A Two-Dimensional Analytical Solution for Remediation of Salt Affected Site through Ditch Drainage System

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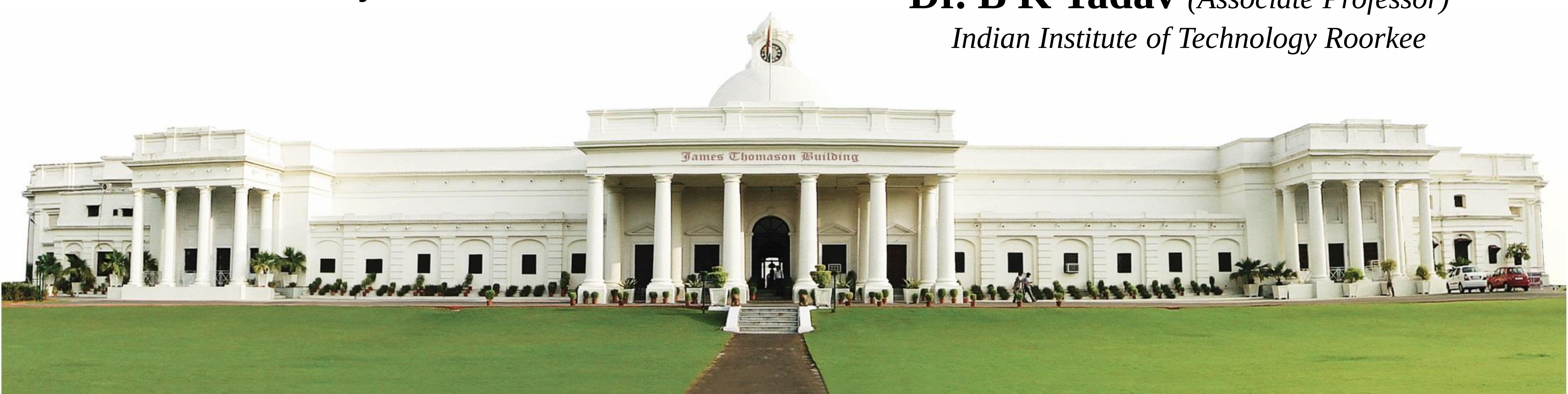


Outline of the presentation

- Introduction
- Aim and objectives
- Formulation
- Solution
- Result and discussion
- Conclusion

Introduction

Drainage system

- A system by which water is drained on or in the soil to enhance agricultural production of crops.

