Thermodynamic consistent formulation for the multiphysics of a brittle ductile lithosphere: semi-brittle semi-ductile deformation and damage rheology

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Motivations

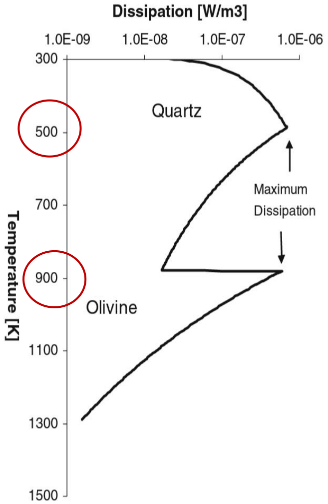
Revisit the concept of crustal rheology

Classical EVP concept

- Peak of stress@ BDT (maximum dissipation/entropy)
- No constraints on stored elastic strain (dissipative fluid behaviour)
- Sharp BDT (fast accomodation of dissipative deformation by thermal activated creep)

Experimental evidence

- Semi brittle behavior @ mid-lower crustal & upper mantle conditions (Mancktelow and Pennacchioni, 2005)
- Dilatant rock behavior @ mid-lower crust T-P conditions (Fischer and Paterson,1989, Violay et al., 2019)
- Transitional, transient semi-brittle/ductile domain due to the system thermodynamic evolution



Regenauer-Lieb and Yuen, 2008

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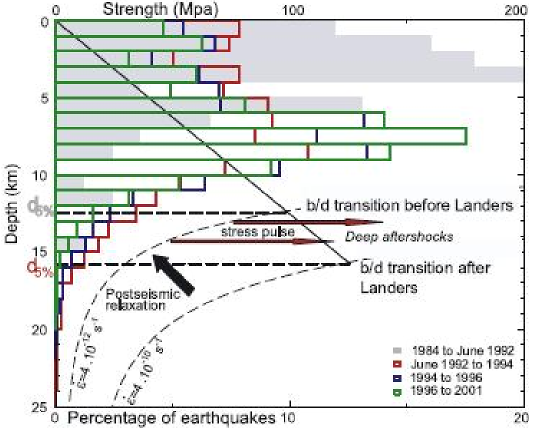
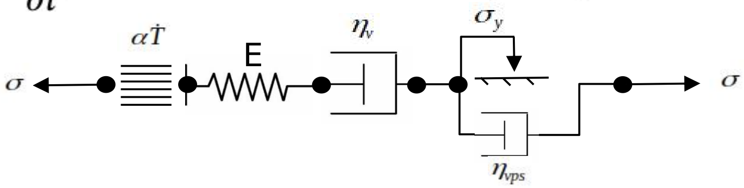


Rolandone et al., 2002

MultiPhysics - part I

$$\nabla \cdot (\boldsymbol{\tau}_{ij} - p\delta_{ij}) + \rho \mathbf{g} = 0 \quad + \quad \frac{\partial c_i}{\partial t} + \boldsymbol{v} \cdot \nabla c_i = 0.$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T - \nabla \cdot (\kappa \nabla T) = \dot{H}_r + \dot{H}_s$$



$$\dot{\varepsilon} = \dot{\varepsilon}^{e} + \dot{\varepsilon}^{v} + \dot{\varepsilon}^{vp} + \dot{\varepsilon}^{\theta}$$

PETSc SNES cycle

$$\bar{p}^{e(n+1)} = p^{(n)} - K\left(\Delta\varepsilon_{kk} - \alpha_\theta \Delta T\right) \qquad \bar{\boldsymbol{\tau}}_{ij}^{e(n+1)} = \tilde{\boldsymbol{\tau}}_{ij}^{(n)} + 2G\Delta\boldsymbol{e}_{ij}$$

$$\bar{\boldsymbol{\tau}}_{ij}^{ve(n+1)} = \bar{\boldsymbol{\tau}}_{ij}^{e(n+1)} - 3G\Delta e^{v} \frac{\bar{\boldsymbol{\tau}}_{ij}^{e(n+1)}}{\bar{\sigma}_e^{e(n+1)}}$$

$$\Delta e^{v} = \frac{\frac{\Delta t}{t^\star}}{1 + \frac{\Delta t}{t^\star}} \frac{\bar{\sigma}_e^{e}}{3G}$$

