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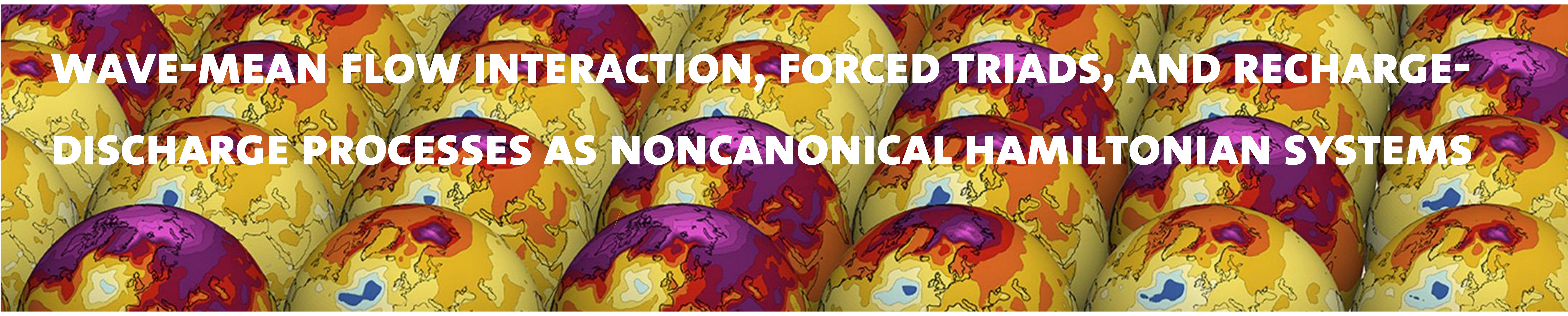


WAVE-MEAN FLOW INTERACTION, FORCED TRIADS, AND RECHARGE-DISCHARGE PROCESSES AS NONCANONICAL HAMILTONIAN SYSTEMS

RICHARD BLENDER AND

JOSCHA FREGIN

RECHARGE PROCESSES REVISITED

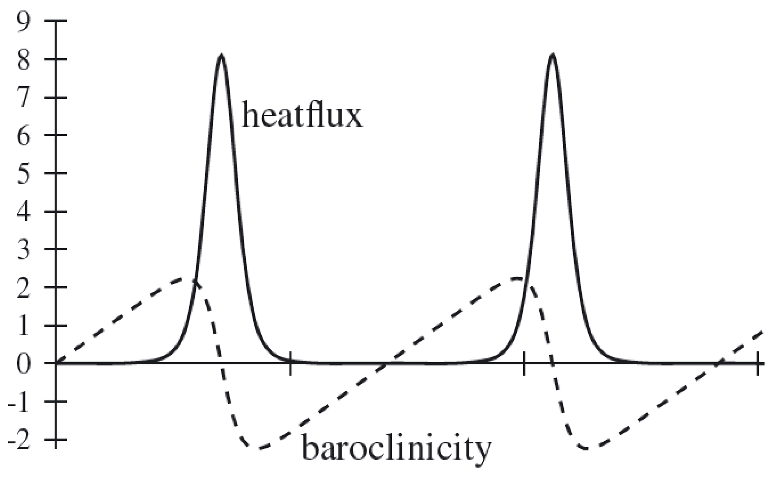
- BAROCLINIC STORMS (AMBAUM AND NOVAK, 2013)
- CONVECTIVE CYCLES (YANO AND PLANT, 2012)

NEW: NAMBU (NONCANONICAL HAMILTONIAN)

- FORCED WAVE-MEAN FLOW INTERACTION MODEL (BLENDER ET AL, 2013)
- FORCED ROSSBY WAVE TRIAD (BLENDER AND FREGIN, 2020).

