Application of polarimetric radar to infer ice fabric anisotropy

Concordia (Dome C)- Antarctica

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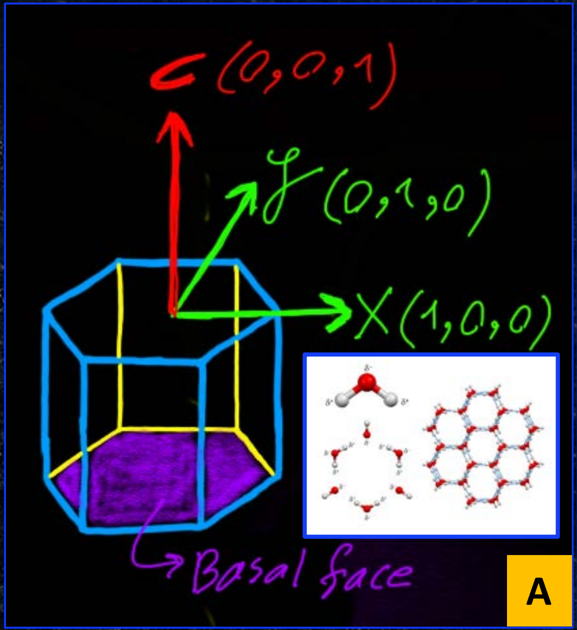
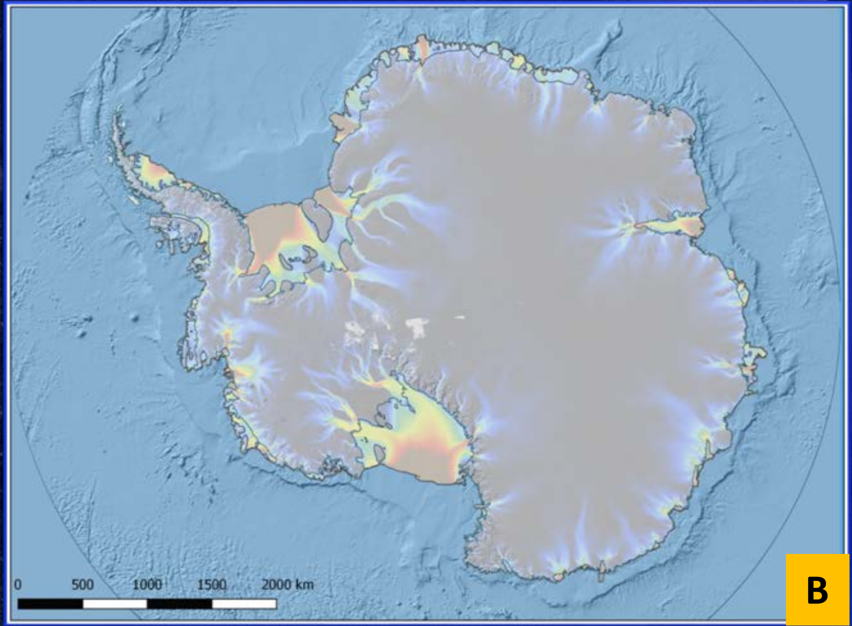
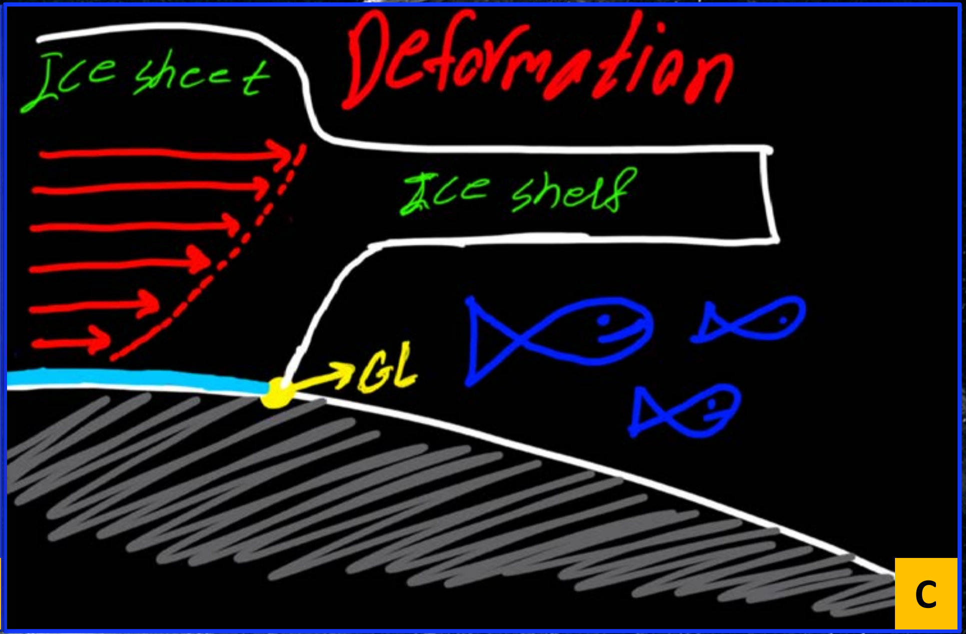
What to expect from this study ?

