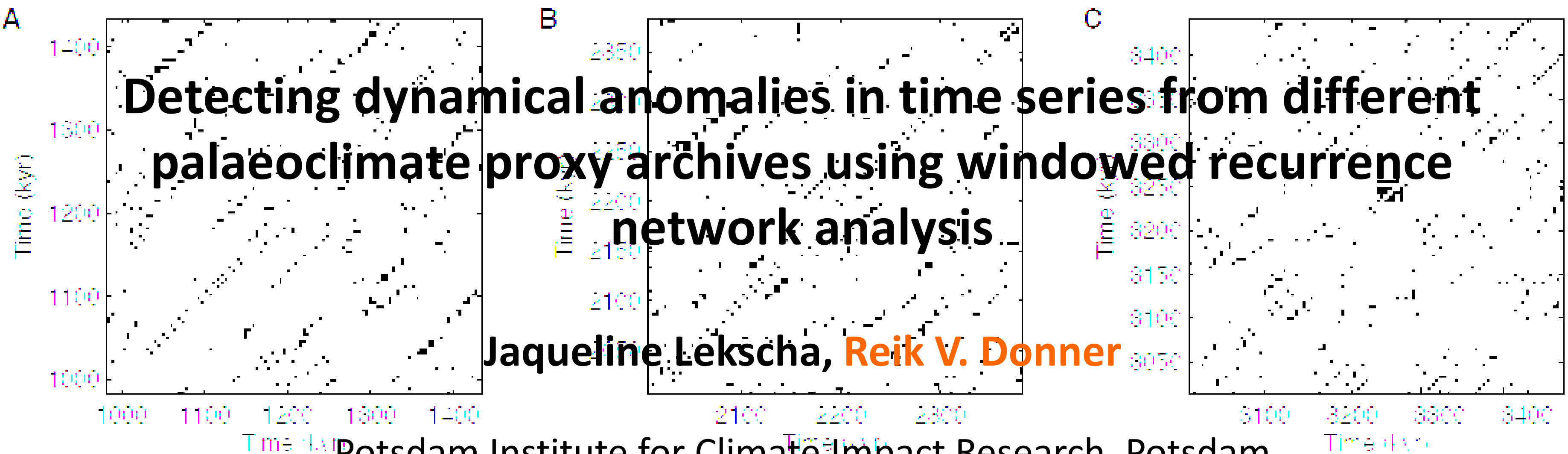

Detecting dynamical anomalies in time series from different palaeoclimate proxy archives using windowed recurrence network analysis

