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Non-iterative numerical model of soil freezing

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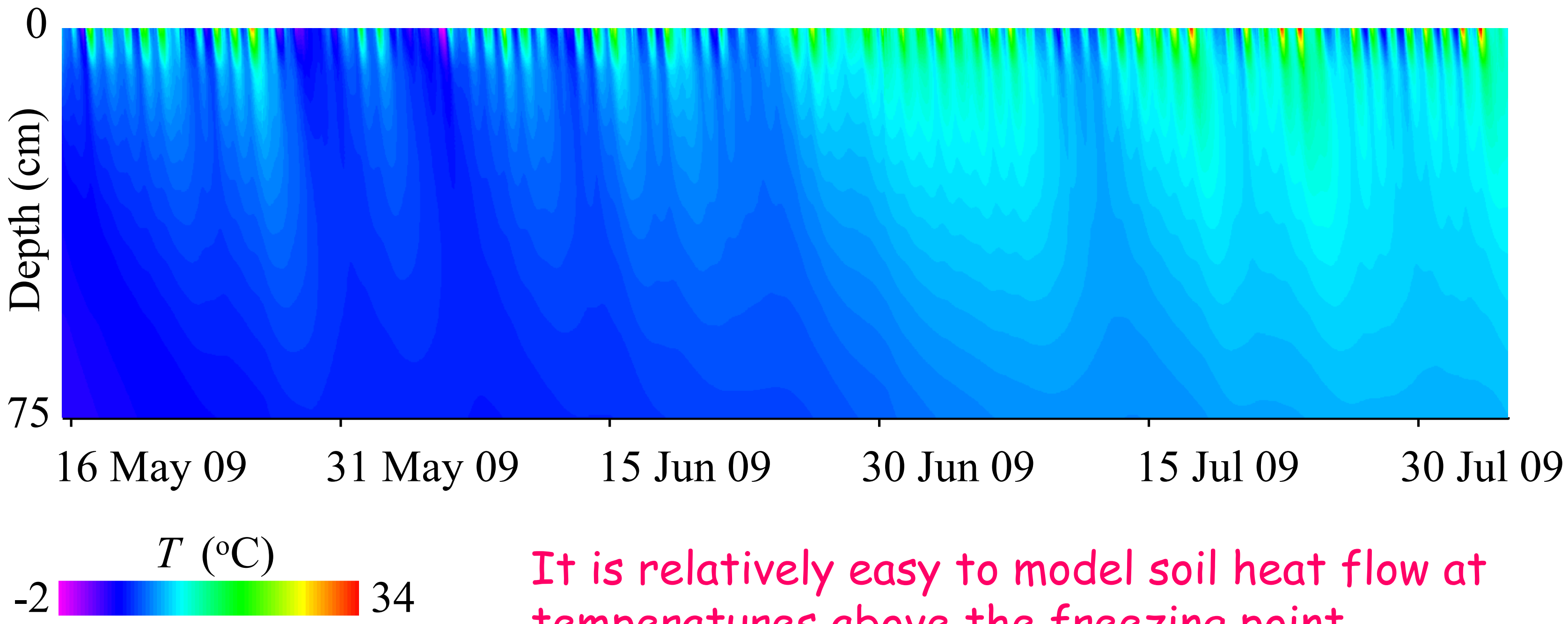

It is relatively easy to model soil heat flow at temperatures above the freezing point, however, phase transitions in freezing soils make modeling much more difficult.

