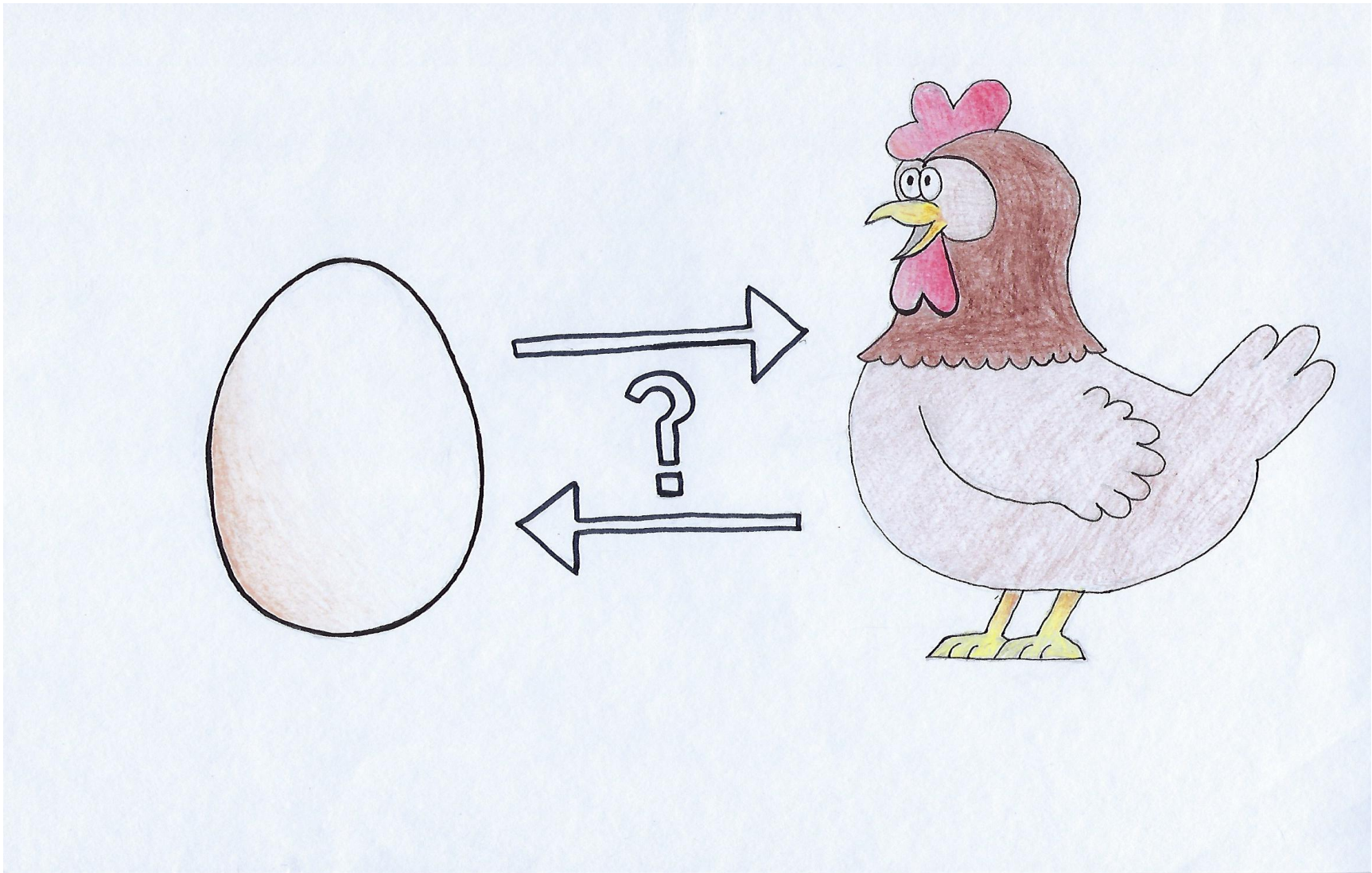
Support:
Praemium Academiae, Czech Academy of Sciences
Czech Science Foundation, project GA19-16066S

Causality analysis:
Can we identify causes of observed phenomena?