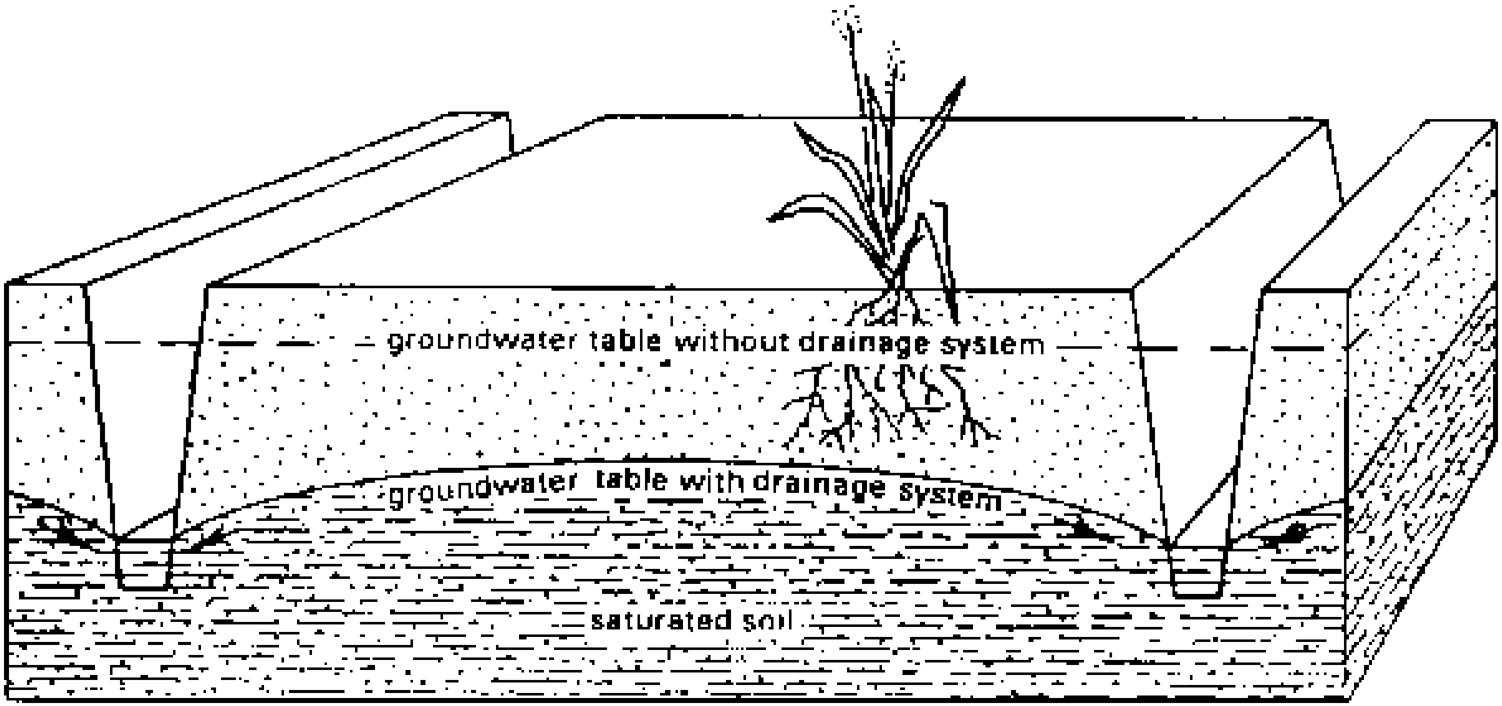
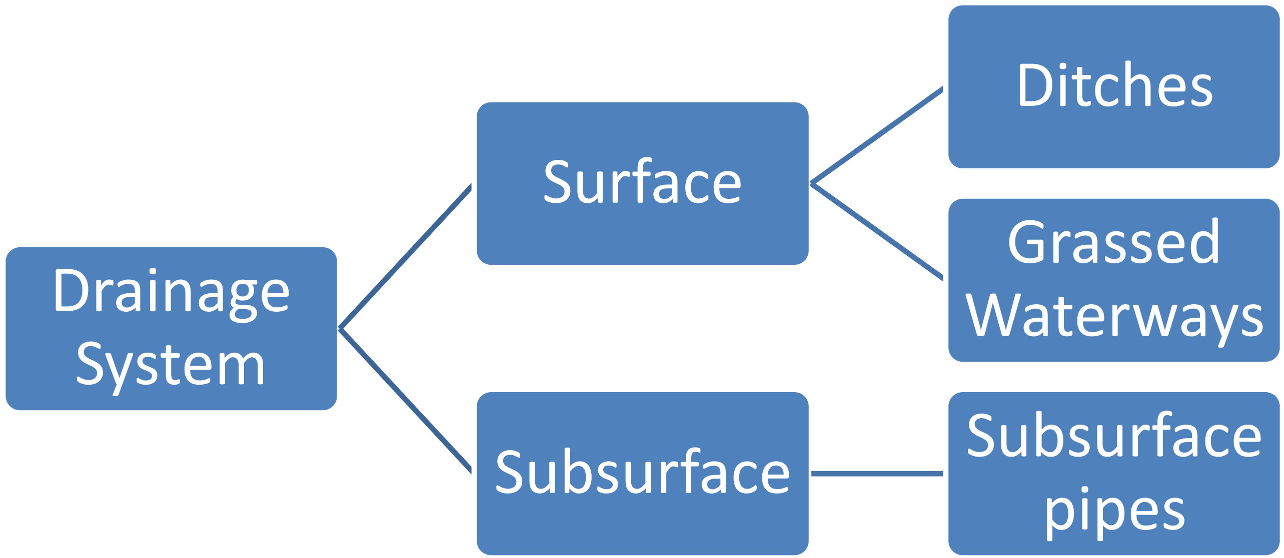


Fig. Control of Groundwater by means of deep open Drains

Introduction

Drainage system



Benefits of Ditch Drainage system

Overcome Soil salinity

- Ponding on the surface of field and then drain out the water from the field

Overcome Water Logging

- Drain out water from the field

Aim

To derive an analytical solution for the prediction of transient seepage of ditch drainage system which is receiving water from a ponded field which is influenced by Source/Sink

Objectives

- To analyze the influence of source on the ditch drainage flow
- To analyze the influence of Sink on the ditch drainage flow
- To analyze the flow for different time values
- To Comparison of the analytical solution of the problem to numerical solution of problem for the same hydraulic parameters