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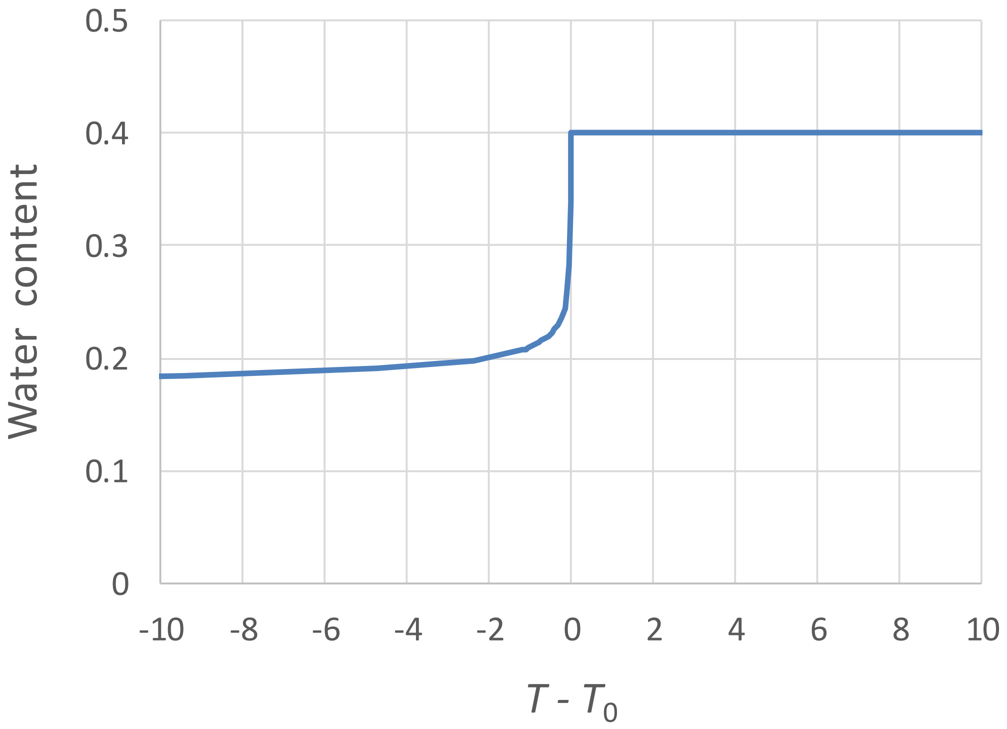
Simplifying assumptions

- Freezing point depression is neglected (the soil freezing curve is approximated by a step function)
- Soil is rigid (no soil heaving)
- Liquid water in frozen soil is immobile

The suggested modeling approach abstracts from many complexities associated with the freezing phenomena in soils, yet preserves the principal physical mechanism of conserving the internal energy of the soil system during phase transitions.

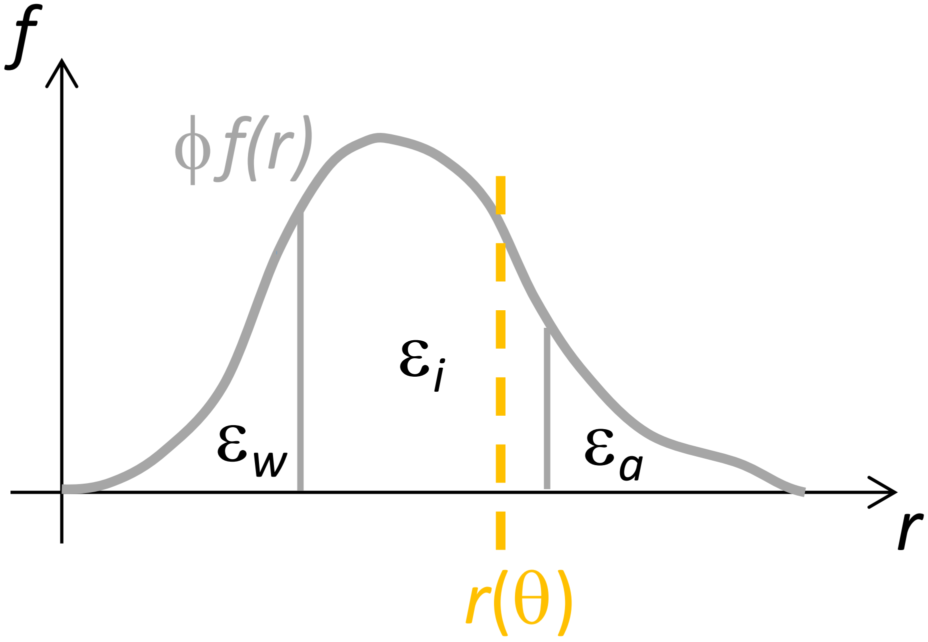


Soil freezing curve of a sandy loam.