Granger causality


C. Granger, 2003 Nobel prize in economy

- causal variable can help to forecast the effect variable after other data has been first used
- generalization using information theory
  conditional mutual information (transfer entropy)
- $I\big(y(t); x(t+\tau)|x(t), x(t-\eta), \ldots, x(t-(n-1)\eta)\big)$
  - $y(t)$ – the cause (predictor)
  - $x(t+\tau)$ – the effect (the future of the influenced variable)
  - $x(t), x(t-\eta), \ldots, x(t-(n-1)\eta)$ – condition – removes the influence of history of the influenced variable
- for Gaussian systems CMI $\equiv$ TE $\equiv$ Granger causality
- causality interpreted as information transfer

Mutual information

- mutual information

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

- average amount of common information, contained in the variables X and Y
- measure of general statistical dependence
- $I(X;Y) \geq 0$
- $I(X;Y) = 0$ iff X and Y are independent

Conditional mutual information

- conditional mutual information $I(X; Y|Z)$ of variables X, Y given the variable Z

$$I(X; Y|Z) = H(X|Z) + H(Y|Z) - H(X, Y|Z)$$

- Z independent of X and Y

$$I(X; Y|Z) = I(X; Y)$$

- $I(X; Y|Z) = I(X; Y; Z) - I(X; Z) - I(Y; Z)$
- "net" dependence between X and Y without possible influence of Z

Entropy definitions

random variable X with sets of values Ξ and PDF's $p(x)$

- Shannon entropy

$$H(X) = -\sum_{x \in \Xi} p(x) \log p(x)$$

- **Rényi entropy**

$$H_\alpha(X) = \frac{1}{1-\alpha} \log \sum_{x \in \Xi} p(x)^\alpha,$$

  where $\alpha > 0$, $\alpha \neq 1$.
- As $\alpha \to 1$, $H_\alpha(X)$ converges to $H(X)$ which is Shannon entropy.
- Rényi's measure satisfies $H_\alpha(x) \leq H_{\alpha'}(x)$ for $\alpha > \alpha'$.

Rényi entropy

Rényi order parameter $\alpha > 0$, $\alpha \neq 1$

$$p(x)^{\alpha}$$

- $\alpha > 1$ “amplifies” center of PDF
- $0 < \alpha < 1$ “amplifies” **tails** of PDF
- tails – extreme events