- Ice internal deformation is very important in glaciology and ice sheet modeling. The temperature-dependent, non-linear and anisotropic rheology of ice governs the way how ice deforms but this quantity is poorly constrained by observation.
- We use polarimetric radar (ApRES) to determine crystal orientation fabric and ice anisotropy.
- Here we are presenting a method to estimate fabric parameters:
  - α: Orientation of fabric horizontal principal axes relative to a geographic coordinate system
  - E2-E1: Horizontal eigen value difference which quantifies the horizontal asymmetry of the ice fabric
  - r: Ratio of the Fresnel reflection coefficients along the principal axes
- We show the correlation between HV power anomaly and α
- We show the correlation between nodes azimuthal difference in HH power anomaly and r
- We applied a nonlinear optimization on the ApRES data and we estimated fabric parameters
  - Using HV power anomaly to find initial value of α
  - Using depth gradient of polarimetric coherence phase difference (Jordan 2019) to find initial value of E2-E1
  - Using HH power anomaly as the optimization cost function
- We designed a stand-alone user-friendly application for this method which will be available in GitHub.
- Our results can explain and reconstruct the ApRES data and match with the ice core observation

Introduction: What is ice anisotropy and why should we care about it?

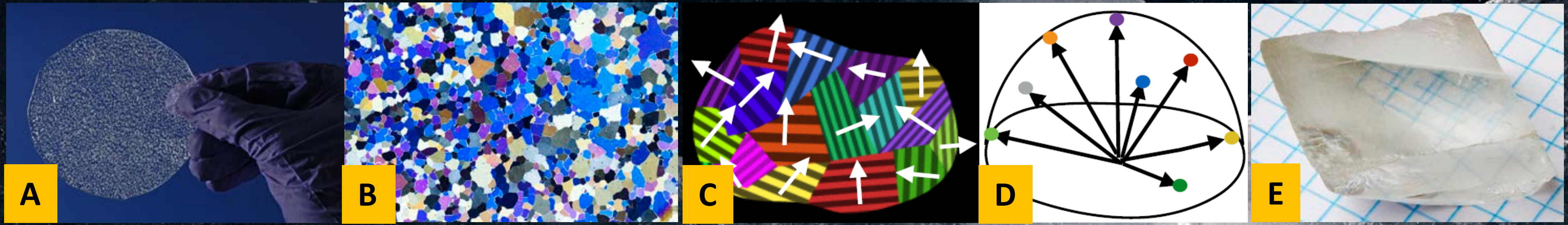


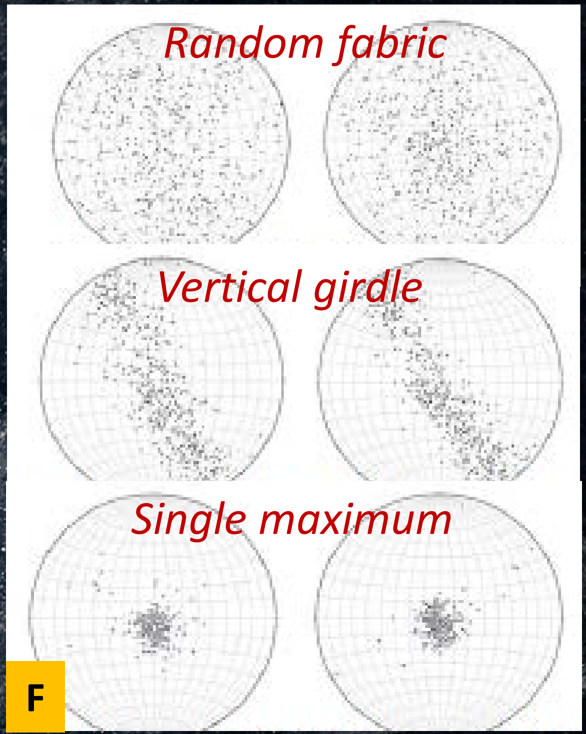




- Gravitational forces drive ice movement in glaciers and ice sheets. This can cause internal ice deformation. In most cases it is difficult to determine the amount this deformation. This increases uncertainties in several areas such as:
  - ice-flow model initialization with data assimilation techniques
  - prediction of erosion rates from surface velocities
  - predicting age-depth relationships required for finding new ice-core drill sites
- The temperature-dependent, non-linear and anisotropic rheology of ice governs the way how ice deforms.
- Due to the limited number of ice cores this quantity is poorly constrained by observations
- Figure A: A single ice crystal and the direction of its c-axis
- Figure B: Ice flow velocity map of Antarctica. The flow velocity increase from the center towards the ocean.
- Figure C: Ice deformation when the bottom of the ice is frozen and the flow velocity decreases from top to bottom

Introduction: What is ice anisotropy and why should we care about it?