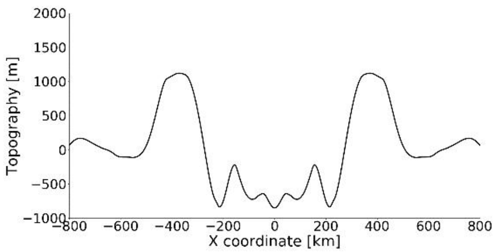
$$\mathcal{F}\left(p, \sigma_e\right) \leqslant 0$$

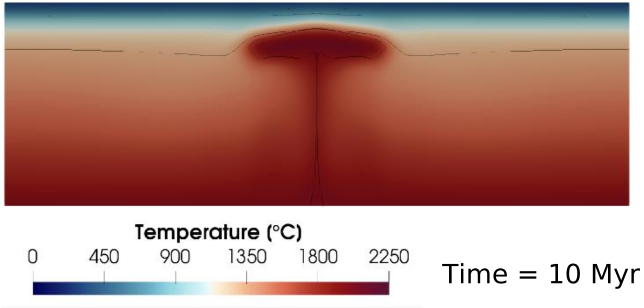
$$p = \bar{p}^{e} + K\beta\Delta\gamma, \quad \sigma_e = \bar{\sigma}_e^{ve} - 3G\Delta\gamma$$

$$\Delta\gamma^{p} = \frac{\langle \mathcal{F}\left(\bar{p}^{e}, \bar{\sigma}_e^{ve}\right) \rangle}{3G + \alpha\beta K + H}$$

$$p^{(n+1)} = \bar{p}^{e(n+1)} + \beta K \Delta\gamma \qquad \boldsymbol{\tau}_{ij}^{(n+1)} = \bar{\boldsymbol{\tau}}_{ij}^{ve(n+1)} - 3G\Delta\gamma \frac{\bar{\boldsymbol{\tau}}_{ij}^{ve(n+1)}}{\bar{\sigma}_e^{ve}}$$

Coupling of mantle plume with a EVP lithosphere

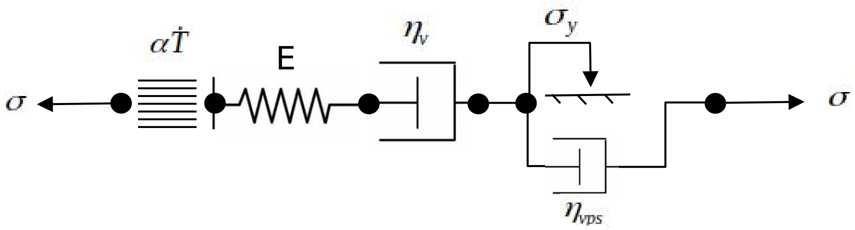




Time = 10 Myr

- Finite elastic strength leads to compressional and extensional stress accumulation
- Viscous weakening (relaxation) features in a decoupling in the middle crust and multi-wavelength topography (basins)

MultiPhysics - part I



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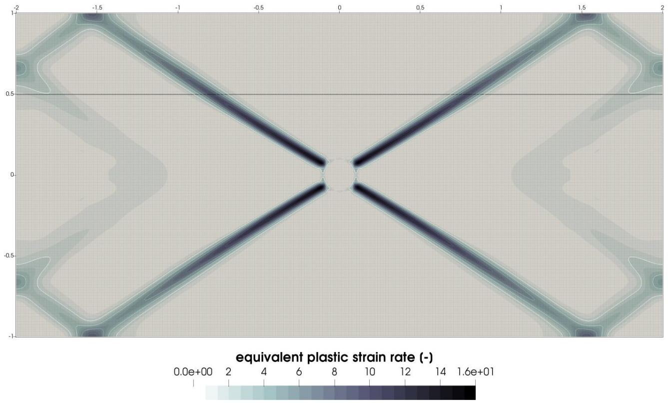
$$p = \bar{p}^{e} + K\beta\Delta\gamma, \quad \sigma_e = \bar{\sigma}_e^{ve} - 3G\Delta\gamma$$