Formulation

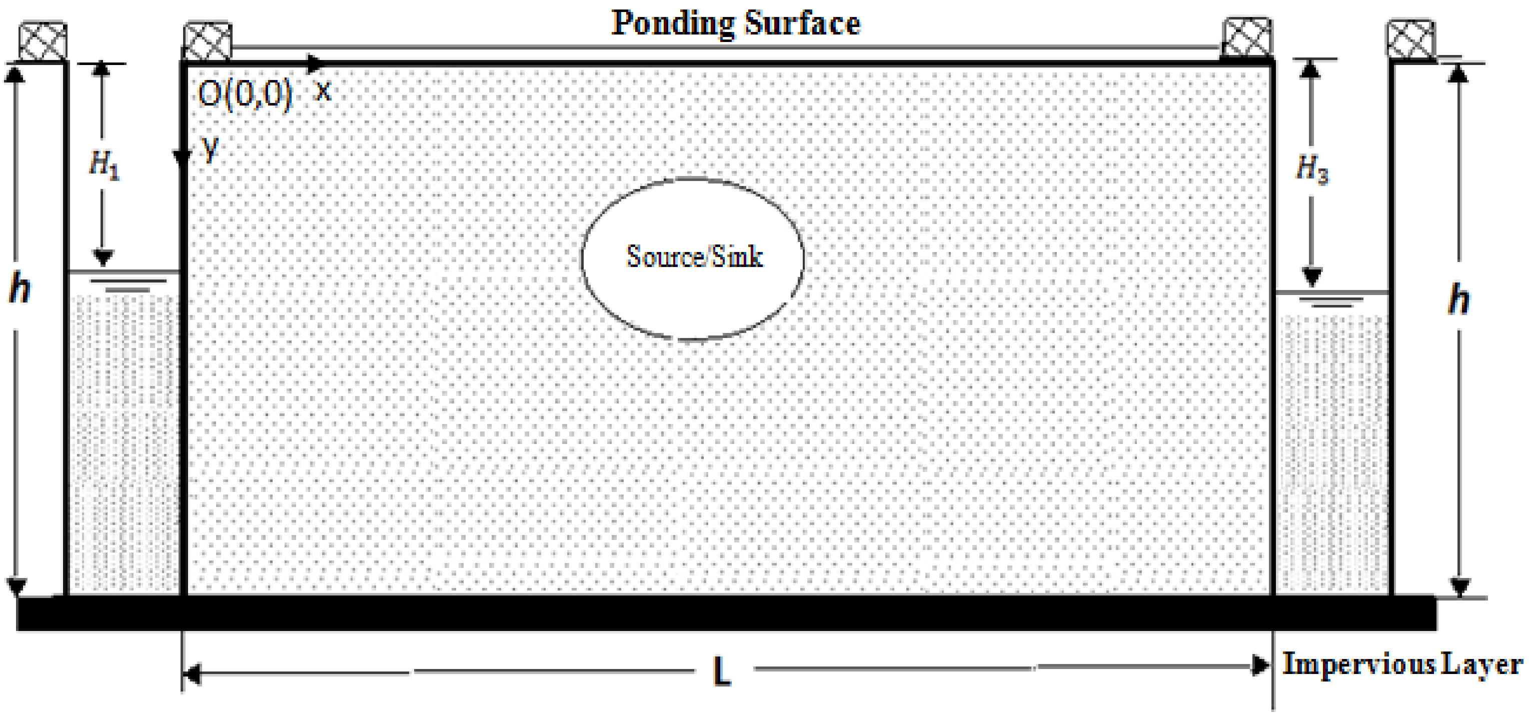


Fig. Geometry of a fully penetrating ditch drainage system in homogeneous and anisotropic soil subjected to a uniform ponding depth at the surface of the soil

Formulation

Parameter	Notification
Spacing between the ditches	L
Depth of the Ditches	h
Ponding depth on the surface	t_o
level of water in the left edge drain	H_1
level of water in the right edge drain	H_2

Assumptions for flow geometry

- Soil medium is homogeneous and anisotropic
- Origin is considered at the top edge of the left hand drain and is demarcated by point O
- The x-axis is taken as positive towards right direction whereas the y-axis is taken to be positive in the vertically downward direction.
- A source/ sink entity of intensity is assumed to be acting throughout the domain.

