Resonant coupling of mode-1 and mode-2 internal waves by topography

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Korteweg-de Vries equation

$$A_t + cA_x + \frac{Q_x}{2Q}cA + \mu AA_x + \delta A_{xxx} = 0$$

The coupled KdV system for this resonance:

$$B_{1T} + \nu_1 B_1 B_{1X} + \lambda_1 B_{1XXX} = \frac{\gamma c_2^{1/2}}{2c_1^{1/2}} h_T B_2 \,,$$

$$B_{2T} - \Delta B_{2X} + \nu_2 B_2 B_{2X} + \lambda_2 B_{2XXX} = -\frac{\gamma c_1^{1/2}}{2c_2^{1/2}} h_T B_1 \,.$$

In this X-T reference frame, $T = \int_0^x \frac{dx}{c_1}, \quad X = T - t$

the linear phase speed for mode-1 and mode-2 wave are 0, -Δ respectively, where

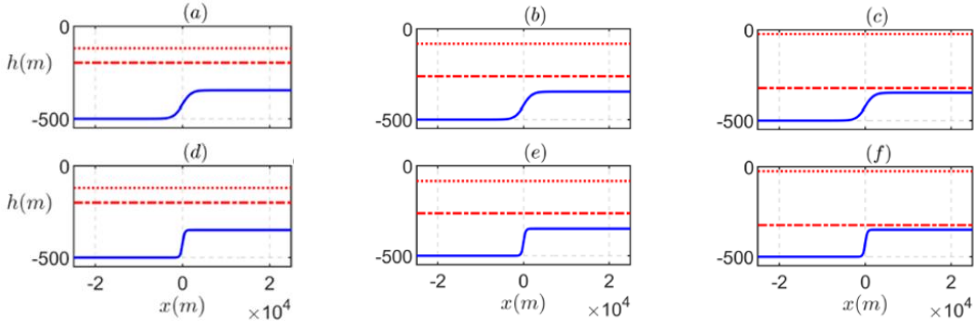
$$\Delta = \{\frac{1}{c_2^2} - \frac{1}{c_1^2}\}\frac{c_1^2}{2} \approx (c_1 - c_2)/c_2$$

ν_i, λ_i (i=1,2) are usual KdV coefficients; depth h is slowly

Varying and parameter γ is $\gamma = \frac{4(c_{1h}c_{2h}c_1c_2)^{1/2}}{c_2^2 - c_1^2}$

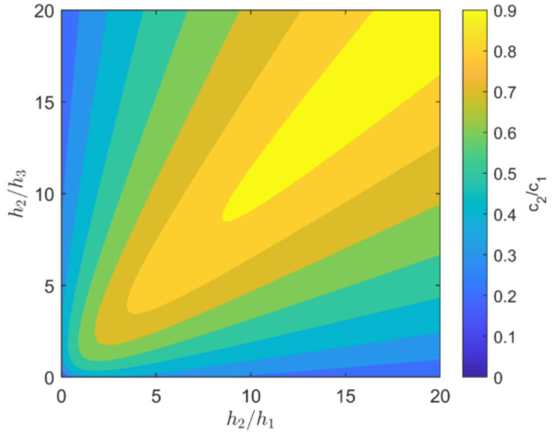
Three-layer fluid system:

$$\rho_0(z) = (\rho_2 + \Delta\rho)\mathcal{H}(-z - h_1 - h_2) + \rho_2\mathcal{H}(-z - h_1)\mathcal{H}(z + h_1 + h_2) + (\rho_2 - \Delta\rho)\mathcal{H}(z + h_1)$$