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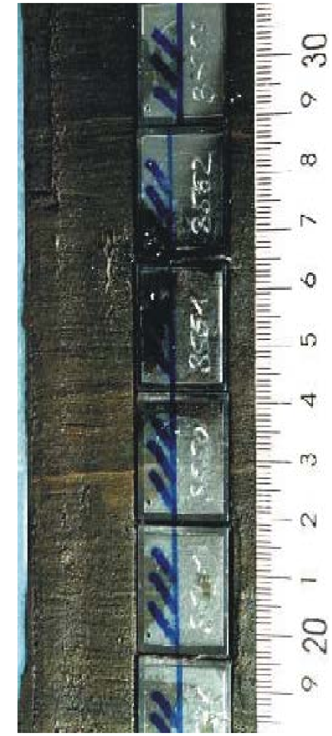
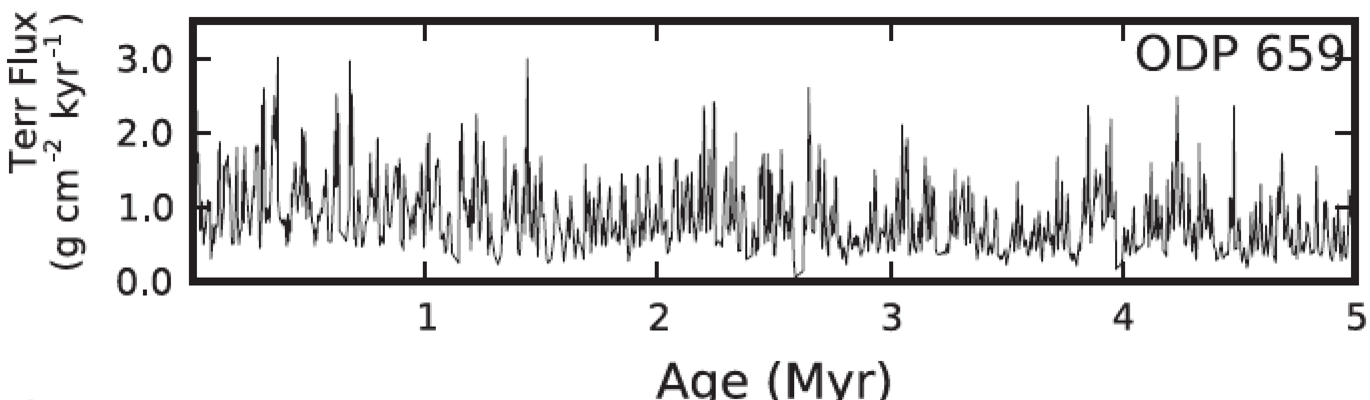
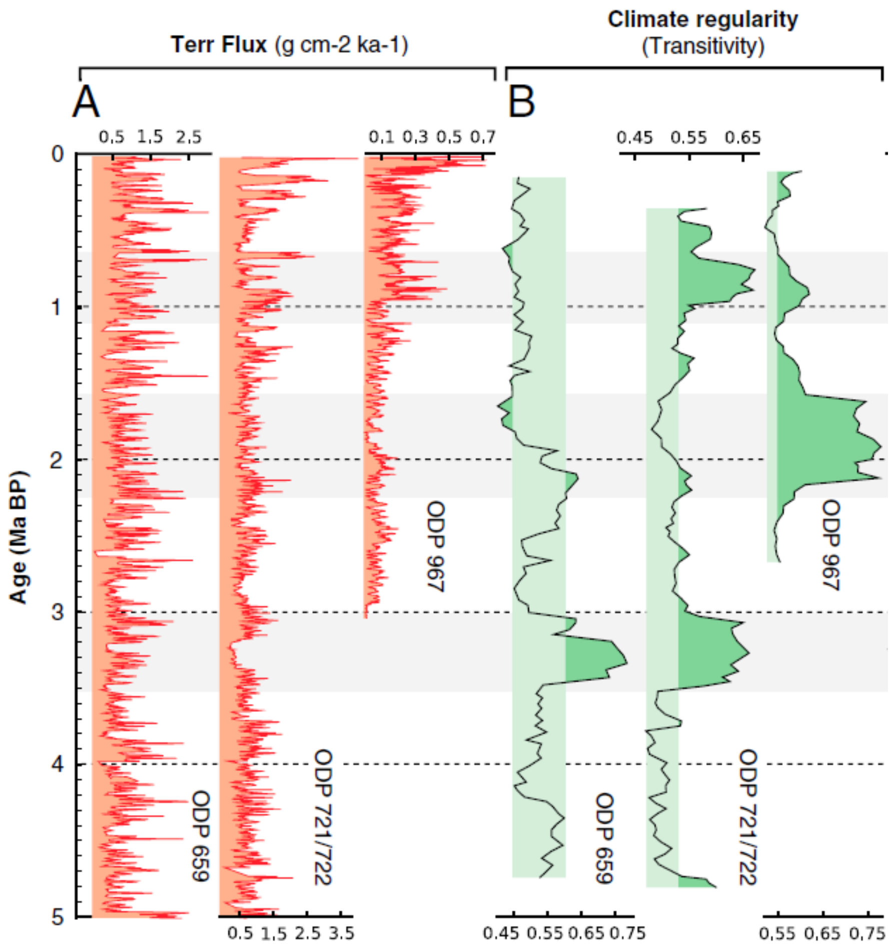
Research Question

What information can we gain from time-dependent nonlinear analysis of palaeoclimate proxy time series?