$$\Delta\gamma^{vp} = \chi^{vp}\Delta\gamma^{p}$$

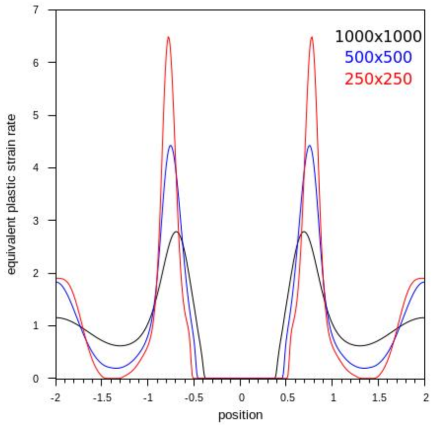
$$\chi^{vp} = \frac{\frac{(3G+\alpha\beta K+H)\Delta t}{\eta_p}}{1 + \frac{(3G+\alpha\beta K+H)\Delta t}{\eta_p}}$$

$$p^{(n+1)} = \bar{p}^{e(n+1)} + \beta K \Delta\gamma \qquad \boldsymbol{\tau}_{ij}^{(n+1)} = \bar{\boldsymbol{\tau}}_{ij}^{ve(n+1)} - 3G\Delta\gamma \frac{\bar{\boldsymbol{\tau}}_{ij}^{ve(n+1)}}{\bar{\sigma}_e^{ve}}$$

Elastic-viscous-viscoplastic shear banding



plastic strain rate within the deformed bands

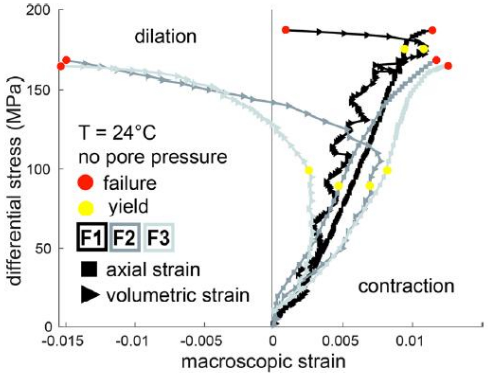


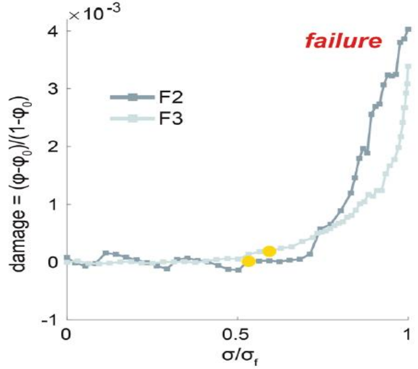
Nonlinear dynamics for the BDT - some aspects

- Transition from brittle faulting to ductile flow controls by porosity evolution upon (quasi) dynamic loading
- Brittle failure associated with dilatancy, cataclastic flow can be sustained under compactive mode

- Microstructure evolution during loading is essential feedback mechanism for the macroscopic evolution
- Brittle failure of rocks (coalescence of microcracks) sensitive to strain rate and effective confining pressure

- Experimental evidence of transient (un)drained response of rocks even at creep loading
- Transient inelastic modes influence subcritical crack growth, precursory phase to earthquake rupture and/or volcanic edifice collapse, recovery of engineered reservoirs





Renard et al. (2018)

porous damage rheology - theoretical framework

Main governing equations

$$\begin{cases} \nabla \cdot \left(\boldsymbol{\tau}_{ij} - \left(p' + \alpha_B p_f\right)\delta_{ij}\right) + \rho \boldsymbol{g} = 0 \\ \frac{1}{M_B}\frac{\partial p_f}{\partial t} + \alpha_B \frac{\partial \varepsilon^e_{kk}}{\partial t} + \frac{\partial \varepsilon^{in}_{kk}}{\partial t} + \nabla \cdot \boldsymbol{q}_D = 0 \\ \frac{\partial \phi}{\partial t} - \left(\alpha_B - \phi\right)\left(\frac{\left(1-\alpha_B\right)}{K}\frac{\partial p_f}{\partial t} + \frac{\partial \varepsilon^e_{kk}}{\partial t}\right) - (1-\phi)\frac{\varepsilon^{in}_{kk}}{\partial t} = 0 \\ \frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T - \nabla \cdot (\kappa \nabla T) = \dot{H}_r + \dot{H}_s \\ \frac{\partial c_i}{\partial t} + \boldsymbol{v} \cdot \nabla c_i = 0. \end{cases}$$