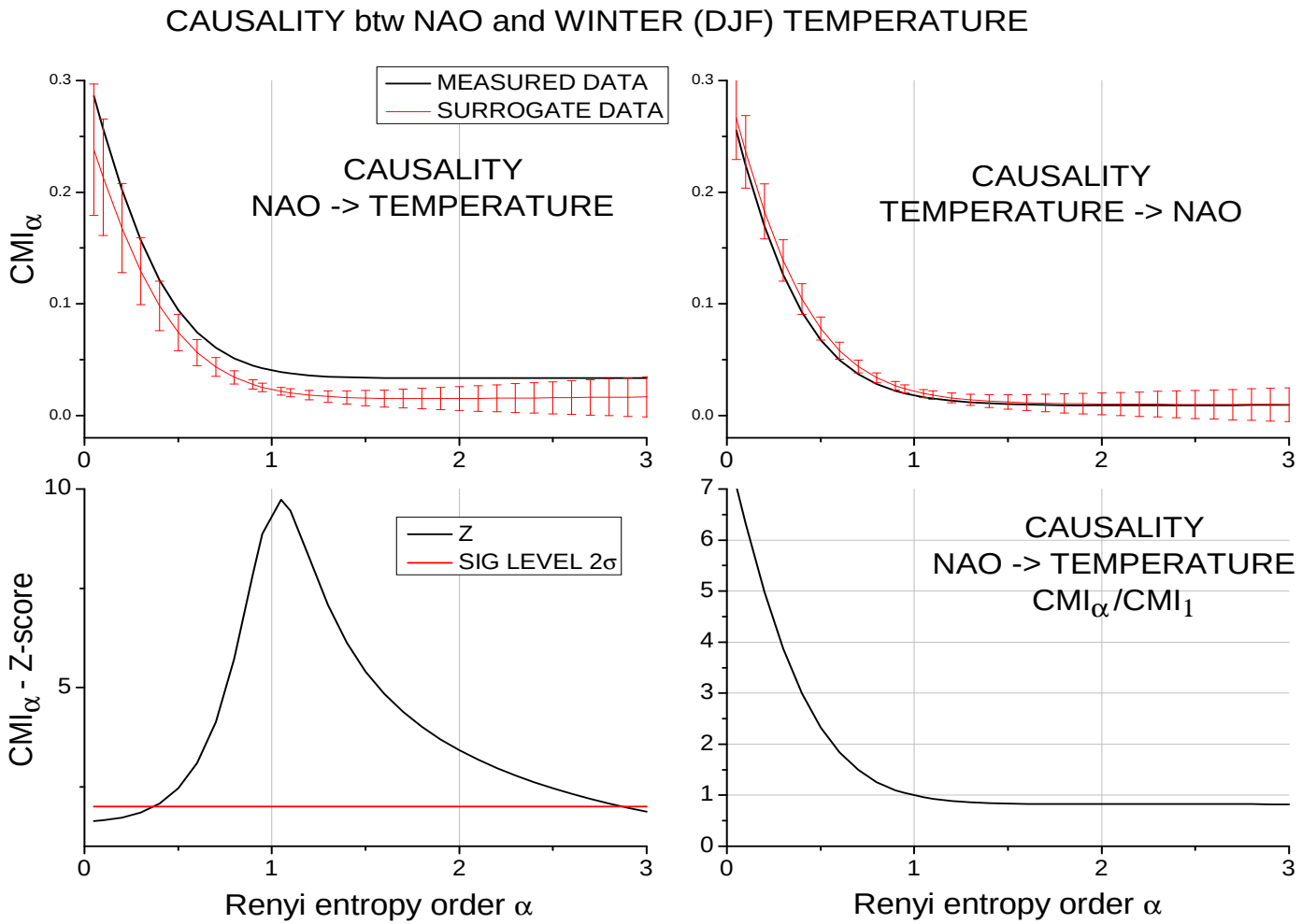
Can conditional mutual information (transfer entropy) in Rényi concept uncover causes of extreme events?

