Nonlinear time series modelling of the North Atlantic Oscillation

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Introduction

The North Atlantic Oscillation (NAO) refers to the fluctuation in sea level pressure (SLP) between the Icelandic low and the Azores high and it is the dominant mode of climate variability over the North Atlantic.

- Controls the strength and direction of westerly winds and location of storm tracks across the North Atlantic.
- Impacts the seasonal climate and surface weather conditions.
- Has irregular behavior, due to complex and nonlinear interactions between many spatiotemporal scales[2], but is known to have extended periods of positive and negative phases[3] associated with a wide range of time scales from days to decades and longer[7].

Objective of the study

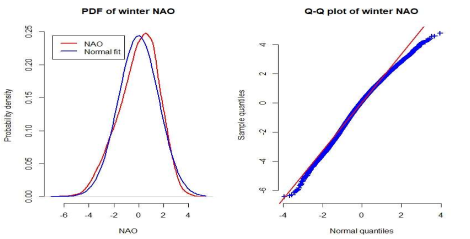
Set up mathematical models for the daily winter NAO index based on these observed properties.

Time series of the daily winter NAO index

The time series for the daily NAO index published by Cropper et al. in 2015[1] has been analyzed. This time series is calculated from actual SLP observations on Iceland and the Azores, but reanalysis data have been used to fill in the gaps in the observations. Indices for the 142 winters between December 1, 1872 and February 28, 2014 are included in the analysis.

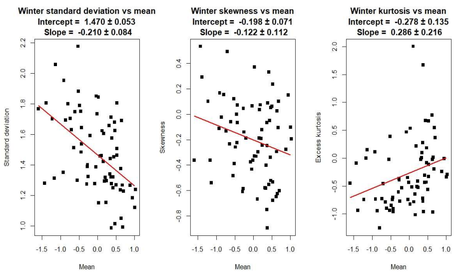
Distribution of the daily winter NAO index

The NAO index shows a characteristic departure from a normal distribution. Large positive values and small negative values are less common for the NAO, whereas large negative values and positive values on the order of one standard deviation are more common for the NAO.

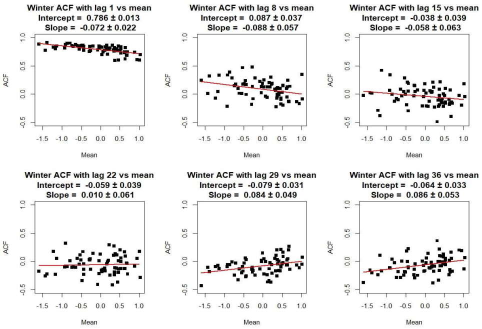


Phase dependence of the NAO distribution

Both higher moments (standard deviation, skewness, kurtosis) and the sample autocorrelation function (ACF) of the NAO depend on the present state of the NAO. We can investigate this dependence by studying the 142 winters in the data. During winters when the negative phase dominates, the standard deviation and skewness are larger, but the kurtosis is smaller.



Similarly, during winters when the negative phase dominates, the sample ACF is larger for lags up to two weeks, but smaller for lags of the order of one month.



A linear and two nonlinear models

A linear **AR(p)** process $\{X_i\}_{i=1}^n$ is defined as

$$X_i = \mu + \varphi_1(X_{i-1} - \mu) + \cdots + \varphi_p(X_{i-p} - \mu) + \varepsilon_i,$$

where p is a positive integer, $\varphi_1, \dots, \varphi_p$ are real constants, μ is the expectation of X_i and $\{\varepsilon_i\}_{i=1}^n$ is white noise.

An AR(p) model cannot capture the following properties of the winter NAO index[4]:

- Skew distribution with fatter negative tail than positive tail[5,6].
- Phase dependence of higher moments and sample ACF.
- Different time scales for the positive and negative phases.

A self-exciting threshold autoregressive process $\{X_i\}_{i=1}^n$, abbreviated **SETAR(p)**, is defined as

$$X_i = \mu_j + \varphi_1^j(X_{i-1} - \mu_j) + \cdots + \varphi_p^j(X_{i-p} - \mu_j) + \varepsilon_i,$$

where p is a positive integer, φ_i^j and μ_j are real and $\{\varepsilon_i\}_{i=1}^n$ is white noise. Here j is determined by the thresholds $-\infty = r_0 < r_1 < \cdots < r_k = \infty$ and the condition $r_{j-1} < X_{i-1} \le r_j$.

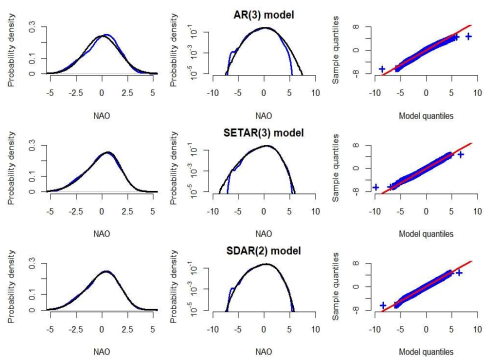
A state-dependent nonlinear autoregressive process $\{X_i\}_{i=1}^n$, abbreviated **SDAR(p)**, is defined as

$$X_i = \mu + \sum_{j=1}^3 \sum_{k=1}^p \varphi_k^j (X_{i-k})^j + \varepsilon_i,$$

where p is a positive integer, φ_k^j and μ are real constants and $\{\varepsilon_i\}_{i=1}^n$ is white noise.

We investigate these three models for the optimal choice of p in terms of the Bayesian information criterion (BIC).

Fit of NAO distribution

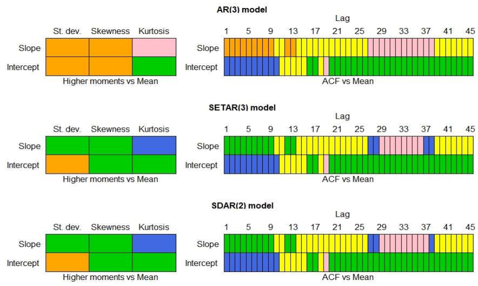


- Both the SETAR(3) and SDAR(2) models reproduce the NAO distribution very well.
- The SETAR(3) overestimates the extreme negative tail slightly.

Fit of phase dependence of the NAO

We have simulated 71 000 winters using the three models and investigated how well the models reproduce the phase dependence of higher moments and sample ACF. For the second, third and fourth moments and the sample ACF with lags 1-45, we have performed linear regressions versus the mean of the NAO. The intercepts and slopes of these simulated regressions are then compared with 95% confidence intervals of the intercepts and slopes obtained from the data (see Figures 2-3).

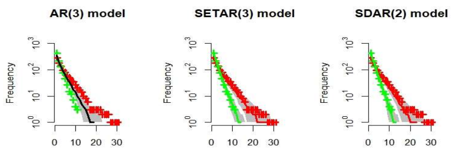
Green, blue and yellow boxes in the plot below indicate that the simulated values are within the confidence bounds and that the confidence intervals are positive (blue), negative (green) or includes zero (yellow). Orange and pink boxes indicate that the simulated values are larger and smaller, respectively, than the simulated ones.



- Both the SETAR(3) and SDAR(2) models reproduce the phase dependence of the NAO very well.

Time scales of the two NAO phases

We have investigated the frequency of positive and negative phase events of the NAO.



- Both the SETAR(3) and SDAR(2) models reproduce the frequency of positive and negative phase events very well.

Summary

Both nonlinear models investigated reproduce the distribution, sample ACF, phase dependence and different time scales of the two NAO phases very well.

References

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