Thermodynamics of damage rheology

Helmholtz Free Energy (Lyakhovsky et al., 1993)

$$\Psi\left(\varepsilon^e_{ij}, \alpha, \zeta, \varepsilon^{in}_v\right) = \Psi_{dr}\left(\varepsilon^e_{ij}, \alpha\right) + \Psi_\phi\left(\varepsilon^e_{ij}, \zeta, \varepsilon^{in}_v\right)$$

$$\Psi_{dr}\left(\varepsilon^e_{ij}, \alpha\right) = \frac{1}{2}K(\alpha)\,\varepsilon^{e2}_v + \frac{3}{2}G(\alpha)\,\varepsilon^{e2}_d - \Gamma(\alpha)\,\varepsilon^e_v \|\varepsilon^e_{ij}\|$$

$$\Psi_\phi\left(\varepsilon^e_{ij}, \zeta, \varepsilon^{in}_v\right) = \frac{1}{2}M_B\left[\alpha_B \varepsilon^e_v - \left(\zeta - \varepsilon^{in}_v\right)\right]^2$$

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \varepsilon^e_{ij}} = \frac{\partial \Psi_{dr}}{\partial \varepsilon^e_{ij}} + \frac{\partial \Psi_\phi}{\partial \varepsilon^e_{ij}} = \sigma'_{ij} - \alpha_B p_f \qquad p_f = \frac{\partial \Psi}{\partial \zeta} = -M_B\left[\alpha_B \varepsilon^e_v - \left(\zeta - \varepsilon^{in}_v\right)\right]$$

$$\sigma'_{ij} = \sigma_{ij} + \alpha_B p_f = \bar{\sigma}'_{ij} - \alpha \sigma^\alpha_{ij}$$

$$\bar{\sigma}'_{ij} = K_0 \varepsilon^e_v \delta_{ij} + 2G_0 \boldsymbol{e}^e_{ij} \qquad \sigma^\alpha_{ij} = \Gamma\left[\|\varepsilon^e_{ij}\|\delta_{ij} + \left(\xi - 2\xi_0\right)\varepsilon^e_{ij}\right]$$

$\xi = \frac{\varepsilon^e_v}{\|\varepsilon^e\|}$ Elastic strain ratio [-√3; √3]

ξ_0 critical strain ratio (internal friction)

porous damage rheology - flow laws and damage correction

Additive decomposition

$$\dot{\varepsilon} = \dot{\varepsilon}^{e} + \dot{\varepsilon}^{v} + \dot{\varepsilon}^{vp} + \dot{\varepsilon}^{\theta}$$

visco-plasticity + damage

$$\mathcal{F} = \sqrt{(\bar{p}' - \bar{p}'^{0})^2 + \left(\bar{\sigma}_e - \bar{\sigma}_e^0\right)^2}$$

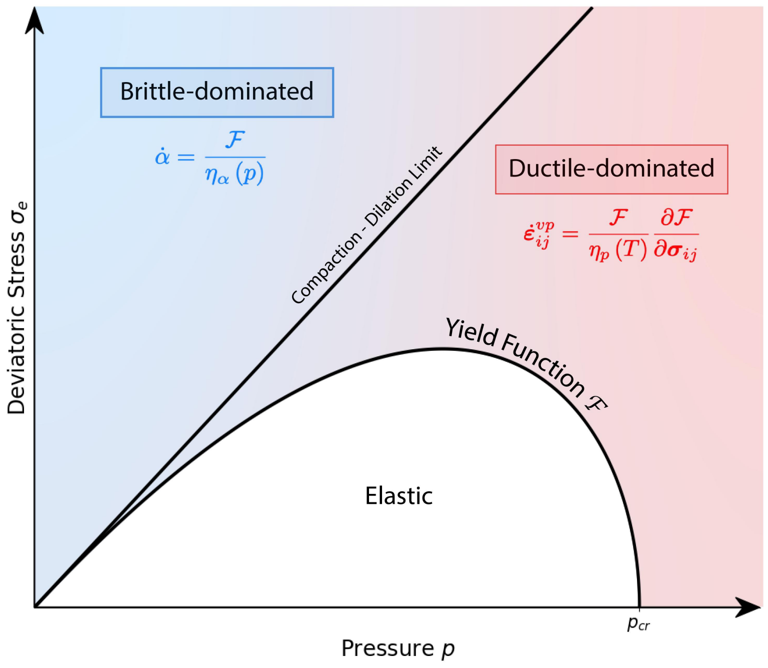
$$\dot{\varepsilon}_{ij}^{vp} = \dot{\gamma}^{vp} \frac{\partial \mathcal{F}}{\partial \bar{\sigma}_{ij}}, \qquad \dot{\gamma}^{vp} = \frac{< \mathcal{F} >}{\eta_p}$$

$$\dot{\alpha} = \frac{< \mathcal{F} >}{\eta_\alpha} = \frac{\eta_p}{\eta_\alpha} \sqrt{(\dot{\varepsilon}_v^{vp})^2 + (\dot{\varepsilon}_d^{vp})^2}$$