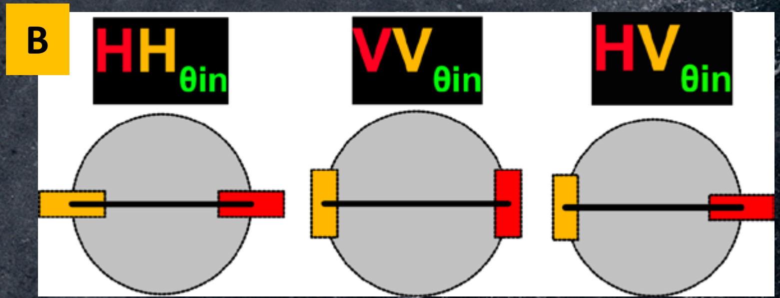


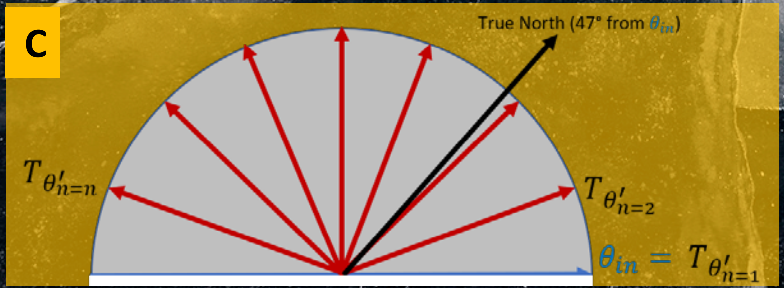
- Determining ice anisotropy by observation using a thin section of ice and a polarized microscope (Weikusat 2017)
- It is possible to transfer the c-axis orientation measured in ice core to the radar relevant dielectric permittivity tensor (Fujita 2006)
- C-axis orientation distribution of the ice fabric:
  - Using second order orientation tensor
  - Eigenvalues describe the relative concentration of c-axes aligned with each principal coordinate direction/eigenvector
  - $E1+E2+E3 = 1$ & $E1<E2<E3$
  - Assuming E3 is vertical with E1 and E2 in the horizontal plane
- Random fabric (near surface) $E1=E2=E3$
- Vertical girdle (horizontal tension at moderate ice depth $E1=0$ & $E2=E3=0.5$
- Single pole (deep ice undergoing vertical compression) $E1=E2=0$ & $E3=1$
- A thin section of ice (fig. A) under a polarized microscope (fig. B).
- Figure C & D: The colors representing the direction of the c-axis at every single crystal.
- Figure E: The effect of birefringence. It happens when the light (wave) is traveling through media with two different velocities.
- Figure F: Projecting the direction of all the c-axis on a Schmidt diagram. It can reveal the type of fabric.

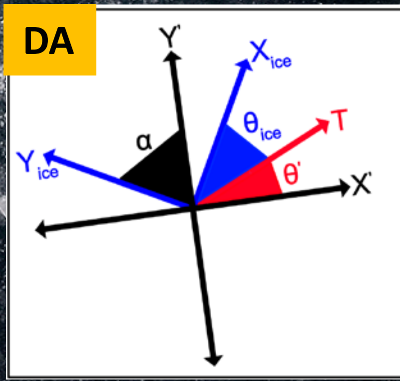
Method: Polarimetric Radar (ApRES)

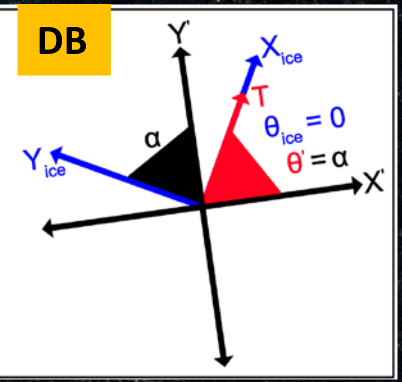
- Frequency-Modulated Continuous-Wave (FMCW) radar
- 200 MHz bandwidth
- Center frequency of 300 MHz
- It can store the de-ramped signal phase (Jordan 2019)
- Using polarimetric radar to determine fabric anisotropy is similar to studying a thin section of ice under a microscope. Here instead of a thin section of ice, we are studying a piece of ice with several hundred meters of thickness and instead of polarized light, we are using electromagnetic waves with much lower frequency compared to a microscope light.

- Figure A: ApRES radar and antenna (field setup)
- Figure B: ApRES antenna orientation
- Figure C: ApRES field data acquisition. To study fabric anisotropy all the antenna combinations in figure B must be obtained for several azimuthal angles between 0 and 180.
- Figure D(A): T is transmitter antenna. There are two coordinate systems. X'Y' is the measurement frame. The known coordinate system on the surface. XiceYice is the fabric principal axes. We named the angle between these two coordinate systems α. The angle between T and X'Y' is defined as θ' and θice is defined as the angle between T and XiYi.
- Figure D(B): T is rotated until it is aligned with one of the principal axis.









Method: Polarimetric radio wave propagation

References on this slide
Fujita 2006

A

$$\begin{bmatrix} s_{HH} & s_{VH} \\ s_{HV} & s_{VV} \end{bmatrix}_N = \left(\frac{exp(jk_0 z)}{4\pi z}\right)^2 * \left[\prod_{i=1}^{N}[RTR']_{N+1-i}\right] * [R\gamma R']_N * \left[\prod_{i=1}^{N}[RTR']_i\right]$$

$$\mathrm{T} = \begin{bmatrix} \mathrm{T}_x & 0 \\ 0 & \mathrm{T}_y \end{bmatrix} \quad \mathrm{R} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \quad \gamma = \begin{bmatrix} \gamma_x & 0 \\ 0 & \gamma_y \end{bmatrix} \quad r = \frac{\gamma_y}{\gamma_x}$$

B

Signal reconstruction for any azimuthal angle

$$s_{HH(\theta')} = s_{HH\,\theta_{in}} cos^2(\theta') + s_{VV\,\theta_{in}} sin^2(\theta') - 2 s_{HV\,\theta_{in}} sin(\theta')\, cos(\theta')$$

$$s_{VV(\theta')} = s_{VV\,\theta_{in}} cos^2(\theta') + s_{HH\,\theta_{in}} sin^2(\theta') + 2 s_{HV\,\theta_{in}} sin(\theta')\, cos(\theta')$$

$$s_{HV(\theta')} = s_{HV\,\theta_{in}} (cos^2(\theta') - sin^2(\theta')) + \left(s_{HH\,\theta_{in}} - s_{VV\,\theta_{in}}\right) sin(\theta')\, cos(\theta')$$

C

Power anomaly [dB]

$$\delta P_{(\theta',z)} = 20 \cdot log_{10}\left(\frac{\left|s_{(\theta',z)}\right|}{\frac{1}{n}\sum_{i=1}^{n}\left|s_{(\theta'_i,z)}\right|}\right)$$

