Global Sensitivity Analysis of Optimal Climate Policies

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Motivation

- Solutions to climate–economic models are often communicated as *optimal climate policies*.
- Yet, the factors that drive these solutions remain obscure.
- Climate–economic models are *highly sensitive* to initial assumptions and calibrations.
- They are also computationally too *expensive* to run large simulations.
- Most of the existing sensitivity analyses are thus limited to *local and nonsystematic*.

Global sensitivity analysis (GSA) establishes the *robustness* of a model's solution and identify the most important *drivers of uncertainty* in its output.

Drivers for optimal climate policies: analytical results

The social cost of carbon is a measure of the seriousness of climate change.

(Anthof and Tol, 2013)

- The factors that determine the cost of carbon in a generic climate–economic framework are (van den Bijgaart et al., 2016):

SCC formula for DICE

$$SCC_t = \frac{1.3 a_2 \lambda^{a_3}}{m} \frac{1}{b_{12} + \sigma} \frac{\xi_1}{\xi_1 + \sigma} Y_t,$$

$$\sigma = \rho + (\alpha - 1) g - l.$$

Parameters for climate, economy, and their interplay:	
ρ	time discount rate
α	elasticity of marginal utility
a_2	relative damage at 1°C
a_3	damage–temperature elasticity
λ	climate sensitivity
b_{12}	depreciation of atm. CO_2
ξ_1	adjustment to eq. temperature

- Most models are much more detailed and therefore too sophisticated to enjoy analytical solutions.
- Advanced methods of GSA provide a computational way to decompose the uncertainty in SCC.

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GSA framework

GSA see a model as a function of a random vector of its parameters $\boldsymbol{\Theta}$ with an output Y,

$$Y = \mathcal{M}(\boldsymbol{\Theta}).$$

Each parameter is assigned a probability density function f_{Θ_i}.

Variance decomposition (Sobol, 1993)

$$\mathrm{Var}[Y] = \sum_{i=1}^{M} \mathrm{Var}[\mathcal{M}_i(\Theta_i)] + \sum_{1\leq i<j\leq M} \mathrm{Var}[\mathcal{M}_{i,j}(\Theta_i, \Theta_j)] + \cdots + \mathrm{Var}[\mathcal{M}_{1,2,\cdots,M}(\boldsymbol{\Theta})].$$

Partial variances measure potential *reductions* in the total variance when the values of the corresponding parameters are known.

Sobol' indices

Sensitivity indices are the shares of partial variances in total variance,

$$S_{\mathbf{u}} = \frac{\mathrm{Var}\left[\mathcal{M}_{\mathbf{u}}(\boldsymbol{\Theta}_{\mathbf{u}})\right]}{\mathrm{Var}[Y]}.$$

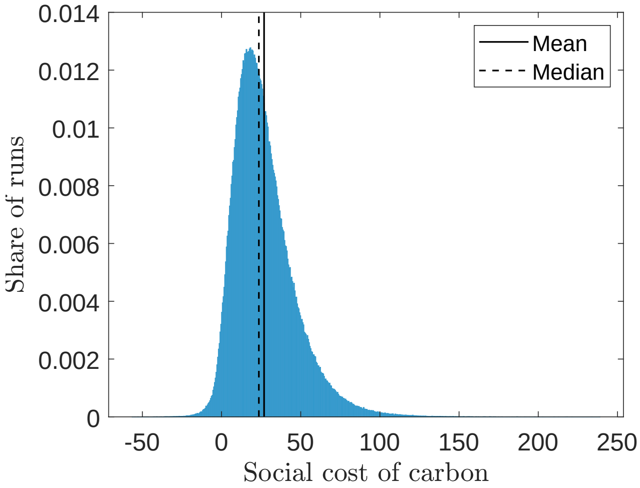
Traditionally sensitivity indices are estimated using **Monte–Carlo** techniques (*very, very expensive*).