Experiment with Rényi CMI

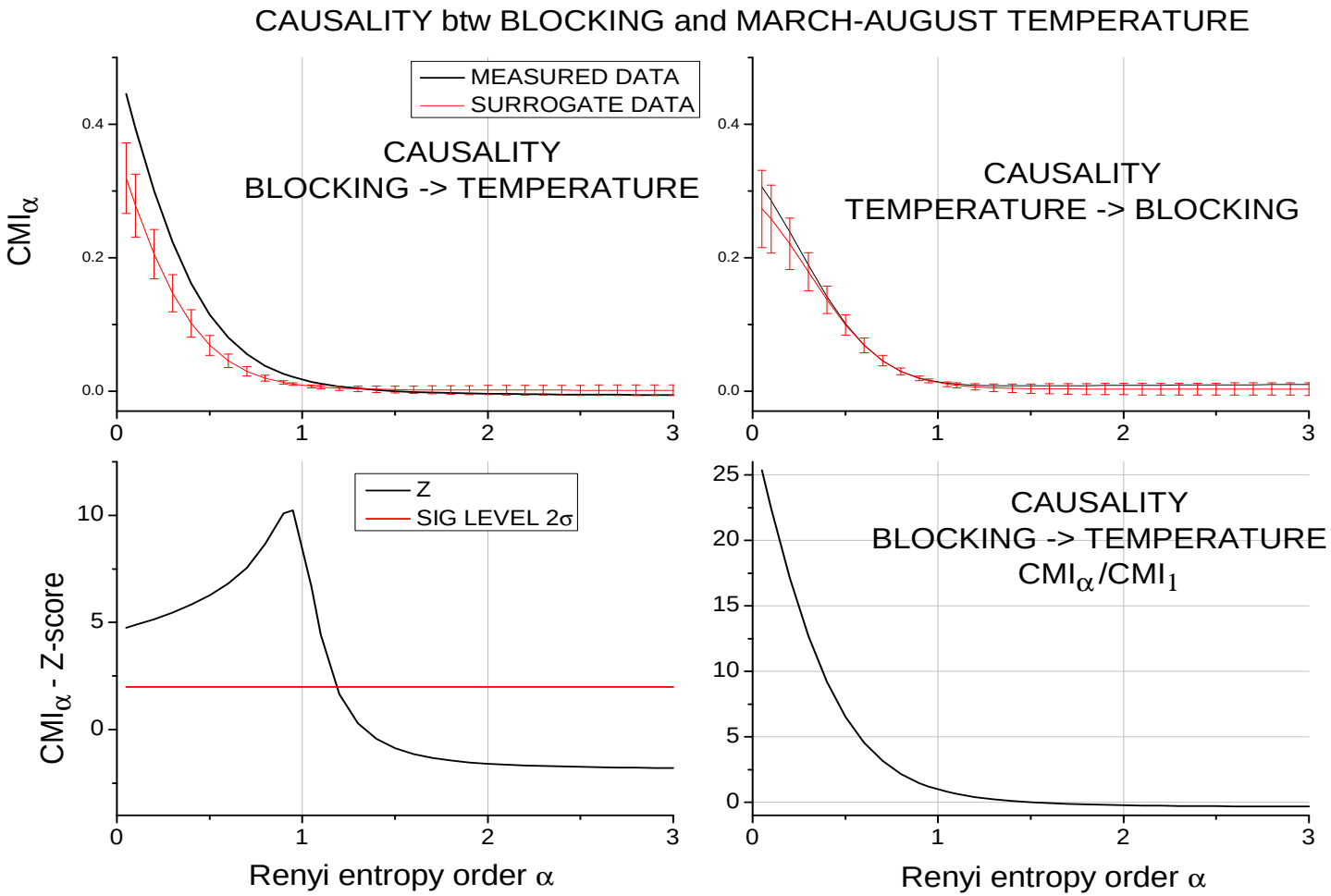
DATA

- near surface air temperature (SAT) anomalies
- daily station data, example Frankfurt
  (50° 02' 47"N, 8° 35' 54"E, 112 m above sea level)
- daily NAO index
- daily blocking index for 9°E
  Tibaldi S, Molteni F (1990) On the operational predictability of blocking. Tellus 42A : 343—365