AMBAUM AND NOVAK (2014): STORM TRACKS

Mechanism: Storms reduce baroclinicity



Baroclinicity

$$s = -kT_y$$

Heatflux, storms

$$f = kl^2\,\overline{v'T'}$$

$$\dot{s} = F - f, \quad \dot{f} = 2(s - s_0)f$$

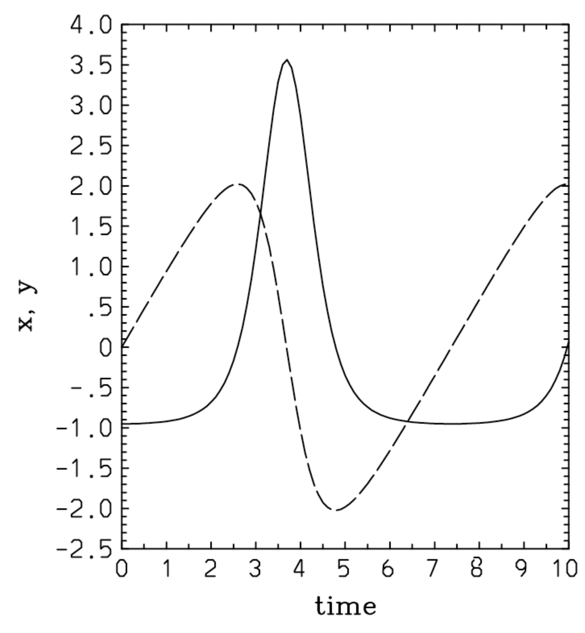
CANONICAL HAMILTONIAN

$$H(s,y) = (s-s_0)^2/2 + f/2 - F/2 \ln f$$

$$\dot{y} = \partial H/\partial s \text{ and } \dot{s} = -\partial H/\partial y \qquad y = (1/2)\ln f$$

YANO AND PLANT (2012): CONVECTION

Mechanism: Convection reduces the work function



Cloud base mass flux, Convection (solid) x

Cloud work function (dashed) y

$$\dot{x} = (1+x)y, \quad \dot{y} = -fx$$

CANONICAL HAMILTONIAN

$$H(y,\eta) = y^2/2 - fe^{\eta} + \eta$$

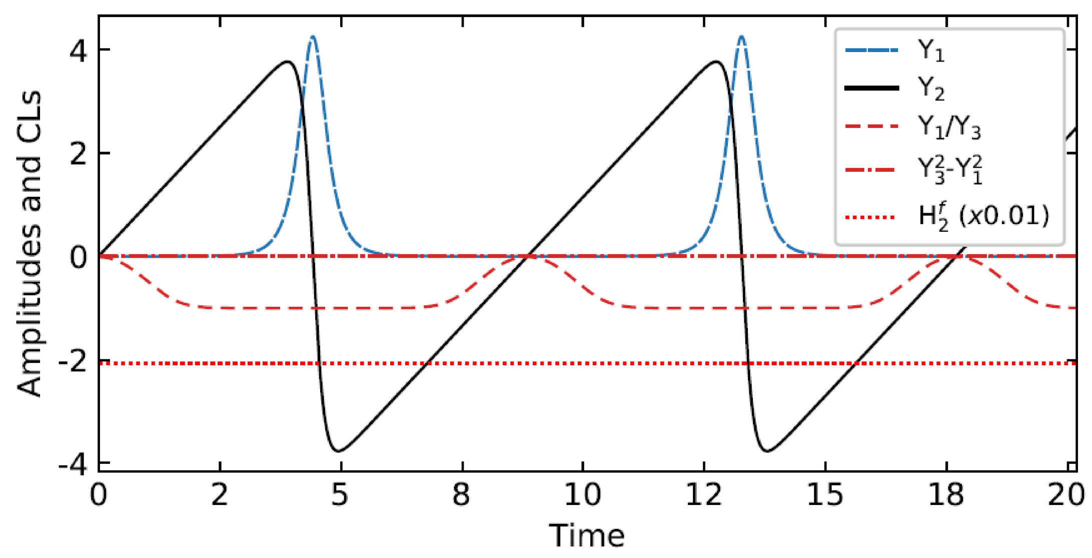
$$\dot{\eta} = \partial H/\partial y \text{ and } \dot{y} = -\partial H/\partial \eta \qquad \eta = \ln(1+x)$$

FORCED WAVE TRIAD

Real Amplitude equations for a resonant wave triad with forcing of the intermediate wavenumber (2)

$$\frac{dY_1}{dt} = -Y_2Y_3$$
$$\frac{dY_2}{dt} = Y_1Y_3 + f_2$$
$$\frac{dY_3}{dt} = -Y_1Y_2$$

The amplitude Y_2 shows a recharge cycle
Unforced waves: grow with opposite sign



Two conservation laws: Hamiltonian H and C

NAMBU FORM OF THE FORCED EQUATIONS

Nambu (1973): Extension of Hamiltonian dynamics with additional conservation laws