Analytical solution

Governing Equation

$$K_x \frac{\partial^2 \phi}{\partial x^2} + K_y \frac{\partial^2 \phi}{\partial y^2} + Q(x, y, t) = S_s \frac{\partial \phi}{\partial t} \qquad \text{Eq. (1)}$$

Where

S_s *Specific Storitivity*

K_x *Hydraulic Conductivity in horizontal direction*

K_y *Hydraulic Conductivity in vertical direction*

Boundary Conditions in flow domain

$\phi(x, y, t = 0) = 0$	$0 < x < L,$	$0 < y < h,$	(I)
$\phi(x, y, t > 0) = -y,$	$x = 0,$	$0 < y < H_1,$	(II)
$\phi(x, y, t > 0) = -H_1,$	$x = 0,$	$H_1 < y < h,$	(III)
$\phi(x, y, t > 0) = -y,$	$x = L,$	$0 < y < H_2,$	(IV)
$\phi(x, y, t > 0) = -y,$	$x = L,$	$H_2 < y < h,$	(V)
$\phi(x, y, t > 0) = t_0,$	$x = L,$	$h = 0,$	(VI)
$\dfrac{\partial \phi(x, y, t > 0)}{\partial y} = 0,$	$0 < x < L,$	$y = h,$	(VII)

Analytical solution

Considering the steady state condition and neglecting source/sink term, the analytical solution to Eq. (1) using method of separation of variable (Kirkham and Powers 1972) is given by

$$\phi_E(x,y) = \sum_{m=1}^{M}\left[A_m \frac{\sinh\left(\sqrt{\frac{K_y}{K_x}} N_m x\right)}{\sinh\left(\sqrt{\frac{K_y}{K_x}} N_m L\right)} \sin(N_m y) \right] + \sum_{n=1}^{N}\left(B_n \frac{\sinh\left(\sqrt{\frac{K_y}{K_x}} N_n (L-x)\right)}{\sinh\left(N_n L\sqrt{\frac{K_y}{K_x}}\right)} \sin(N_n y) \right)$$

$$+\sum_{p=1}^{P}\left(C_p \frac{\cosh\left(\sqrt{\frac{K_x}{K_y}} N_p (h-y)\right)}{\cosh\left(\sqrt{\frac{K_x}{K_y}} N_p h\right)} \sin(N_p x) \right)$$

Eq. (2)

Where,

$$N_m = \frac{(1-2m)\pi}{2h};$$

$$N_n = \frac{(1-2n)\pi}{2h};$$

$$N_p = \frac{p\pi}{L};$$

A_m, B_n and C_p are the Fourier coefficient

Applying condition (II) and (III) to Eq. (2), we get

$$\sum B_n \sin(N_n y) = -y, \qquad 0 < y < H,$$

$$\sum B_n \sin(N_n y) = -H_1, \qquad H_1 < y < h,$$

Applying Fourier series to in the domain $0 < y < h$, we yield

$$B_n = \frac{-2}{h}\left[\left(\int_0^{H_1} y \sin(N_n y)dy\right) + \left(\int_{H_1}^{h} H_1 \sin(N_n y)dy\right)\right]$$

Integrating the above equation, the expression for B_n is given by

$$B_n = \left(\frac{-2}{h}\right)\frac{\sin(N_n H_1)}{N_n^{\,2}}; \qquad \text{Eq. (3)}$$

ANALYTICAL SOLUTION

Similarly we can calculate the other coefficients as below

$$A_m = \left(\frac{-2}{h}\right)\frac{\sin(N_m H_2)}{N_m{}^2} \qquad \text{Eq. (4)}$$

$$C_p = \left(\frac{2}{L}\right)\left[\frac{1-\cos(N_p L)}{N_p}\right]t_0 \qquad \text{Eq. (5)}$$

To satisfy the homogenous boundary condition, assuming $V(x,y,t)$ to be the transient solution

$$V(x,y,t) = \phi(x,y,t) - \phi_E(x,y) \qquad \text{Eq. (6)}$$

ANALYTICAL SOLUTION

By its very definition, $V(x, y, t)$ is bound to satisfy homogeneous boundary condition because both ϕ and ϕ_E are satisfying same set of boundary conditions; i.e.,

$$V(x, y, t) = 0 \qquad x = 0, \qquad 0 < y < h \qquad \text{(VIII)}$$

$$V(x, y, t) = 0 \qquad x = L, \qquad 0 < y < h \qquad \text{(IX)}$$

$$V(x, y, t) = 0 \qquad y = 0, \qquad 0 < x < L \qquad \text{(X)}$$

$$\frac{\partial V}{\partial h} = 0 \qquad y = h, \qquad 0 < x < L \qquad \text{(XI)}$$