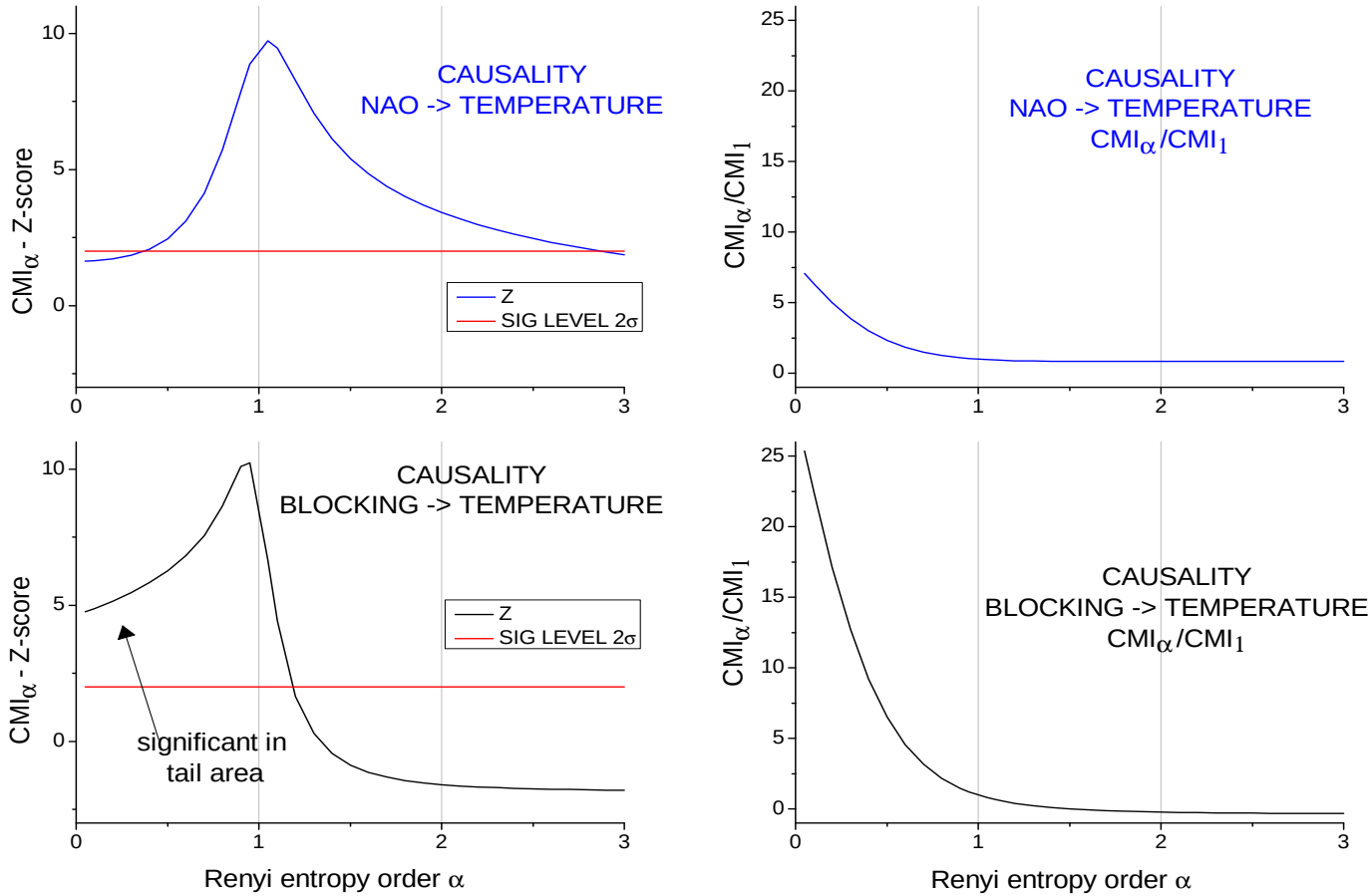
Causality in Rényi concept



Causality in Rényi concept



Causality in Rényi concept: NAO vs blocking



Experiment with Rényi CMI

RESULTS

- NAO significant causality mainly for $\alpha > 1$
- blocking significant causality mainly for $\alpha < 1$
- Rényi CMI normalized by Shannon CMI for $\alpha < 1$ much larger for blocking