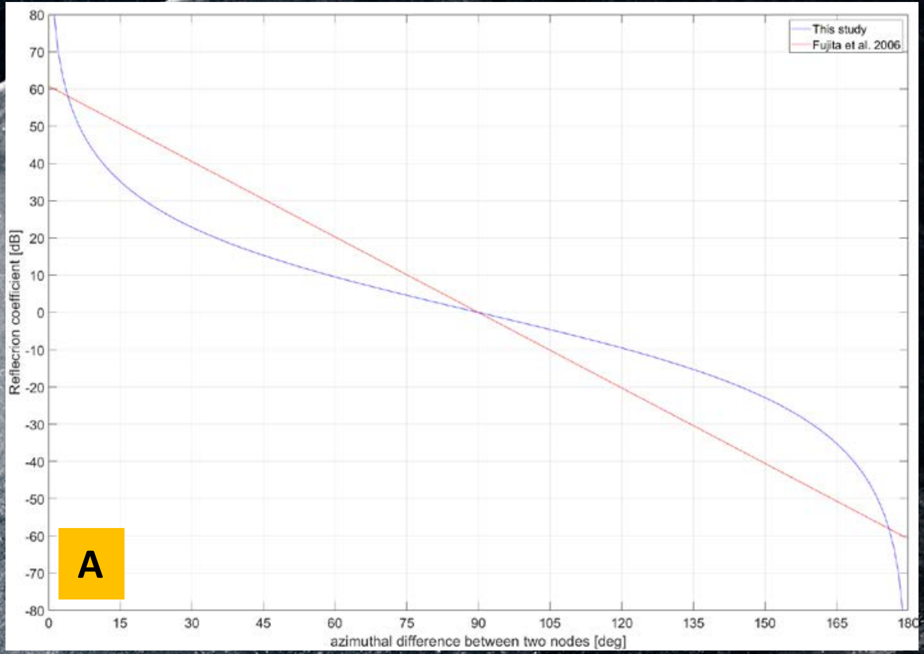
- Equation A: We use matrix based polarimetric radio wave propagation method to model the backscattered signal in all the antenna orientation at each depth and azimuth
- Equation B: Instead of measuring the signal at every azimuthal angle, we can measure HH, HV and VV at only one angle and reconstruct the rest of the azimuthal angles using the reconstruction equations. These equations are derived from eq. A.
- Equation C: Calculating power anomaly from the backscattered signal.

Method: The importance of HV & HH power anomaly

- There are specific depth and azimuth that the HH and HV power anomaly from the backscattered signal reach to their minimum values. These points are defined as nodes.

- HV power anomaly:
  - The minimum of HV power anomaly is happening at/around the orientation of E2
  - $\delta_{HV_{(\theta',z)}} = 20 * log_{10}\left(\frac{\sin(\theta'-\alpha)\cos(\theta'-\alpha)}{\frac{1}{n}\sum_{i=1}^{n}\sin(\theta'_i-\alpha)\cos(\theta'_i-\alpha)}\right)$

- HH Power anomaly:
  - HH power anomaly is sensitive to the scattering ratio
  - $[r] = \propto log_{10}\left(\frac{\gamma_y}{\gamma_x}\right) = \propto log_{10}\left(\frac{1}{tan^2\left(\frac{\Delta\vartheta}{2}\right)}\right)$



- Figure A: The nodes in HH power anomaly can be used to determine the reflection coefficients ratio. We have proven there is a correlation between the azimuthal difference of the two nodes and the scattering ratio. The red line is suggested by Fujita 2006. The blue line is from this study.

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Emmy Noether-Programm DFG

EGU European Geosciences Union

Method: The importance of polarimetric coherence phase shift

References on this slide
Fujita 2006
Dall 2010
Jordan 2019
Jordan 2020

Phase difference between waves along the two principal axes

$$\emptyset = \frac{4\pi f}{c} * \int_{z_N}^{0} \left(\sqrt{\varepsilon'_X} - \sqrt{\varepsilon'_Y}\right) * dz + (\Delta\emptyset_X + \Delta\emptyset_Y)$$

$$C_{HHVV}(\theta', z) = \frac{\sum_{j=1}^{M} s_{HH,j} \cdot s_{VV,j}}{\sqrt{\sum_{j=1}^{M} \left|s_{HH,j}\right|^2} \sqrt{\sum_{j=1}^{M} \left|s_{VV,j}\right|^2}}$$ *hhvv Polarimetric coherence*

$$\emptyset_{HHVV}(\theta', Z) = arg(C_{HHVV})$$ *Statistical estimation of the phase difference between hh and vv*