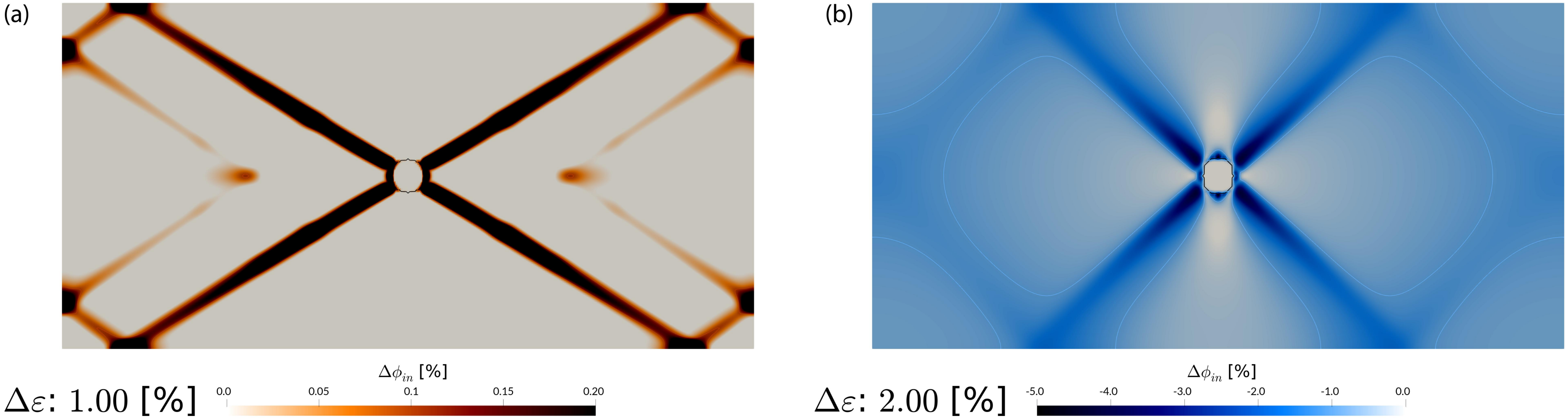
$$\eta_p = \eta_p^0 \exp\left(\frac{Q}{RT}\right)$$

$$\eta_\alpha = \eta_\alpha^0 \exp\left(\frac{p_c}{p^\star}\right)$$

Yield function (undamaged space)