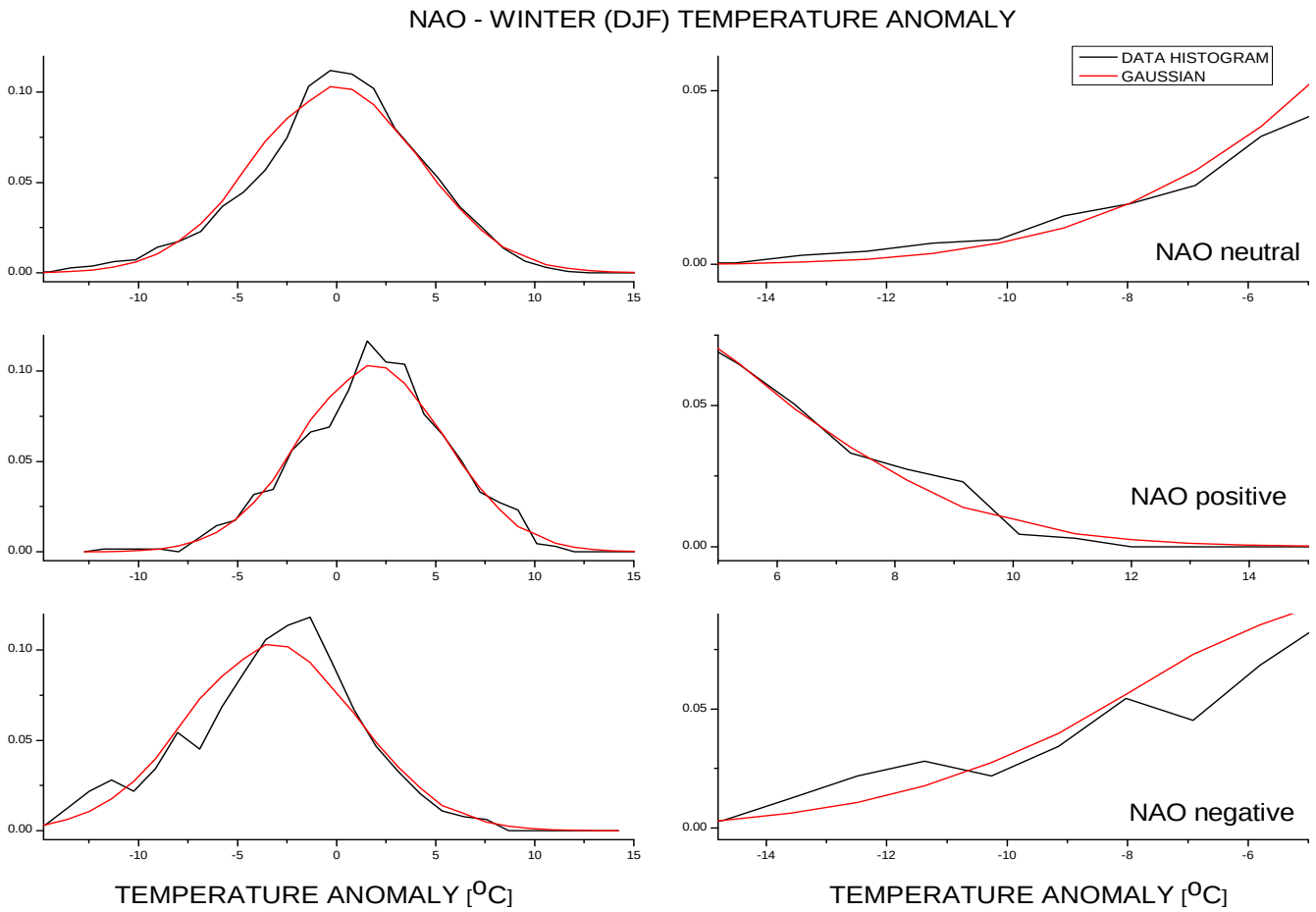
CAN WE INFER

- NAO is causal to temperature, but not causing extremes
- blocking is causal mainly with respect to extremes