ANALYTICAL SOLUTION

For mathematical simplicity,

expressing $V(x, y, t)$ in a double Fourier series

$$V(x, y, t) = \sum\sum D_{uv}(t)\sin(N_u x)\sin(N_v y). \quad \text{Eq. (7)}$$

and the source/sink term is also expressed in a double Fourier series as expressed by

$$Q(x, y, t) = \sum\sum q_{uv}(t)\sin(N_u x)\sin(N_v y) \quad \text{Eq. (8)}$$

where

$$N_u = \frac{u\pi}{L};$$

$$N_v = \frac{(1-2v)\pi}{2h};$$

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ANALYTICAL SOLUTION

Substituting the expression of Eq.(7) and Eq.(8) in Eq. (1)

Applying double Fourier series

$$S_s \frac{d}{dt}(D_{uv}) + (K_x N_u{}^2 + K_y N_v{}^2) D_{uv}(t) = q_{uv}(t)$$

Multiplying $e^{\left(\frac{K_x N_u{}^2 + K_y N_v{}^2}{S_s}\right)t}$ on both side

$$\frac{d}{dt}\left[D_{uv}(t) e^{\left(\frac{K_x N_u{}^2 + K_y N_v{}^2}{S_s}\right)t} \right] = \frac{q_{uv}(t)}{S_s} e^{\left(\frac{K_x N_u{}^2 + K_y N_v{}^2}{S_s}\right)t}$$

On simplifying we get,

$$D_{uv}(t) = D_{uv}(0) e^{-\left(\frac{K_x N_u{}^2 + K_y N_v{}^2}{S_s}\right)t} + e^{-\left(\frac{K_x N_u{}^2 + K_y N_v{}^2}{S_s}\right)t} \int_0^t \frac{q_{uv}(t)}{S_s} e^{\left(\frac{K_x N_u{}^2 + K_y N_v{}^2}{S_s}\right)t} dt$$

ANALYTICAL SOLUTION

where,

$D_{uv}(0)$ is evaluated by putting $t=0$ in Eq.(6); yields

$$V(x,y,0)=\sum\sum D_{uv}(0)\sin(N_u x)\sin(N_v y)$$

upon further simplifying, we get

$$\phi\,(x,y,0)-\phi_E(x,y)=\sum\sum D_{uv}(0)\sin(N_u x)\sin(N_v y)$$

Applying double Fourier run to in the domain $0<x<L$ and $0<y<h$,

$$D_{uv}(0)=\left(\frac{2}{L}\right)\left(\frac{2}{h}\right)\int_0^L\int_0^h\left(f(x,y)-\phi_E(x,y)\right)\sin(N_u x)\sin(N_v y)\,dxdy$$