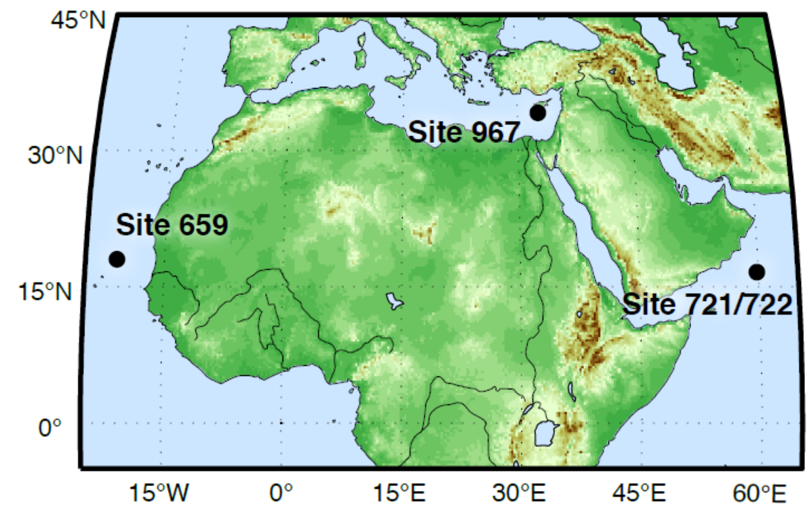




Example: Recurrence network analysis



Example: analysis of terrigenous dust flux in marine sediment cores around Northern Africa (Donges et al., PNAS 2011, NPG 2011)



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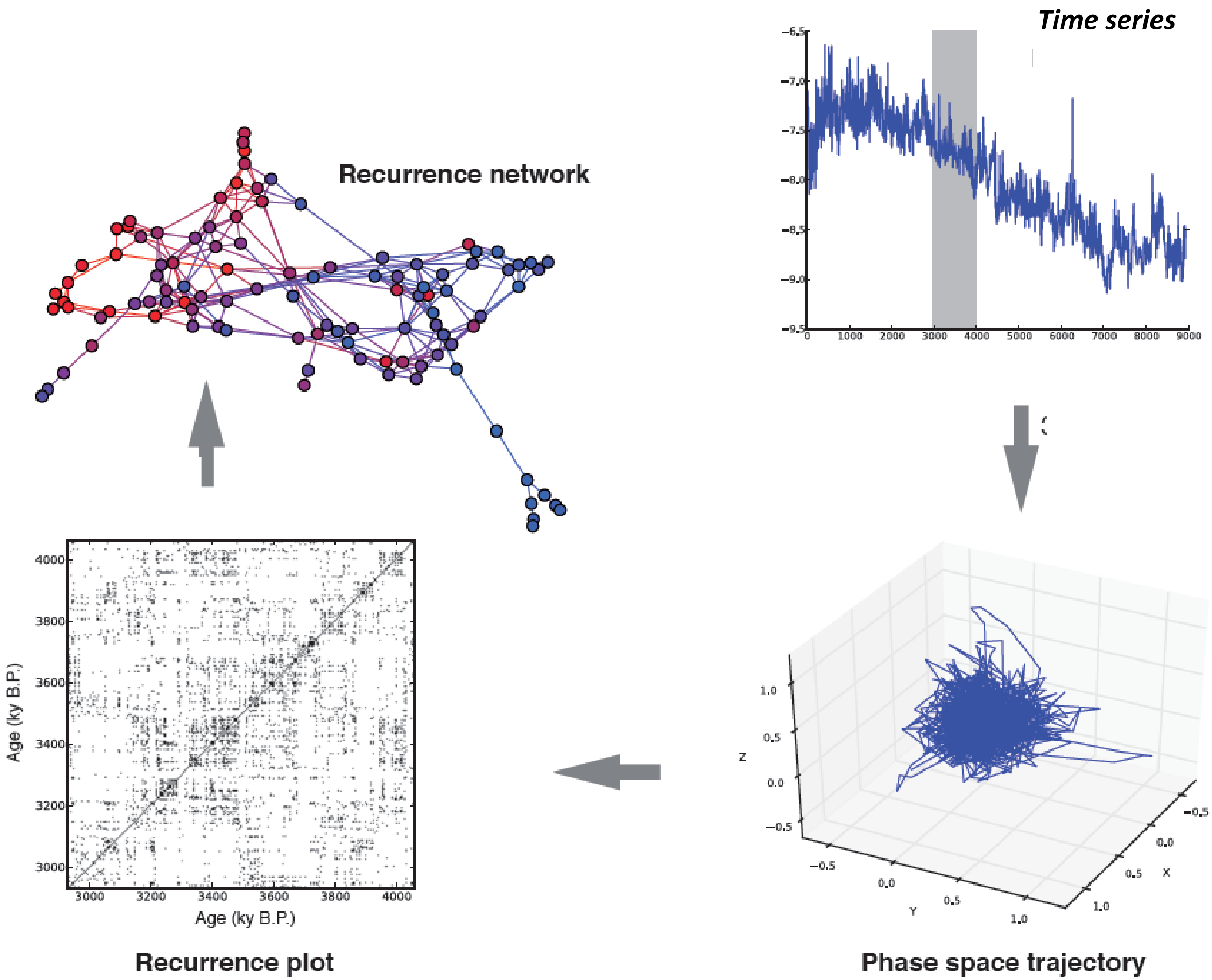
Recurrence networks: General idea