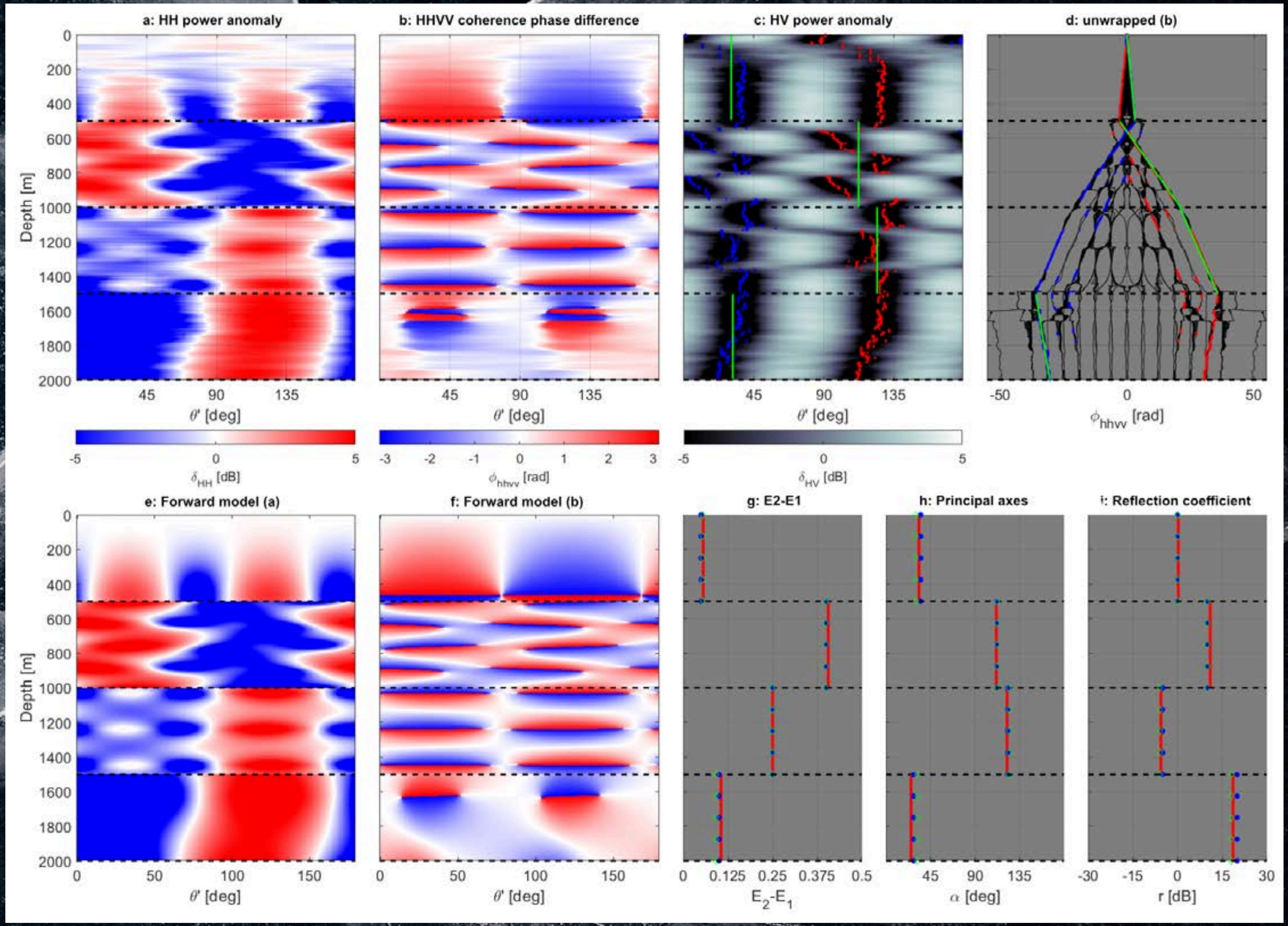
Horizontal asymmetry of the ice fabric

$$E2 - E1 = \pm \frac{2c}{4\pi f_c} \frac{\sqrt{\varepsilon'}}{\Delta\varepsilon'} \cdot \frac{d\emptyset_{HHVV(0°,90°)}}{dz}$$

- Polarimetric coherence (C_{HHVV}) measures the polarimetric phase shift and infer horizontal anisotropy in the ice fabric. It measures the relative phase shift between HH and VV as a function of depth.

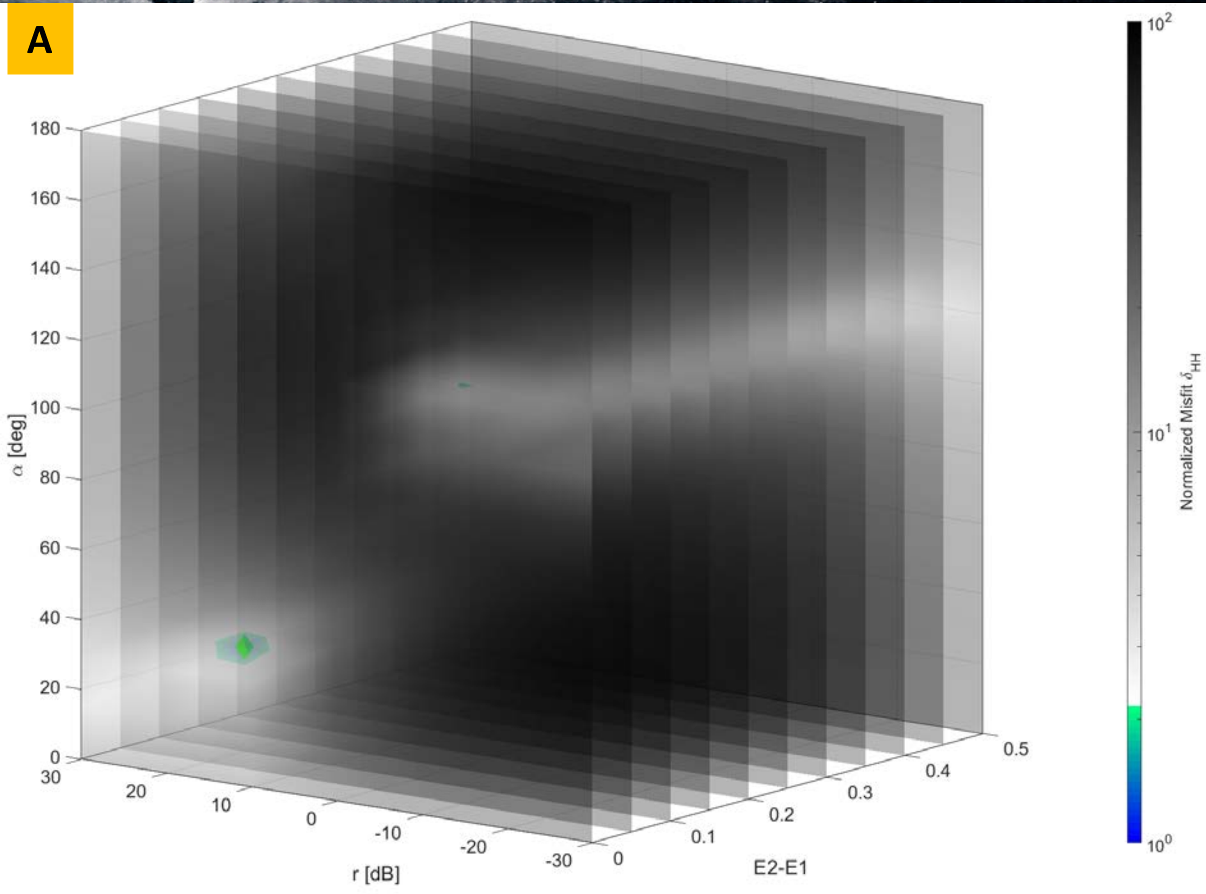
Results: Inferring fabric parameters (4 layers synthetic model)