Shear bands evolution under dilation (a) and compactant (b) mode

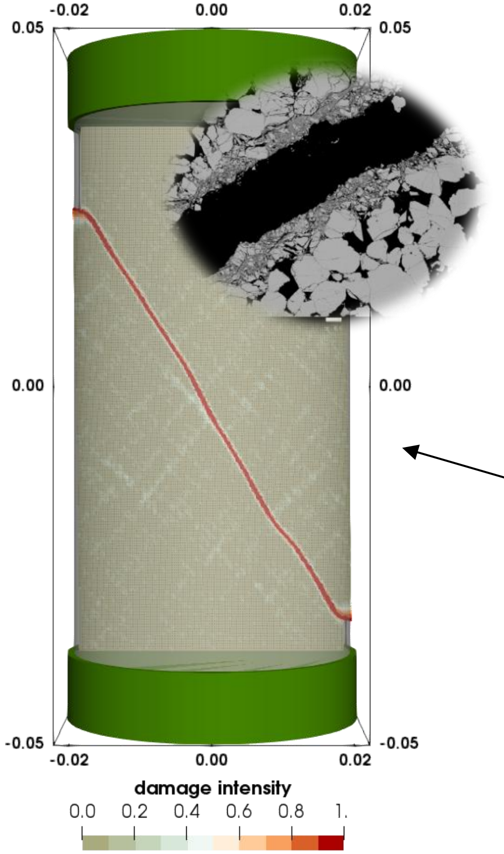


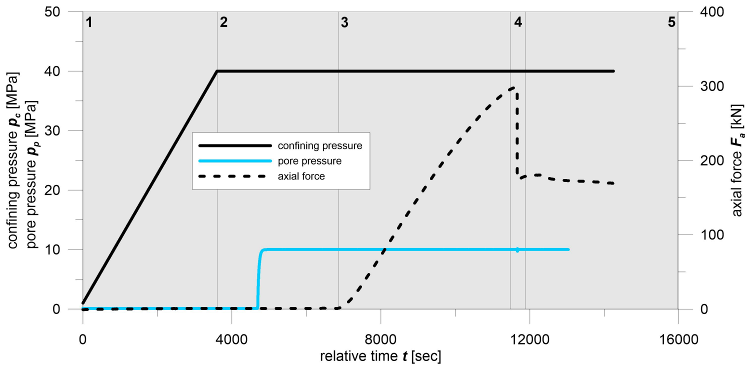
- increase in inelastic porosity
- localized deformation
- bands orientation at approx. 33°

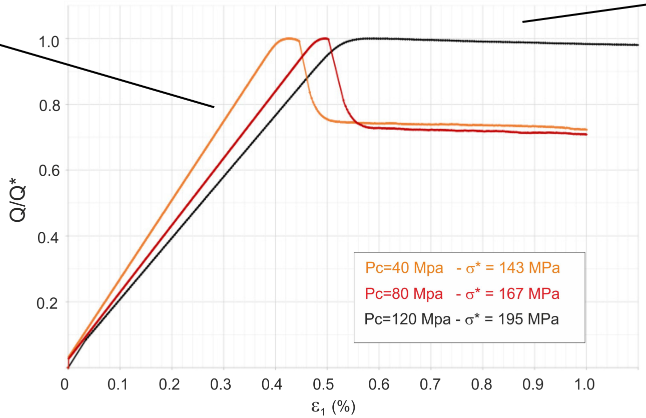
- decrease in inelastic porosity
- diffuse deformation
- bands orientation @ approx. 45°