Step 1: Trajectory unfolding by embedding

Common approach: Time-delay embedding

$$\boldsymbol{x}(t) = (x(t), x(t-\tau), \dots, x(t-(m-1)\tau))$$

Embedding delay τ: minimize redundancy of information in different coordinates

Embedding dimension m: minimize projection effects due to insufficient unfolding

Trade-off: given time series length restricts maximum reasonable embedding dimension

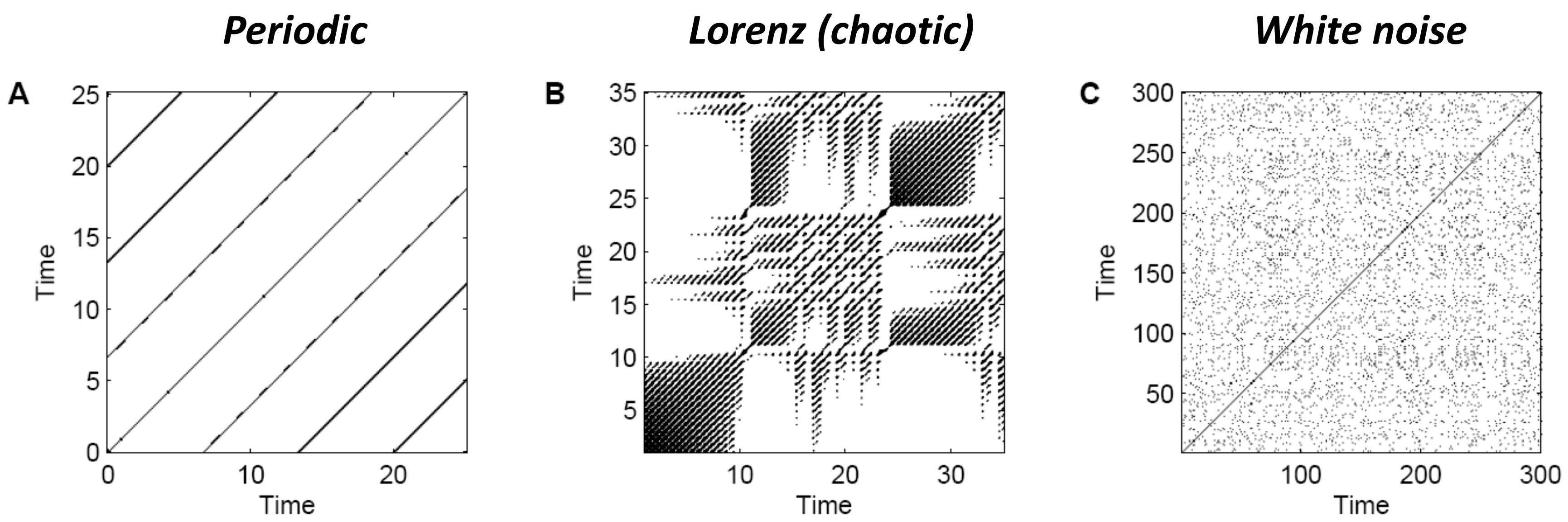
Often rather moderate dependence of results on delay (Donges et al., NPG 2011; Lekscha & Donner, Chaos 2018 & Proc. R. Soc. A, 2019)