Case A Case B Case C

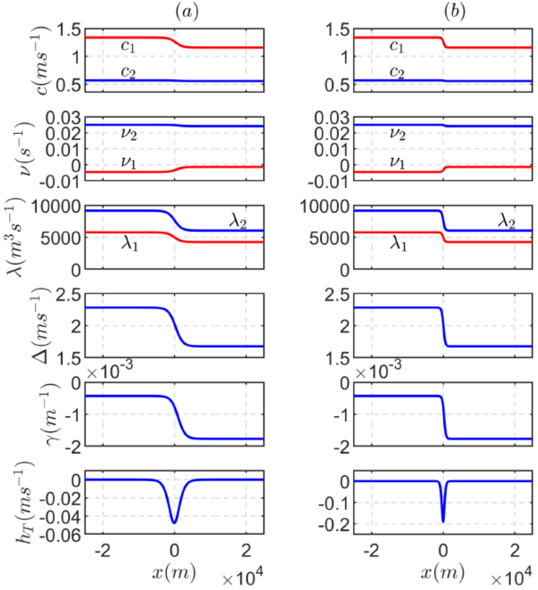
Panels (*a; b; c*) and (*d; e; f*) are for gentle and steep slopes, respectively.



The ratio of mode-2 and mode-1 wave speeds (c_2/c_1) for a three-layer fluid model as a function of h_2/h_1 and h_2/h_3.

Case A： weak resonant coupling

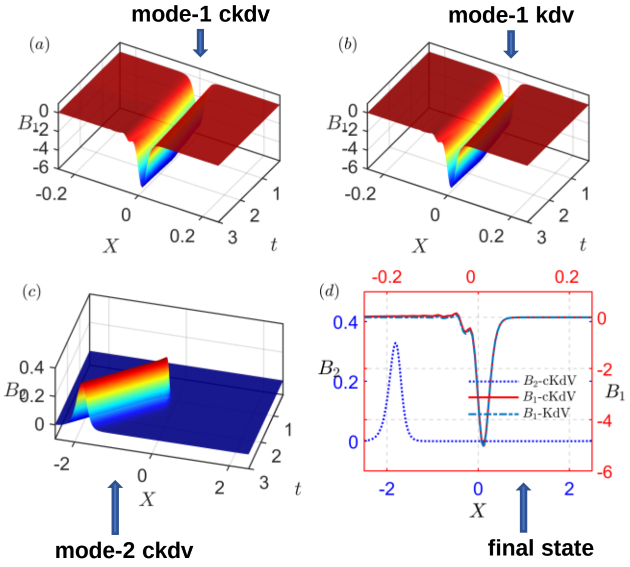
Coefficients of the coupled-KdV model



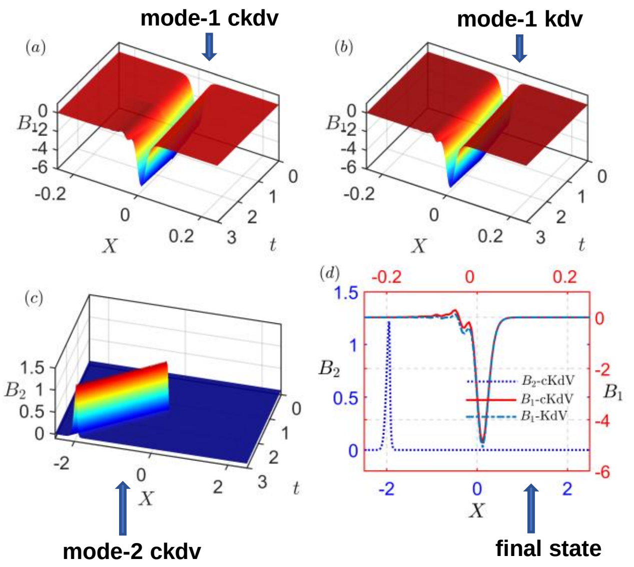
Gentle slope　　　　Steep slope

Wave evolution under the KdV and coupled-KdV model

Gentle



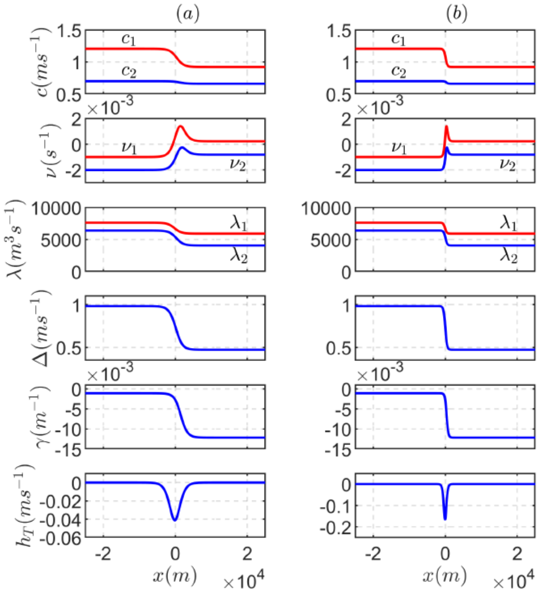
Steep



- A small-amplitude convex mode-2 wave is generated by a depression incident mode-1 solitary wave.
- The feedback on the mode-1 wave is negligible.