Drained triaxial setting - modelling results

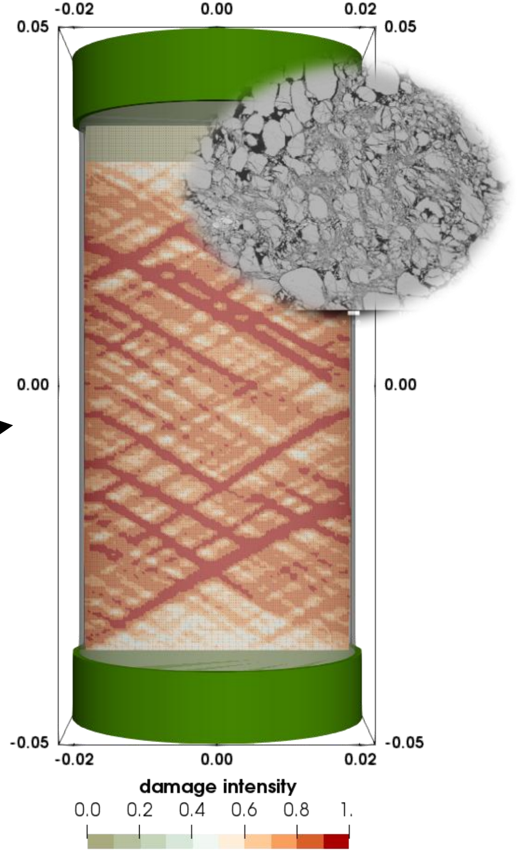
Triax model - 40 MPa confinement





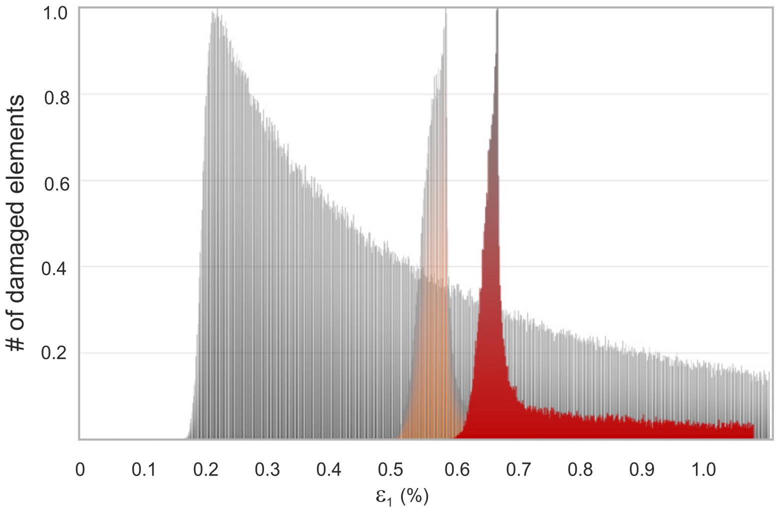


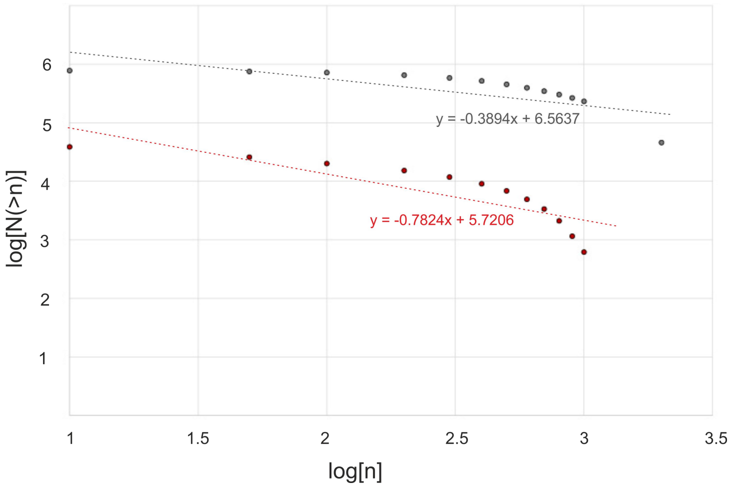
Triax model - 120 MPa confinement



Cacace and Jacquey, EuroConference, 2019

Triaxial setting - ongoing modelling results





Proxy for AEs

$$\log[N(n)] = a - b\log[n]$$

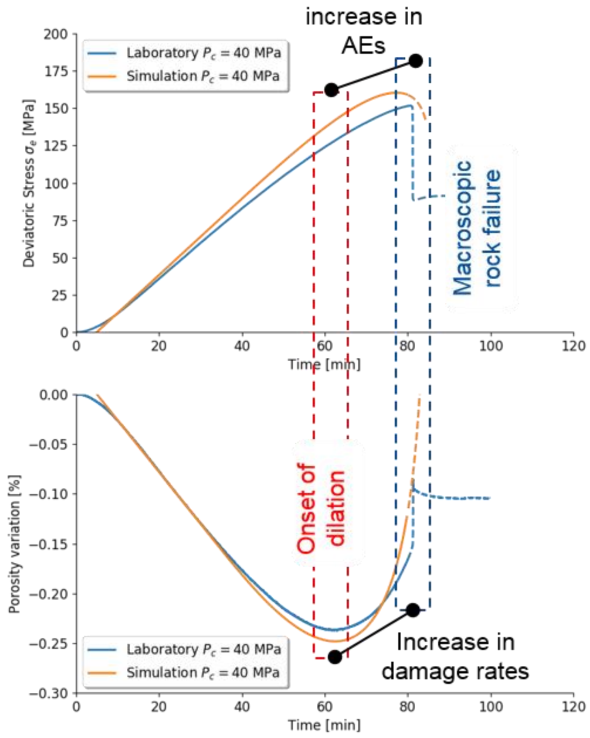
- High confinement - ductile onset at inelastic stage, power law distribution (b-value ~ 0.3)
- Low confinement - brittle power law, but macroscopic peak @ failure (b-value ~ 0.62-0.8)

similarities to Omori's law and rate and state frictional behaviour

Conclusions, ongoing works and more

- Thermodynamic consistent framework for multiphysics at BDT conditions
- Frictional damage rheology == function to the state (p, T) and rates of deformation
- Self-promoting (porosity dependent) damage weakening on friction and capped yield
- Porosity evolution as precursor to run-away instabilities (though look at current work on brittle creep!)

- Across scale studies required linking to existing approaches and community build up:
  - Upscaling from strain rate dependency
  - Rate and state friction (see Lyhakovsky et al., 2005)
  - Long-term geodynamic approaches



Cacace and Jacquey, 2019