Polynomial chaos expansions approximate the outcome with a sum of polynomials of parameters' values,

$$Y \approx \sum_{\alpha \in \mathcal{A}} y_\alpha \Psi_\alpha(\boldsymbol{\Theta}).$$

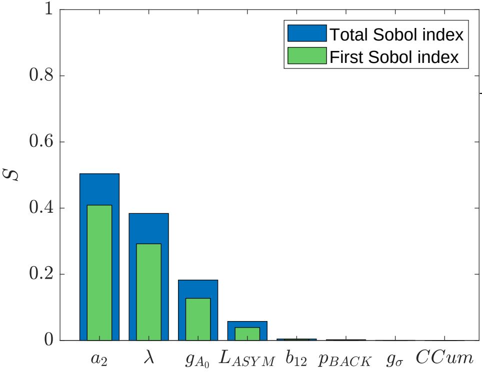
They provide a way to compute the full set of sensitivity indices at a *much lower cost* (Sudret, 2008).

Simulated distribution of optimal carbon tax in DICE



Note: reporting the *mean* value as optimal is hardly informative.

Variance decomposition à la Nordhaus (2008)



The set of parameters limited to 8	
a_2	Damage function coefficient
λ	Climate sensitivity
g_{A_0}	Initial growth rate of TFP
L_{ASYM}	Asymptotic population
b_{12}	Carbon transition
p_{BACK}	Cost of backstop technology
$g_{\sigma 0}$	Init. change of decarbonization
$CCum$	Maximum extraction of carbon

- *Global* sensitivity analysis overcomes the limitations of the local approach and challenges the pre-selection of “important” parameters.
- BUT has to be applied carefully...

The issue of dependence

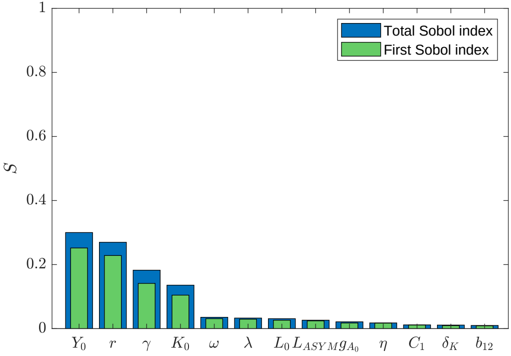
- Variance-based GSA relies on the assumption of independence of all parameters.
- Spurious significance of the sensitivity indices is one result of misapplication (Saltelli and Tarantola, 2002).
- In DICE some relationships are fundamental to the model's structure and have to be preserved under the analysis,

  e.g. Ramsey rule, $r^* = \rho + \alpha g^*$.

- Reformulate the alanysis such that the parameters that can be considered independent are sampled *in the place* of dependent ones.

Variance decomposition, full set of parameters

The most influencial out of *all* parameters:



Y_0	Initial gross output
r	Rate of return on capital
γ	Capital share in production
K_0	Initial level of capital
ω	Damage at 3°C
λ	Climate sensitivity
L_0	Initial population
L_{Asym}	Asymptotic population
g_{A_0}	Initial growth rate of TFP
η	Radiative forcing, $2\times CO_2$
ξ_1	Climate coefficient, upper
δ_K	Depreciation rate of capital
b_{12}	Carbon transition

The ranking is consistent with the *analytical* formula above.

SCC formula

Conclusions

Climate-economic models are the scientific base for climate policies, but even for very stylized models, the factors that determine the outcome for the optimal policy are nontrivial to infer.

- Methods of global sensitivity analysis offer a detailed decomposition of uncertainty in a model's outcome.
- The use of highly efficient GSA method based on polynomial chaos expansions drastically reduces the computational cost.
- Only global and comprehensive—as opposed to local or selective—sensitivity analysis gives a trustworthy picture of uncertainty in a model.
- The relationships fundamental to the model's structure define which parameters can be sampled as independent.

References

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