Hamiltonian

$$H_2^f = H + f_2(D_2/2)\ln(Y_1+Y_3)^2$$

Casimir

$$C_2 = \frac{1}{2D_2}(Y_3^2 - Y_1^2)$$

Nambu Dynamics
Divergence-free with two stream-functions

$$\frac{d\mathbf{Y}}{dt} = \nabla C_2 \times \nabla H_2^f$$

Nambu Bracket

$$dF/dt = \{F, C_2, H_2^f\}$$

Rigid body Nambu bracket

$$\{F, A, B\} = \nabla F \cdot \nabla A \times \nabla B$$

Reduces to a *Lie-Poisson* bracket with a Casimir C

WAVE-MEAN FLOW INTERACTION

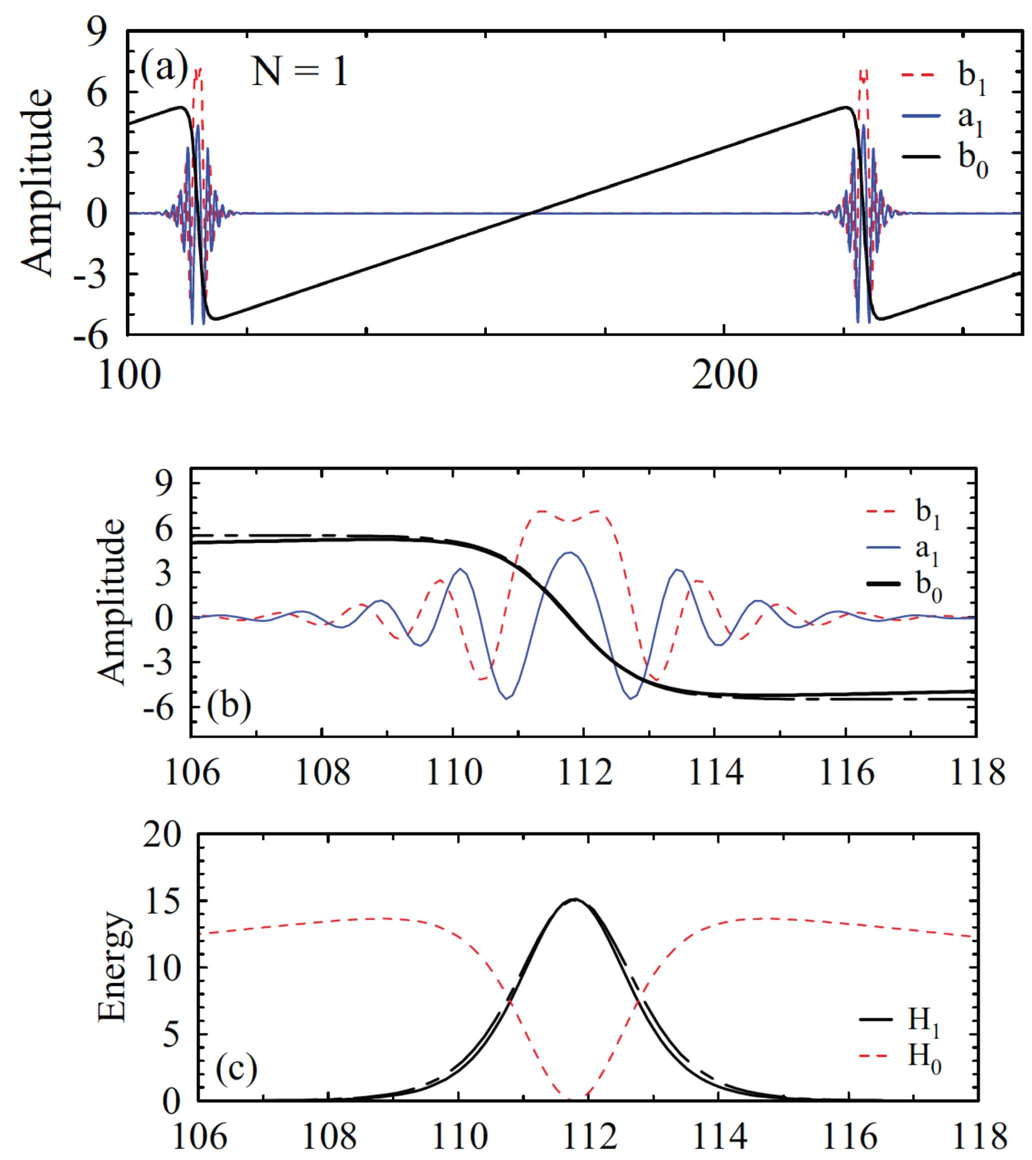
Flow: Discharge by waves and Recharge by forcing f

$$\frac{du}{dt} = -\frac{v^2+w^2}{12} + f$$

Waves amplification and oscillation

$$\frac{dv}{dt} = \frac{uv}{6} + uw$$
$$\frac{dw}{dt} = \frac{uw}{6} - uv$$

Recharge cycle



NAMBU FORM WITH A HAMILTONIAN AND TWO CASIMIRS, A DIVERGENCE FACTOR IS NECESSARY

$$H_f = H_0 + H_w - 3f\ln H_w$$

$$\frac{dX}{dt} = g(v,w)\nabla C_{RD} \times \nabla H_f + 2\nabla C_w \times \nabla H_f$$

$$C_{RD} = w/v \qquad C_w = H_w$$

$$g(v,w) = v^2/2$$

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