Can stochastic resonance explain the amplification of planetary tidal forcing?

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Long-period cycles in solar activity

Gleissberg  de Vries

Solar activity from $^{14}\text{C}$ and $^{10}\text{Be}$ proxies.

Planetary torque

Hypothesis of planetary influence

Abreu et al. 2012
Stochastic Resonance

Transition probabilities:

\[ W_{\pm}(t) = \frac{1}{2}(\alpha_0 \mp \alpha_1 \cos \omega_S t) \]

Output power spectrum:

\[ S(\Omega) = \left(1 - \frac{\alpha_1^2}{2(\alpha_0^2 + \omega_S^2)}\right) \left(\frac{4c^2\alpha_0}{\alpha_0^2 + \Omega^2}\right) + \frac{\pi c^2\alpha_1^2}{\alpha_0^2 + \omega_S^2} \delta(\Omega - \omega_S) \]

Archetypical example of a particle, subject to random fluctuations, in a double-well potential weakly modulated by an external force.

McNamara Wiesenfeld 1989
Babcock-Leighton-type dynamo models

\[ \dot{B}(t) = -\frac{\omega}{L}A(t - T_0) - \tau^{-1}B(t), \]

\[ \dot{A}(t) = \alpha_0 f(B(t - T_1))B(t - T_1) - \tau^{-1}A(t), \]

\[ \Rightarrow \tau^2 \ddot{B}(t) + 2\tau \dot{B}(t) + B(t) = -Nf(B(t - T))B(t - T), \]

\[ N = \frac{\omega \alpha_0 \tau^2}{L}, \quad T = T_0 + T_1, \]

\[ f(B) = \frac{1}{4} \left( 1 + \text{erf} \left( B^2 - B_{\text{min}}^2 \right) \right) \left( 1 - \text{erf} \left( B^2 - B_{\text{max}}^2 \right) \right). \]

The sources of the Alpha and Omega effects are spatially segregated, which is modeled by effective delays in the ODE model.

The Alpha effect is assumed to be limited to \( B_{\text{min}} \leq B \leq B_{\text{max}} \).

Wilmot-Smith et al. 2006
Babcock-Leighton-type dynamo models

Peaks as function of the dynamo number:

- **Peak B**
- **N**
- **Lower critical point**
- **Upper critical point**
- **Range of bi-stability**
- **Period doubling bifurcations and chaos**
- **Strong overshoot oscillation**

$B^2$
• Three types of stable solutions:
  • Zero solution $B = 0$.
  • Oscillations with little overshoot w.r.t. $B_{\text{max}}$, a base frequency close to the linear solution, modulations due to period doubling bifurcations or chaos.
  • Periodic oscillations with strong overshoot.
• Near the upper critical point, all three solutions are stable.
• Adding noise allows the dynamo to switch between these modes.
• Near the critical points, the dynamo is very susceptible to external modulations (Stochastic Resonance).
Evidence from records of solar activity

• Solar activity switches between different modes of oscillation: Both exceptionally large cycles and cycles with almost no sunspots are often clustered (Grand Maxima/Minima).

• Grand Minima themselves are clustered. During periods with Grand Minima, long-period cycles (e.g. 208y and 350y) are stronger than during periods without Grand Minima (McCracken et al 2013).
Evidence from records of solar activity

- Solar cycles observed through sunspot numbers (red dots) are characterized by a steep ascent, and a descent with a “kink”. This shape fits well to the “strong overshoot” solution of the model, with a dynamo number near the upper critical point (blue line):
Conclusions

- BL-type dynamos display a critical point at high dynamo numbers, near which two stable oscillatory modes co-exist. The strong mode could be related to the strong cycles observed during Grand Maxima.

- Near this critical point, Stochastic Resonance could explain some of the long-period modulations of solar activity as an effect of a weak planetary forcing.

- The strong mode shows a characteristic kink in the falling limb (caused by the overshooting over $B_{\text{max}}$), which is also evident in the sunspot records of the modern Grand Maximum.

- Observations show a weak negative correlation between cycle amplitude and length. While our simple noisy dynamo model shows such a negative correlation, when in its low-overshoot mode, the strong-overshoot cycles are longer than the low-overshoot ones.