- HV power anomaly
  - To infer fabric orientation
- Polarimetric coherence phase difference
  - To infer fabric anisotropy
- Picking horizontal layer boundaries
- HH power anomaly
  - Sensitive to scattering ratio
  - Optimization cost function
- Figure a-c: A 4 layers synthetic model made by radio wave propagation method. Black dashed lines are the horizontal layer boundaries.
- Figure c: Red and blue lines are the two possible fabric orientation (α and $\alpha + 90$). Green lines are the average of them in the direction of positive coherence phase difference. This direction is determined using the sign of the polarimetric phase difference gradient (fig. d). We use the green lines in figure c as the initial values for the optimization. They are the nearest guess to the true orientation of fabric principal axes for each layer.
- Figure d: The red and blue lines are the corresponding coherence phase difference at the selected azimuths in figure c. Green lines are the positive phase gradient which is corresponding to the green lines in figure c. They are used to calculate E2-E1 at each layer. We use these values as initial values of E2-E1 for the optimization.
- Figure e-f: Forward model made from the estimated values. The estimated values are calculated by a nonlinear optimization method with initial values explained above.
- Figure g-i: Blue dots are the true model parameters. Green diamonds are the initial values for the optimization and the red lines are the estimated values. The red lines are used in a forward model to generate figure e and f.



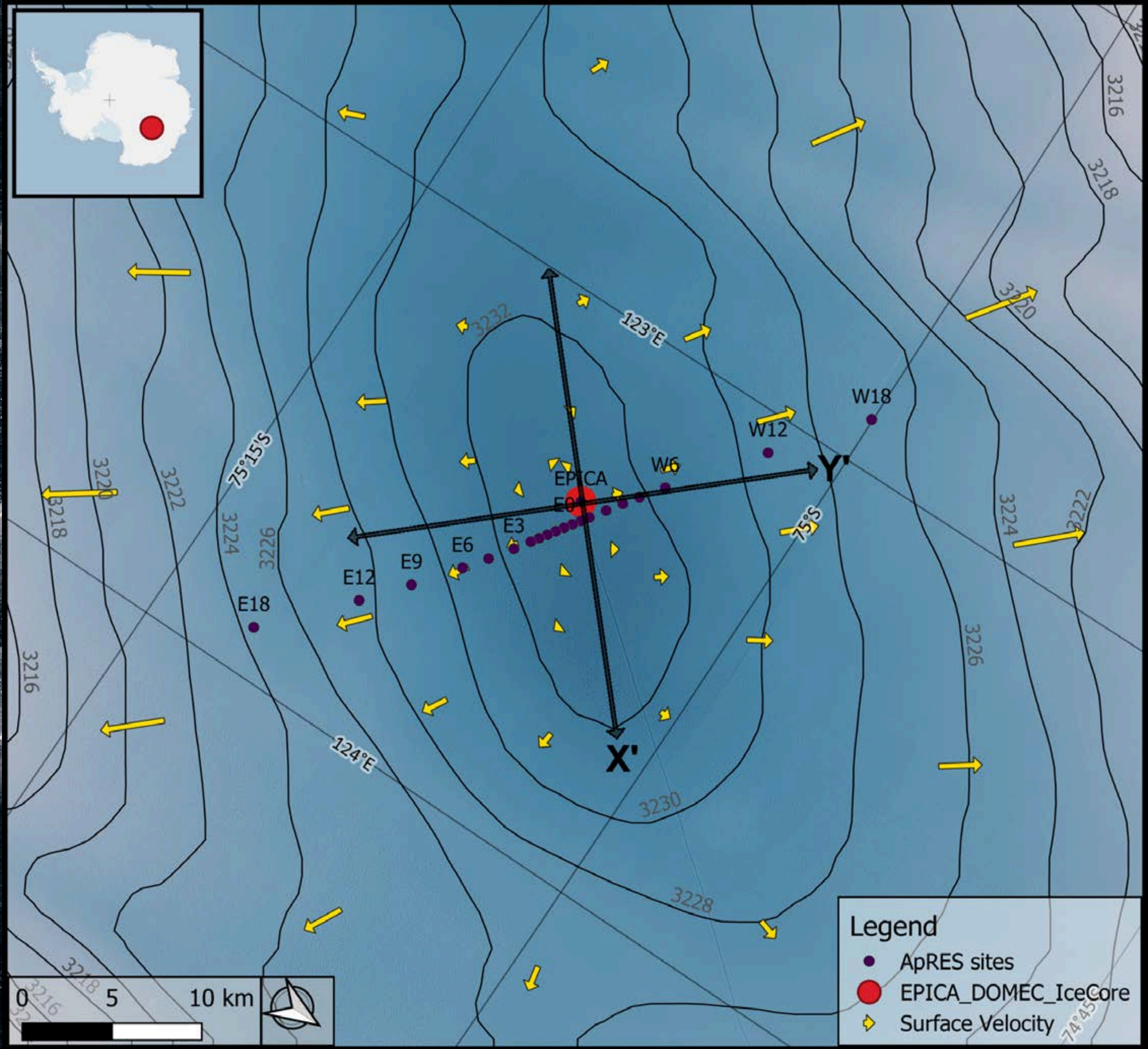
Results: Optimization misfit (Why HH power anomaly)