Alternative: Derivative embedding (Lekscha & Donner, Chaos, 2018) or unfolding based on time series decomposition (Alberti et al, in rev.) – account for uneven sampling of palaeoclimate records in time domain

Step 2: Identify recurrent states

Eckmann *et al.* 1987: Visualization of nearby states (Poincarè recurrences) in state space in terms of "recurrence plots" based on the binary recurrence matrix

$$R_{i,j} = \Theta\left(\varepsilon - d(\vec{x}_i, \vec{x}_j)\right)$$



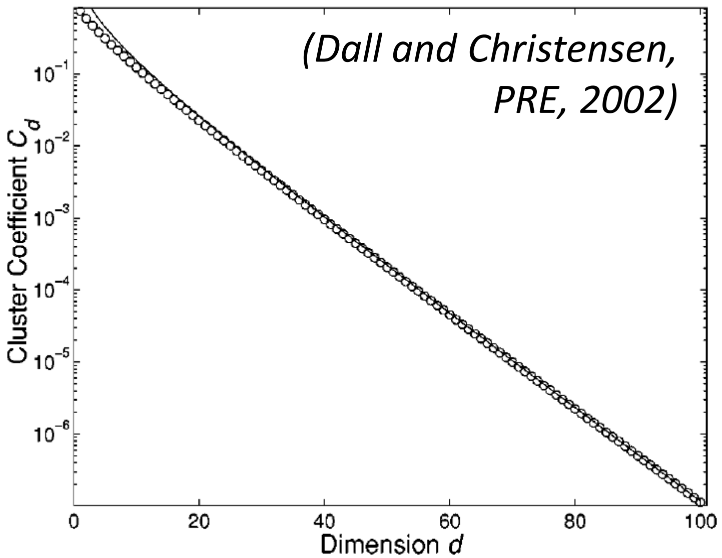
(Donner et al., IJBC, 2011)

Step 3: Random geometric graph analogy

Recurrence relation among state vectors = linkage relations among points in state space: recurrence network with adjacency matrix $A_{ij} = R_{ij} - \delta_{ij}$

Interesting property describing the attractor geometry: network transitivity

$$\mathcal{T}(\varepsilon) = \frac{\sum_{i,j,k} A_{ij}(\varepsilon) A_{jk}(\varepsilon) A_{ki}(\varepsilon)}{\sum_{i,j,k} A_{ij}(\varepsilon) A_{ki}(\varepsilon)}$$



Classical result: for random geometric graphs in a metric space of integer dimension d, transitivity scales exponentially with d (Dall and Christensen, PRE 2002)

Donner et al., EPJB 2011: for maximum norm: $\mathcal{T}=(3/4)^d$

Inversion of this relationship: (fractal) transitivity dimension

How much information can we actually infer from such an analysis?

Problem: palaeoclimate records are proxies of past climate variations, which obey a complex relationship with the unknown variable of interest
(archives like trees, sediments, ice deposits: complex filters)
⇒ How reliable are time-dependent recurrence network analyses?