Case B： moderate resonant coupling

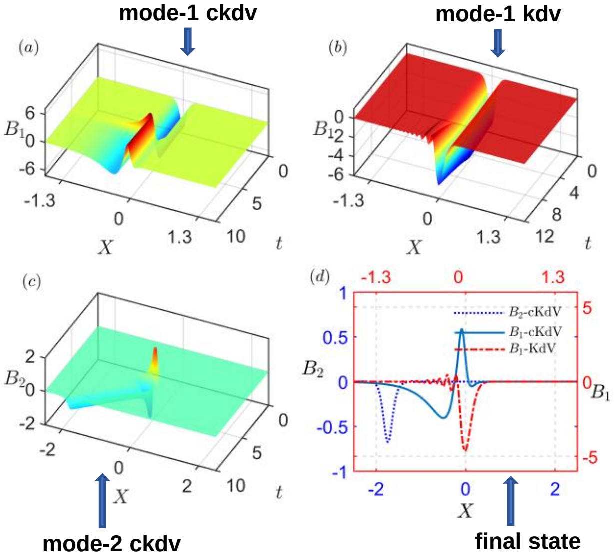
Coefficients of the coupled-KdV model



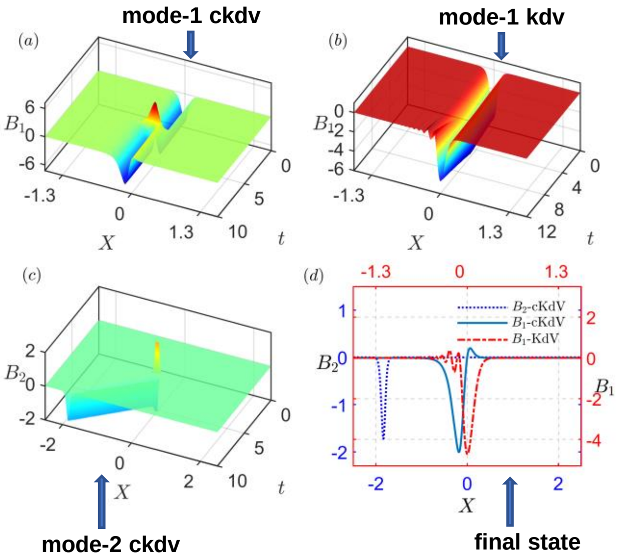
Gentle slope Steep slope

Wave evolution under the KdV and coupled-KdV model

Gentle



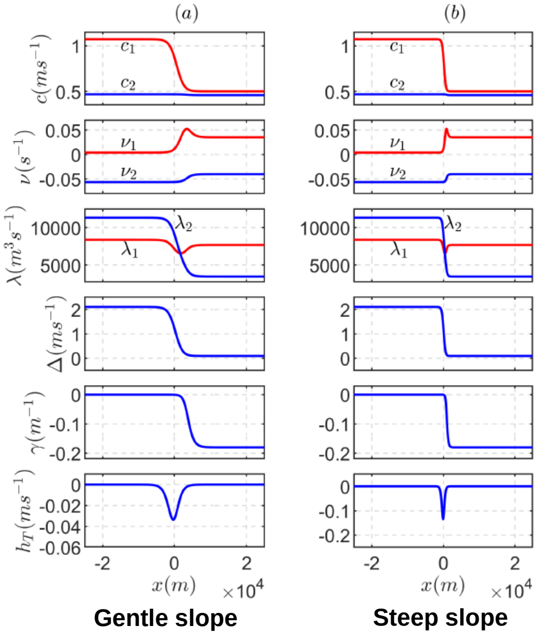
Steep



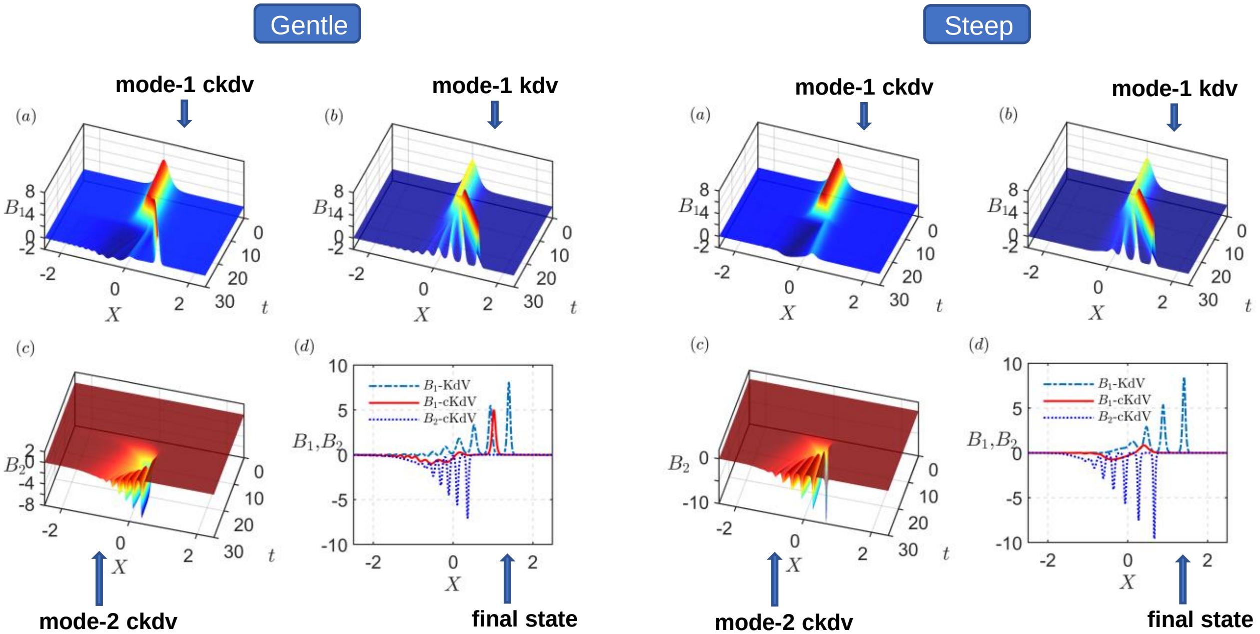
- A concave mode-2 wave of comparable amplitude to the depression incident mode-1 wave is formed.
- The mode-2 wave is concave, although a convex mode-2 wave appears momentarily when mode-1 wave first comes up against the slope.
- Strong feedback enhances the polarity change process of the mode-1 wave.

Case C: strong resonant coupling

Coefficients of the coupled-KdV model



Wave evolution under the KdV and coupled-KdV model



- A large-amplitude concave mode-2 wave is produced by an elevation incident mode-1 wave.
- Strong feedback suppresses the fission of the mode-1 wave.

Summary and conclusions

- The coupling parameter γ in the coupled KdV system varies and increases around tenfold between each case.

- As expected , the relative mode-2 wave speed Δ with respect to the mode-1 wave speed approximates to 0 as the speed ratio (c_2/c_1) increase.

- The mode-2 wave amplitudes are larger when the ratio c_2/c_1 and/or the topographic slope increase, indicating that it is a combination of near-resonance and topographic slope that will lead to significant observable mode-2 waves generated by a mode-1 wave propagating over topography.

- As the generated mode-2 wave amplitude becomes larger, its effect on the incident mode-1 wave is enhanced, either promoting or suppressing the evolution of mode-1 wave.

- It is not necessary to be very close to resonance for there to be significant mode-2 wave amplitudes and feedback onto the mode-1 wave.

Remains to do

- Apply this theory on other density stratifications.
- Compare these analytical results with numerical experiments.
- Quantify the impact of parameter c_2/c_1.