Results and Discussion

Comparison of analytical and numerical results

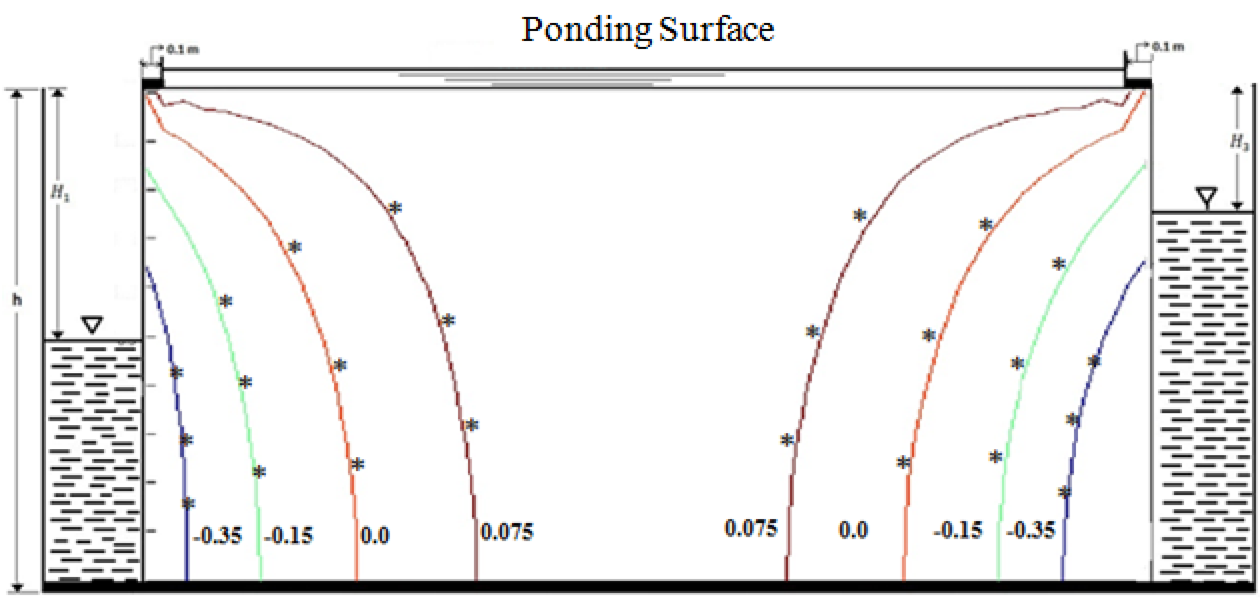

Fig. 2 Comparison of hydraulic head contours as obtained from the proposed analytical solution of the flow problem in Fig. .1 with the corresponding Numerical solution generated contours for the flow situation when L=5 m, $H = 1\ m$, $H_1 = 0.5\ m$, $H_2 = 0.75\ m$, $K_x = 1 m/day$, $K_y = 1 m/day$, $S_s = 0.0001/day$, $dx = 0.5\ m$, $dy = 0.1\ m$; $t = 5\ sec$, asterisks mark hydraulic head contours as generated by numerical solution; solid lines denote hydraulic head contours as generated by the proposed analytical solution; depth of ponding and height of ditch bund are not to scale; all other dimensions are to scale

Results and Discussion

Comparison of analytical results with Barua and Alam (2013)

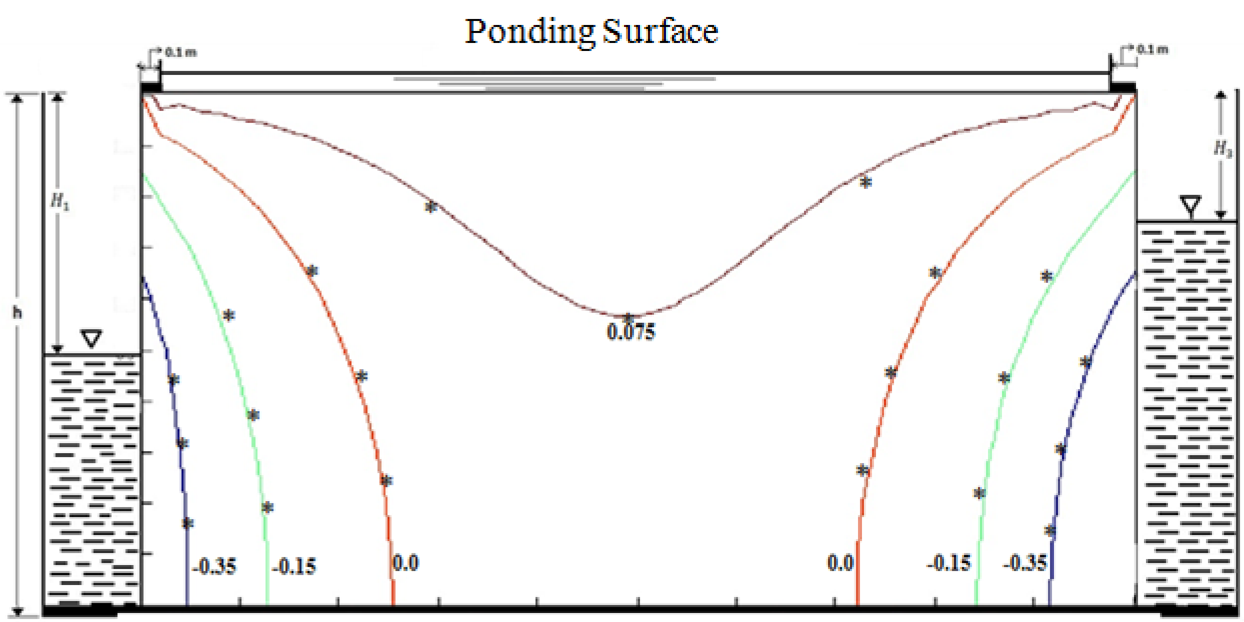

Fig. 3 Comparison of hydraulic head contours as obtained from the proposed analytical solution of the flow problem in Fig. 1 with the solution obtained by Barua and Alam (2013) generated contours for the flow situation when L=5 m, $H = 1\,m$, $H_1 = 0.5\,m$, t=5 sec $H_2 = 0.75\,m$, $K_x = 1 m/day$, $K_y = 1 m/day$, $S_s = 0.0001/day$, $dx = 0.5\,m$, $dy = 0.1\,m$; asterisks mark hydraulic head contours as generated by solution of Barua and Alam (2013); solid lines denote hydraulic head contours as generated by the proposed analytical solution; depth of ponding and height of ditch bund are not to scale; all other dimensions are to scale