Pore space partitioning

$$\varepsilon_w + \varepsilon_i + \varepsilon_a = \phi$$

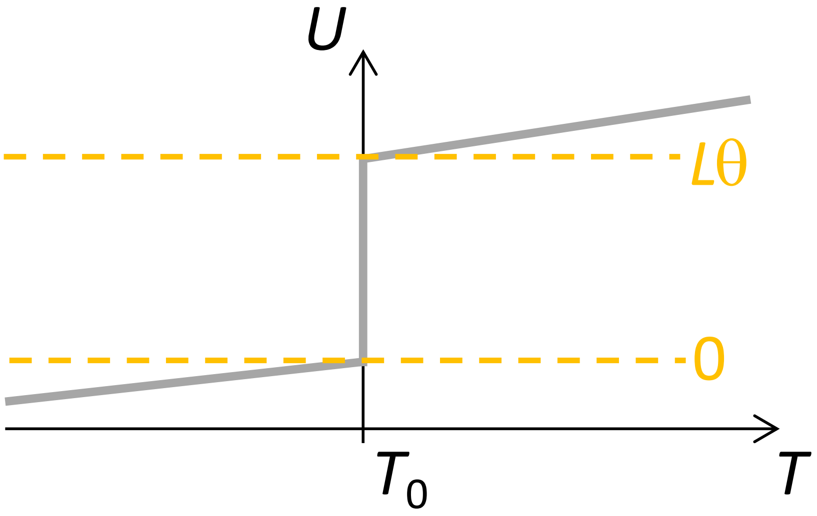
$$\rho_i \varepsilon_i + \rho_w \varepsilon_w = \rho_w \theta$$



Partitioning of pore space between liquid water, ice and air (ε_w, ε_i and ε_a are the volumetric fractions of liquid water, ice and air, $f(r)$ is the pore size distribution function, r is the pore radius, ϕ is the porosity, and θ is the equivalent liquid water content).

Internal energy

$U = L\varepsilon_w + C(T - T_0)$ = {Latent heat} + {Sensible heat}



Internal energy as a function of temperature and liquid water content (T_0 is the freezing point temperature, L is the product of the specific latent heat of fusion and liquid water density, and θ is the equivalent liquid water content).

Internal energy balance

$$\frac{\partial U}{\partial t} + \frac{\partial j_L}{\partial z} + \frac{\partial j_H}{\partial z} = -S_L - S_H$$

$$j_L = qL \qquad j_H = qc_w(T - T_0) - \lambda \frac{\partial T}{\partial z}$$

… Heat fluxes

$$S_L = S_w L \qquad S_H = S_w c_w (T - T_0)$$

… Heat sinks

$$L\frac{\partial \varepsilon_w}{\partial t} + \frac{\partial CT}{\partial t} + c_w \frac{\partial qT}{\partial z} - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + L\frac{\partial q}{\partial z} = -S_w L - S_w c_w T$$

Decomposition of the energy balance equation into separate equations for latent and sensible heat

$$\underbrace{L\frac{\partial \varepsilon_w}{\partial t} + L\frac{\partial q}{\partial z} + S_w L = \boxed{Q}}_{\text{Latent heat balance}}$$

$$\underbrace{\frac{\partial CT}{\partial t} + c_w\frac{\partial qT}{\partial z} - \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) + S_w c_w T = \boxed{-Q}}_{\text{Sensible heat balance}}$$

$$\boxed{Q} = -\frac{\rho_i}{\rho_w} L \frac{\partial \varepsilon_i}{\partial t}$$

Sink/source of
sensible heat due to
phase transition

Freezing cell conditions

$$0 < U < L\theta$$

$$T = T_0$$

$$\cancel{\frac{\partial CT}{\partial t}} + c_w \cancel{\frac{\partial qT}{\partial z}} - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \cancel{S_w c_w T} = -Q$$

$$L\frac{\partial \varepsilon_w}{\partial t} + \cancel{L\frac{\partial q}{\partial z}} + \cancel{S_w L} = Q$$