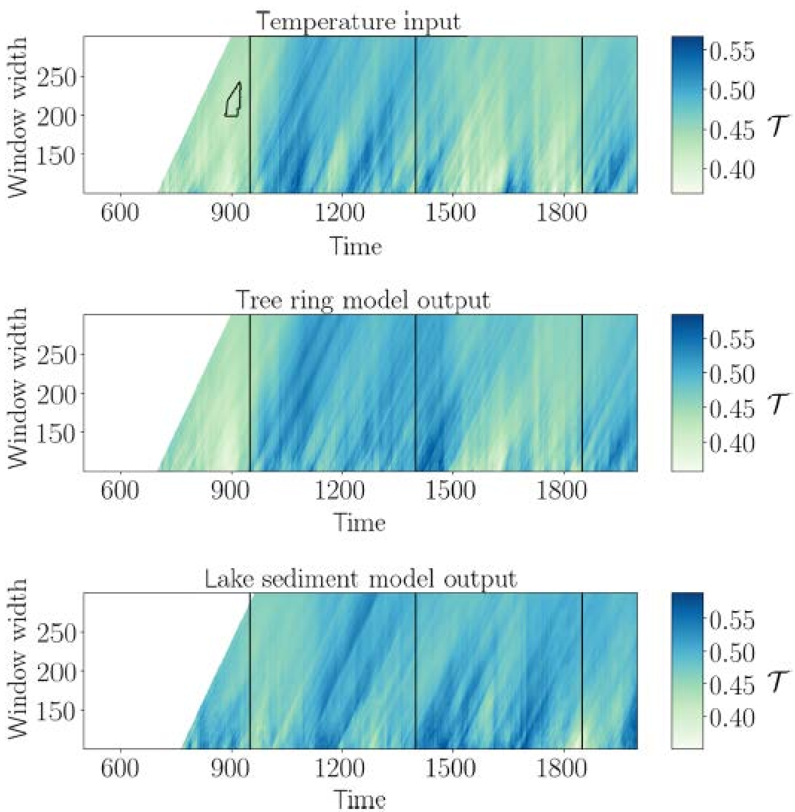
Controlled experiments using four mechanistic proxy system models (details: Lekscha & Donner, NPG 2020) for

- **Tree rings**
- **Lake sediments**
- **Speleothems**
- **Ice cores**

Input series: white/red noise, nonstationary Lorenz/Rössler systems, realistic past climate variability from Last Millenium Reanalysis

How much information can we actually infer from such an analysis?

Example: Last Millenium Reanalysis input



No robust detection of any transition associated with known succession of climate phases

Deterministic/stochastic input series: speleothem model tend to generate artificial signatures (complex nonlinear filters), while lake sediment model lacks sensitivity (no detection of prescribed transitions)

$\Rightarrow$ **Potential implications for interpretations of analysis of real-world palaeoclimate proxies!**

Conclusions and Outlook

Time-dependent recurrence network analysis can be a powerful tool for detecting changes in dynamical complexity of geoscientific time series.

However, for paleoclimate time series, the practical implications of such analyses in terms of actual climate variability still require further study.

Systematic analysis of ensembles generated by proxy system models can be one way to gain better understanding.

In general, the effects of nonlinear filters on recurrence network analysis need to be better understood (similar to effect of variable selections in terms of their observability, cf. Portes et al., Chaos, 2019).

Similar cautionary notes probably also apply to other types of nonlinear time series analysis methods.

Further reading

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Unified functional network and nonlinear time series analysis for complex systems science: The pyunicorn package

Jonathan F. Donges, Jobst Heitzig, Boyan Beronov, Marc Wiedermann, Jakob Runge, Qing Yi Feng, Liubov Tupikina, Veronika Stolbova, Reik V. Donner, Norbert Marwan, Henk A. Dijkstra, and Jürgen Kurths

Citation: Chaos **25**, 113101 (2015); doi: 10.1063/1.4934554

Check it out yourself!