Towards non-linear inverse problem for atmospheric source term determination

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Problem formulation

we assume linear model of atmospheric dispersion using a source-receptor sensitivity (SRS) matrix \( M \) as

\[
y = Mx + e, \tag{1}
\]

\( y \in \mathbb{R}^p \) is a vector aggregating measurements
\( M \in \mathbb{R}^{p \times n} \) is the SRS matrix
\( x \in \mathbb{R}^n \) is a vector of the unknown release to be estimated
\( e \in \mathbb{R}^p \) is error model
Atmospheric model error

- SRS matrix $M$ is traditionally assumed to be correct, which may be misleading.
- Here, we consider (in general) bi-linear model of the source term estimation problem in the form

\[ y = (M + \Delta_M)x + e, \]  

where $\Delta_M$ is the deviation of $M$ from the “correct” SRS fields.
- The deviation $\Delta_M$ can express, e.g., temporal shift and/or spatial shift.
Bi-linear formulation

- bi-linear formulation of the problem

\[
y = \left( \begin{array}{c}
M + \text{diag}(h_t) (M_{t-shift+} - M_{t-shift-}) \\
H_t & S_t
\end{array} \right) x + e, \tag{3}
\]

- \( h_t \in [-1; +1] \) are (unknown) coefficients
- \( M_{t-shift+} \) and \( M_{t-shift-} \) are shifted SRS matrices
Variational Bayes solution (in short)

- prior $p(y)$ is modeled as Gaussian with estimated scalar precession
- $p(H_t)$ is modeled according to the sparse Bayesian learning [Tipping, M. E. Sparse Bayesian learning and the relevance vector machine. Journal of machine learning research, 1, 211-244, 2001.]
Synthetic example

- **Release rate (kg/h)**: The diagram shows release rates over time. The blue line represents bilinear estimation, the green line denotes linear estimation, and the red dashed line indicates the true simulated release.

- **Time (h)**: The x-axis represents time in hours, ranging from 0 to 10.

- **Measurement no.**: The y-axis on the right represents the number of measurements, ranging from -1 to 200.

- **Estimated shift (H values)**: The rightmost diagram displays estimated shifts against measurement numbers, comparing estimation (blue) and simulation (red dashed) results.
ETEX example
Preliminary conclusions

- it is possible to estimate parametric corruptions the SRS fields and correct them
- better measurements fit is observed (indeed, also overfitting in specific cases)

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