Numerical solution of the energy balance equation

$$A_{1n}T_{n-1} + A_{2n}T_n + A_{3n}T_{n+1} = B_n - Q_n$$

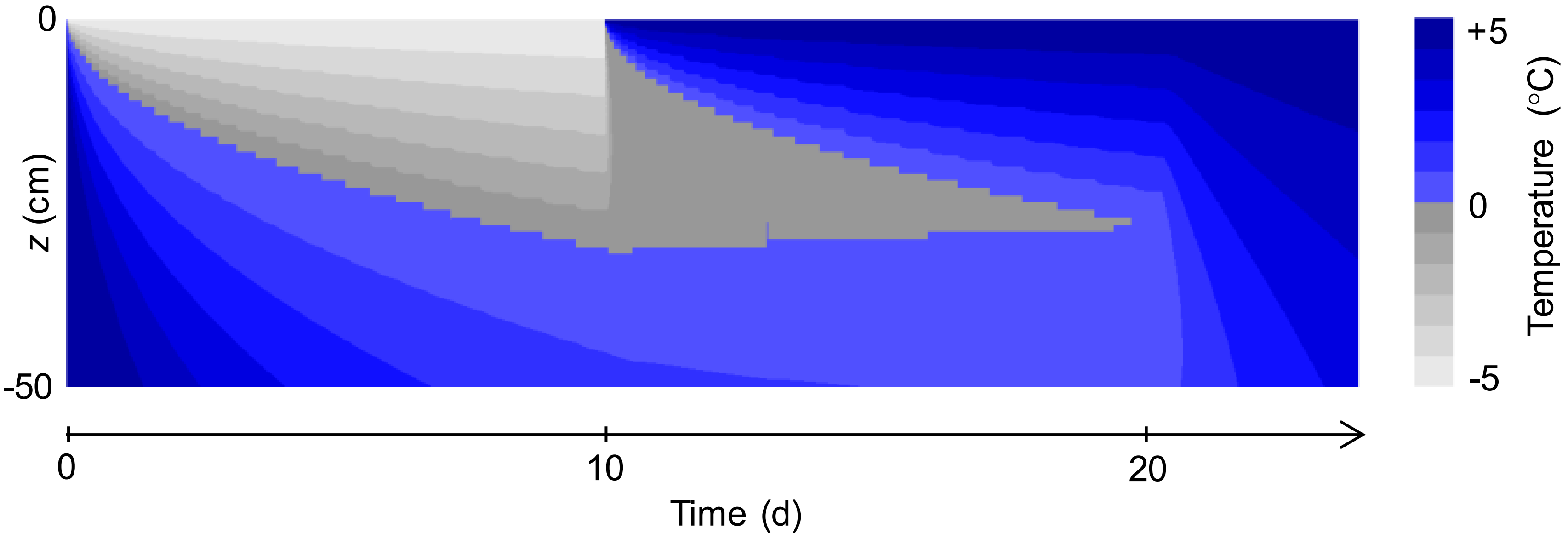
A and *B* are coefficients computed from the local soil thermal properties and soil water fluxes.

If U_n drops below $L\theta$ by cooling of unfrozen soil or rises above zero by warming of frozen soil at any nodal point *n*, the coefficients of the *n*-th row of the matrix system are stored and replaced by the following dummy values to force $T_n = T_0$:

$$A_{1n}^* = 0 \qquad A_{2n}^* = 1 \qquad A_{3n}^* = 0 \qquad B_n^* = T_0 \qquad Q_n^* = 0$$

The resulting set of equations is solved for the vector of nodal temperatures T_n. The original coefficients are then used to evaluate Q_n. After that, the values of Q_n are used to determine the changes of liquid water content ε_w caused by the phase transition at each nodal point of the freezing/thawing zone:

$$\left(\frac{\Delta\varepsilon_w}{\Delta t}\right)_n = -\frac{\rho_i}{\rho_w}\left(\frac{\Delta\varepsilon_i}{\Delta t}\right)_n = \frac{Q_n}{L}$$



Simulated development of the soil temperature profile during a freezing–thawing event. The soil, with the initial temperature of +5°C, responds to freezing conditions at the soil surface, represented by the surface temperature of -5°C over the period of 10 days, followed by warm conditions with the surface temperature of +5°C.

Further details can be found in:

Vogel T, Dohnal M, Votrubova J, Dusek J, 2019, Soil water freezing model with non-iterative energy balance accounting, *Journal of Hydrology*, Volume 578, 124071, https://doi.org/10.1016/j.jhydrol.2019.124071.