Results and Discussion

Comparison for with source and without source at *t= 5 sec*

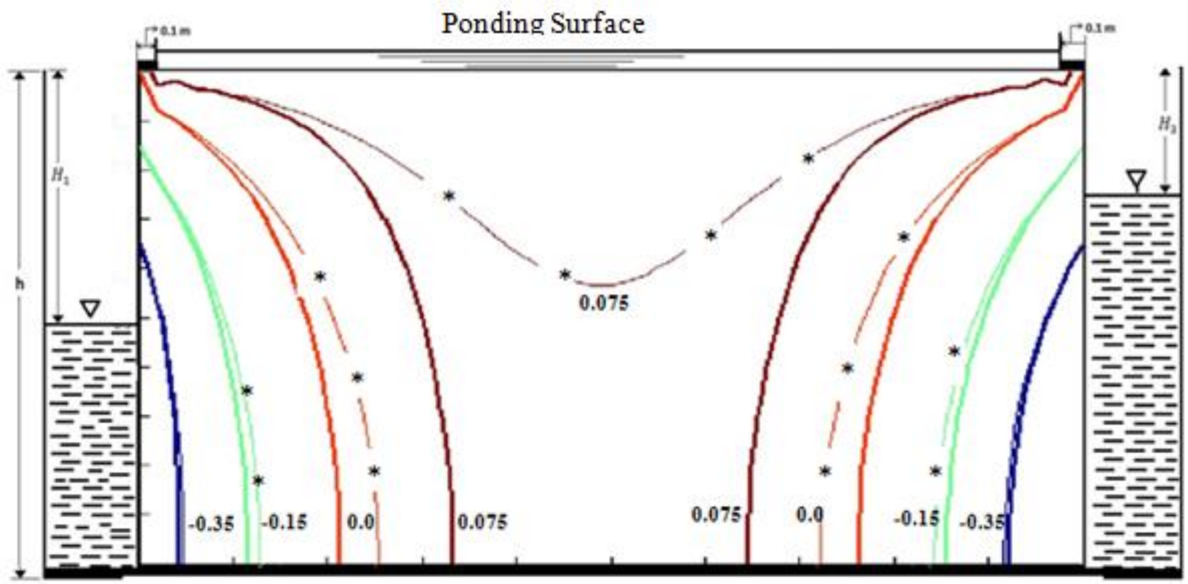

Fig. 4 Comparison of hydraulic head contours as obtained from the proposed analytical solution of the flow problem with influence of source in Fig.1 with the corresponding solution without considering any influence generated contours for the flow situation when $L=5\ m$, $H = 1\ m$, $H_1 = 0.5\ m$, $H_2 = 0.75\ m$, $t=5sec$, $K_x = 1m/day, K_y = 1m/day$, $S_s = 0.0001/day$, $dx = 0.5\ m$, $dy = 0.1\ m$; asterisks marked line hydraulic head contours obtained for without influence of source solution; solid lines denote steady-state hydraulic head contours as generated by the proposed analytical solution with influence of source; depth of ponding and height of ditch bund are not to scale; all other dimensions are to scale