- We use HH power anomaly as the cost function of the optimization. At the beginning of the optimization we have a good initial guess for E2-E1 and the fabric orientation. The only unknown parameter is the reflection coefficient ratio (r). As we explained before HH power anomaly is sensitive to *r* and the other two parameters. This makes it the best choice to be used as the cost function.
- The initial values in this method are crucial. The optimization can stuck in a local minima if the initial values are not good enough.
- Figure A: This figure explains why the initial values are crucial in this method. We used the bottom layer of the previous slide to see how the misfit values are changing. The green diamond in the lower left of the cube is the correct answer which has a misfit around zero. Although there is only one area with low misfit values which is around the correct answer (blue area), there are areas which the misfit is not changing around them. This is the problematic part. For example, if the initial values are selected in the middle part of the right side of the cube (bright zone) the optimization will lead to a wrong answer. The misfit values in this area are not low and not high but it is more or less constant for any E2-E1 while the other areas around it have very high misfit values (dark zones). This is the reason that the optimization can not find a better solution and get stuck in this white area. Therefore the initial values in this method are very important.



Study site

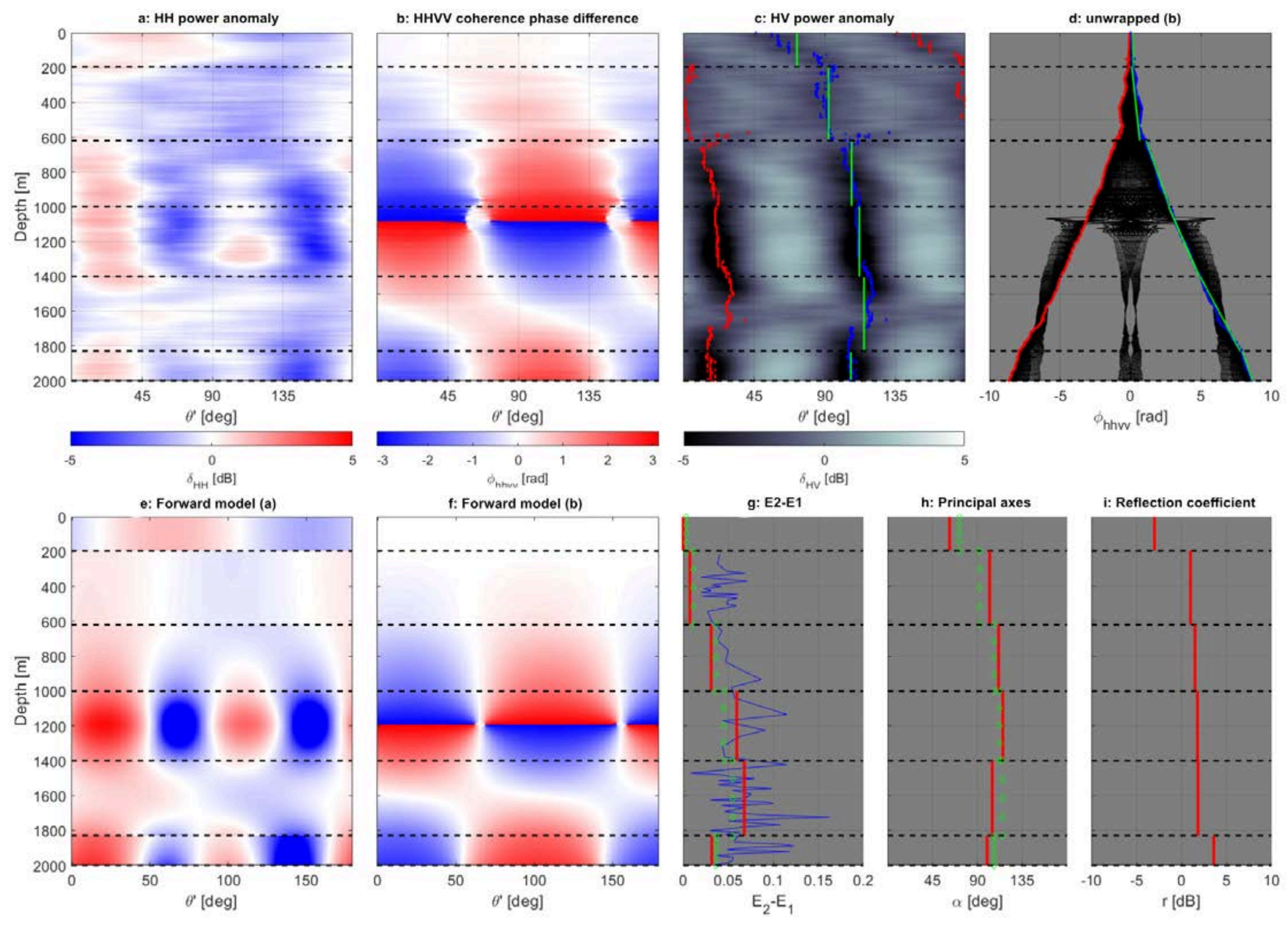
- Location: Concordia Dome C
- 21 ApRES stations
  - 1 station next to the ice core (EPICA)
  - 20 stations aligned in a profile
- X'Y' is the measurement system defined on the surface. HH, VV and HV is obtained along the X' and reconstructed counterclockwise for half a circle.
- We estimated the fabric principal axes orientation (α) relative to X' in the next slide.



