Evaporation (ET) is a dynamic and nonlinear process that incorporates various internal transport mechanisms, which is essential in the unsaturated zone in and regions under low soil moisture conditions. FAO Penman-Monteith (PM) equation is the most widespread method to estimate the evaporation rate in saturated soils. This approach can be implemented as a boundary condition for the Richards’ equation and related to the evaporation rate with the water content in the soil. However, the PM equation is not valid when the soil moisture is low, and the vapor flux is an essential component of the total water flux. In this case, the governing equations are formed out of the coupled Richards’ equation with the heat transport, where the boundary conditions originate from the surface energy balance and the evaporation rate (Saito et al., 2006; Sakai et al., 2011).

Introduction

Two models were implemented in the Dual Richards’ Unsatuated Equation Solver (DRUUES). The first model accounts for surface evaporation by coupling the Richards’ equation and the Penman-Monteith equation

\[ \nabla \cdot (K(h) \nabla h) + \frac{\partial K_s(u)}{\partial z} = C(h) \frac{\partial h}{\partial t} \]  

The initial condition

\[ h(x, t_0) = h_0(x) \quad \forall x \in \Omega \]  

The surface boundary condition

\[ K(h) \left( \frac{\partial h(x, t)}{\partial n} + n_3 \right) = q_{cr}(t) \quad \forall (x, t) \in \Gamma_{surf} \times [0, T) \]  

Including the actual evapotranspiration

\[ q_{cr}(t) = \begin{cases} r(t) - ET_a(t) & \text{if } r(t) - ET_a(t) \geq 0 \\ r(t) - ET_a(t) & \text{if } r(t) - ET_a(t) < 0 \end{cases} \]  

And the Penman-Monteith equation for the evaporation rate

\[ ET_a = \frac{0.408(\Delta (R_g - G) + \frac{\gamma}{\rho_{air}} (e_s - e_h))}{\Delta + \gamma (1 + 0.34u_2)} \]  

The second model accounts for sub-surface evaporation by coupling a modified Richards’ equation and the heat equation

\[ \frac{\partial h}{\partial t} = \nabla \cdot (K(h) \nabla h) + \nabla \cdot (K_s(u) \nabla T) \]  

\[ + \nabla \cdot (K_s(u) \nabla z) - \frac{\partial T}{\partial t} \quad \forall x \in \Omega \]  

\[ \frac{\partial T}{\partial t} = \nabla \cdot (C_p \nabla T) + \nabla \cdot (B_n \nabla h) \]  

\[ - \nabla \cdot [(C_t q_i + C_r q_i') T)] - L \frac{\partial T}{\partial t} \quad \forall x \in \Omega \]  

Initial condition for both partial differential equations

\[ h(x, t_0) = h_0(x) \quad \forall x \in \Omega \]  

\[ T(x, t_0) = T_0(x) \quad \forall x \in \Omega \]  

The surface boundary condition for the water equation

\[ \left\| \mathbf{q}_i (\mathbf{x}, t) \right\|_{x=a} + \left\| \mathbf{q}_i (\mathbf{x}, t) \right\|_{x=a} = E_i(t) \quad \forall x \in \Gamma_{surf} \times [0, T) \]  

Including the evaporation rate

\[ E_i(t) = \frac{\rho (h, T_i) \rho (\rho (T_i) - RH_{air} (T_i))}{\rho_{air}} \]  

The surface boundary condition for the heat equation

\[ - \frac{\partial T}{\partial n} + C_t \left[ \mathbf{q}_i \right]_{x=a} - G - L (\mathbf{q}_i) \left[ \mathbf{q}_i \right]_{x=a} \quad \forall x \in \Gamma_{surf} \times [0, T) \]  

Including the surface energy balance

\[ \dot{R}_s - \dot{H} - \dot{L}_E + G = 0 \]  

Results

A 20 cm long soil profile was used to perform the numerical experiments in 1D. The simulated experiments had total simulation time of 14 days, where the hydraulic properties and the numeric parameters were consistent across all the simulations. Two scenarios were proposed under controlled meteorological conditions.

- The first scenario is known as a dark condition where no incoming shortwave radiation was considered. \( R_s = 0 \).
- The second scenario is characterized by a constant incoming shortwave radiation, \( R_s = \) constant.

References