RESULTS AND DISCUSSION

Influence of Source

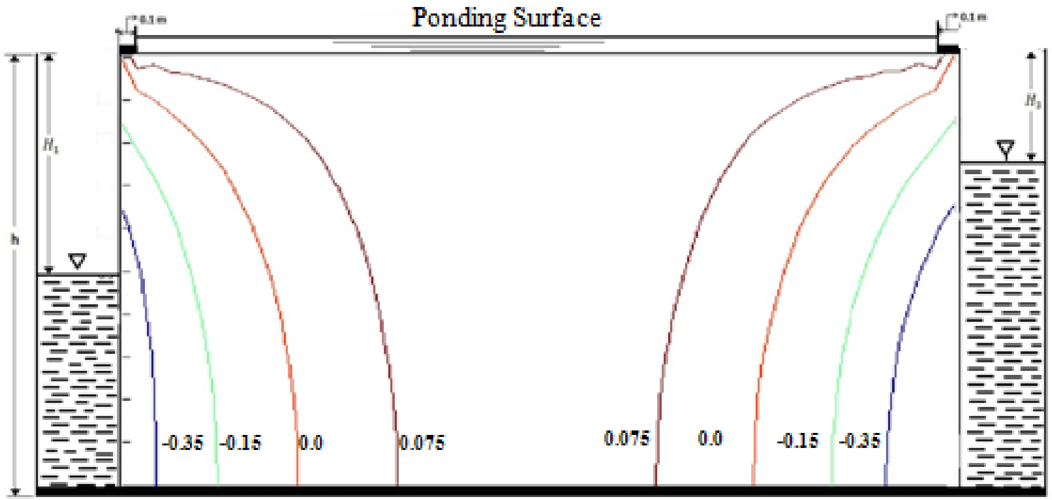

Fig. 5 Hydraulic head contours as obtained from the proposed analytical solution of the flow problem in Fig.1 with the influence of Source for the flow situation when $L=5\ m$, $H = 1\ m$, $H_1 = 0.5\ m$, $t=5\ sec$, $H_2 = 0.75\ m$, $K_x = 1 m/day$, $K_y = 1 m/day$, $S_s = 0.0001/day$, $dx = 0.5\ m$, $dy = 0.1\ m$; solid lines denote hydraulic head contours as generated by the proposed analytical solution; depth of ponding and height of ditch bund are not to scale; all other dimensions are to scale

Results and Discussion

Influence of Sink

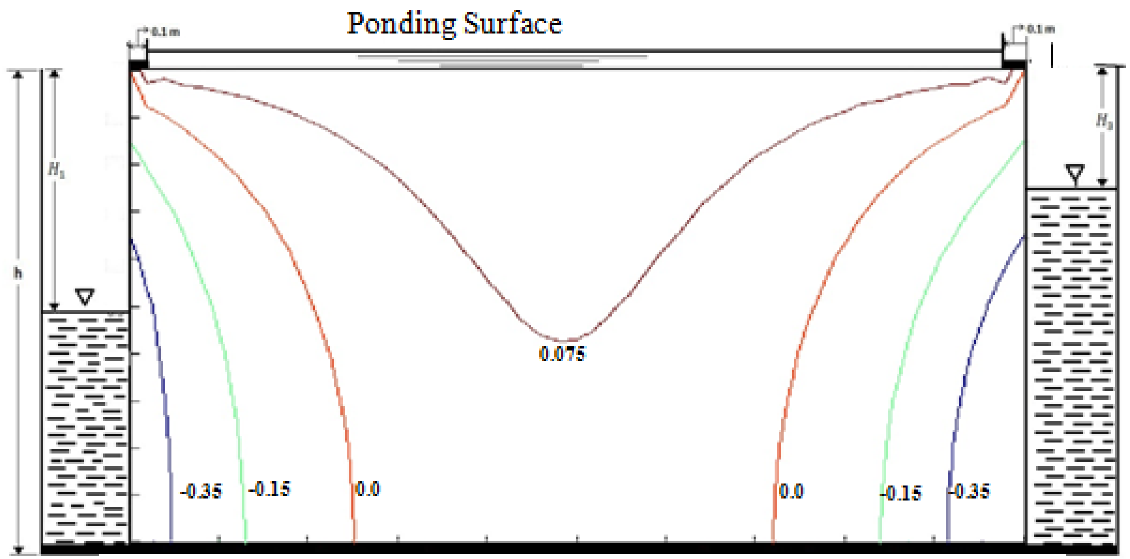

Fig. 6 Hydraulic head contours as obtained from the proposed analytical solution of the flow problem in Fig.1 with the influence of Sink for the flow situation when L=5 m, $H = 1\ m$, $H_1 = 0.5\ m$, t=5 sec, $H_2 = 0.75\ m$, $K_x = 1 m/day, K_y = 1 m/day$, $S_s = 0.0001/day$, $dx = 0.5\ m$, $dy = 0.1\ m$; solid lines denote hydraulic head contours as generated by the proposed analytical solution; depth of ponding and height of ditch bund are not to scale; all other dimensions are to scale

CONCLUSION

- In the proposed model an analytical solution is provided which is obtained by using separation of variable method.
- Obtained solution is tested for the influence of source and influence of the sink in the flow domain.
- When the influence of source is taken in consideration the hydraulic head values in the flow domain are raised and vice versa due to influence of sink hydraulic head values decreased.
- Comparing the proposed solution with Barua and Alam (2013) solution is also similar.

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