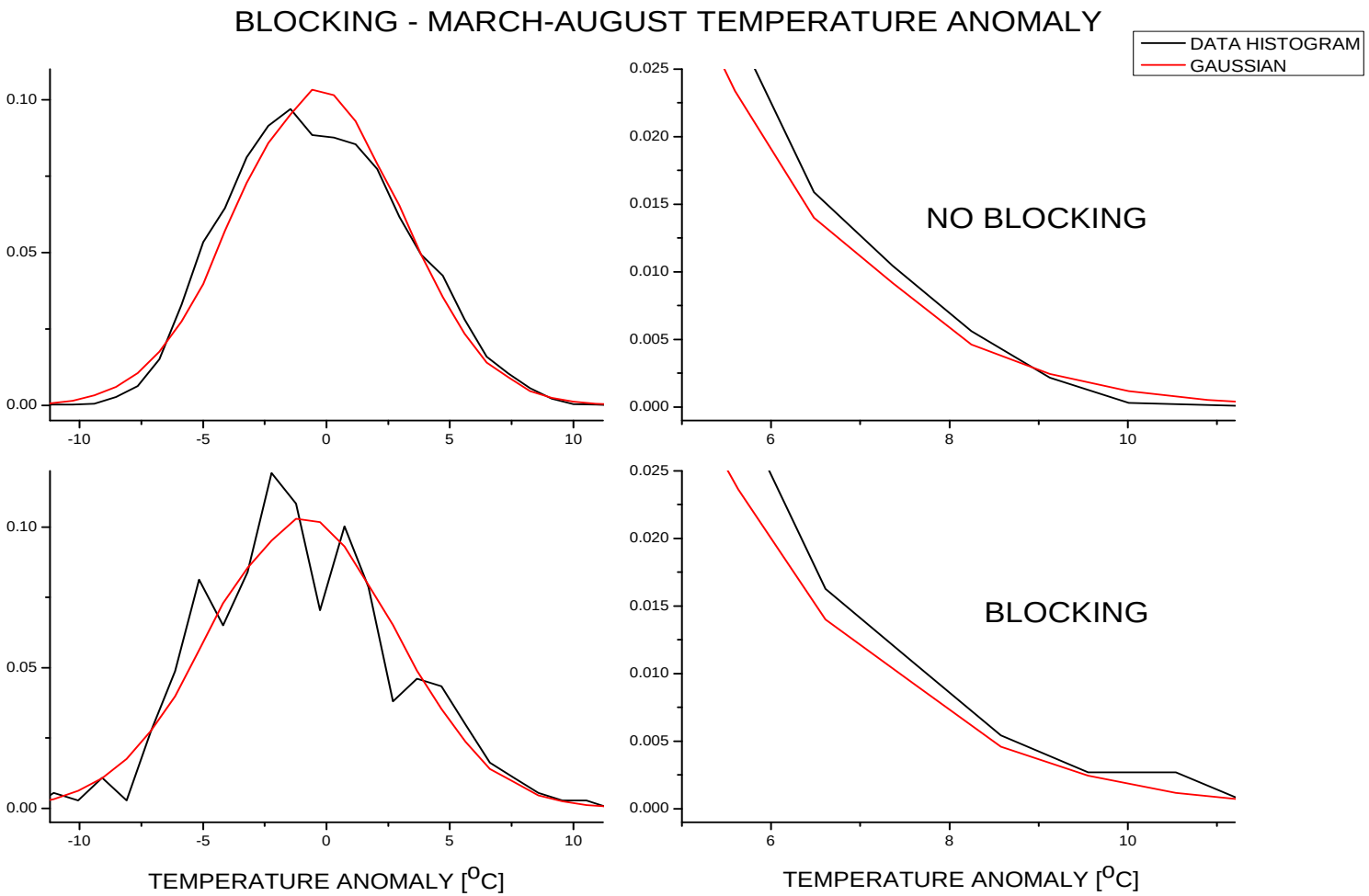
COMPARE

- distribution of SATA for different NAO/blocking conditions
- SATA distribution with Gaussian with the same mean and variance
- tails of data vs Gaussian distribution

Conditional distributions



Conditional distributions



Conditional distributions

RESULTS

- NAO shifts means/whole histogram
- NAO does not specifically cause tails higher than normal
- blocking only slightly shifts means/whole histogram
- blocking causes tails higher than normal

- Iinformation-theoretic approach to causality in Rényi concept seems promising in identification of causes of extreme events
- many technical problems, however, should be solved in order to avoid false results
- research in progress, publication in preparation

CONCLUSION

Thank you for your attention

Interested in postdoc, PhD position?

Interested in part-time long distance job?
(Applicable in Covid-19 border closure conditions.)

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