References on this slide
Vittuari 2004
Quantarctica3
Norwegian Polar Institute in your work

Results: Inferring fabric parameters (EPICA station)

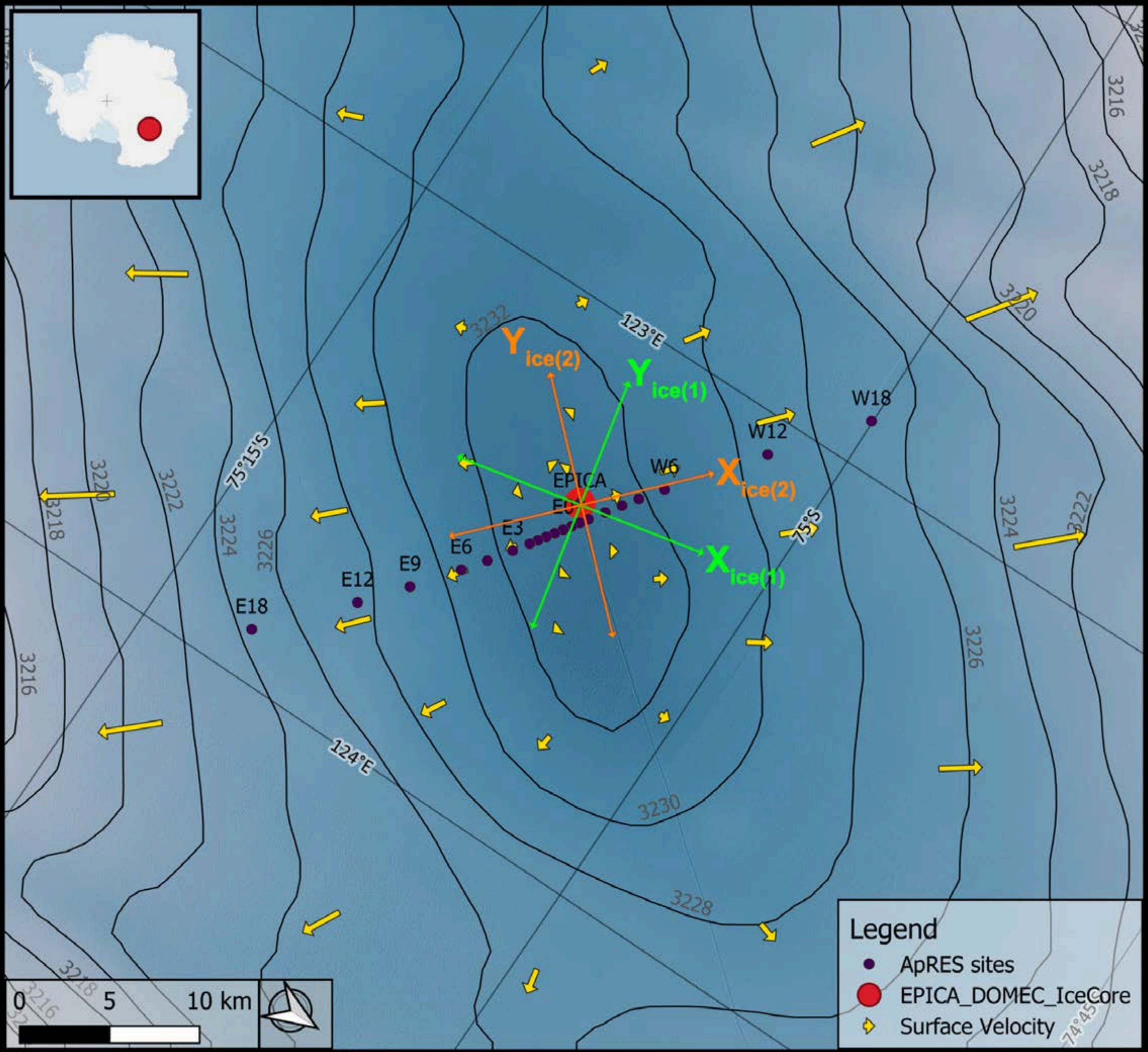
- Estimated E2-E1 is very close to the ice core observation (blue line) (fig. g)
- Rotation in fabric principal axes can be seen in figure. h
- Reflection coefficient ratio is more or less constant and isotropic with depth (fig. i)
- Forward model generated by the estimated parameters (fig. e,f) matches the ApRES data (fig a,b)
- The direction of fabric principal axes is shown in the previous slide (map-green arrows)



- Figure g: The blue line is the data obtained from the EPICA Dome C ice core. The green diamonds are the initial values and the red lines are the estimated values.

Results: Fabric orientation

- Average orientation of fabric principal axes:
  - Green: Depth 0-200 m
  - Orange: Depth 200-2000 m
- The fabric principal axes in below 200 m is rotated around 40° relative to the top 200 m.
- We believe the direction of the strongest eigen vector in a horizontal plane (E2) is same as the direction of Y in each fabric system.
- Here the direction of horizontal anisotropy is approximately parallel to the ice divide and perpendicular to the ice flow direction. (Yice2)



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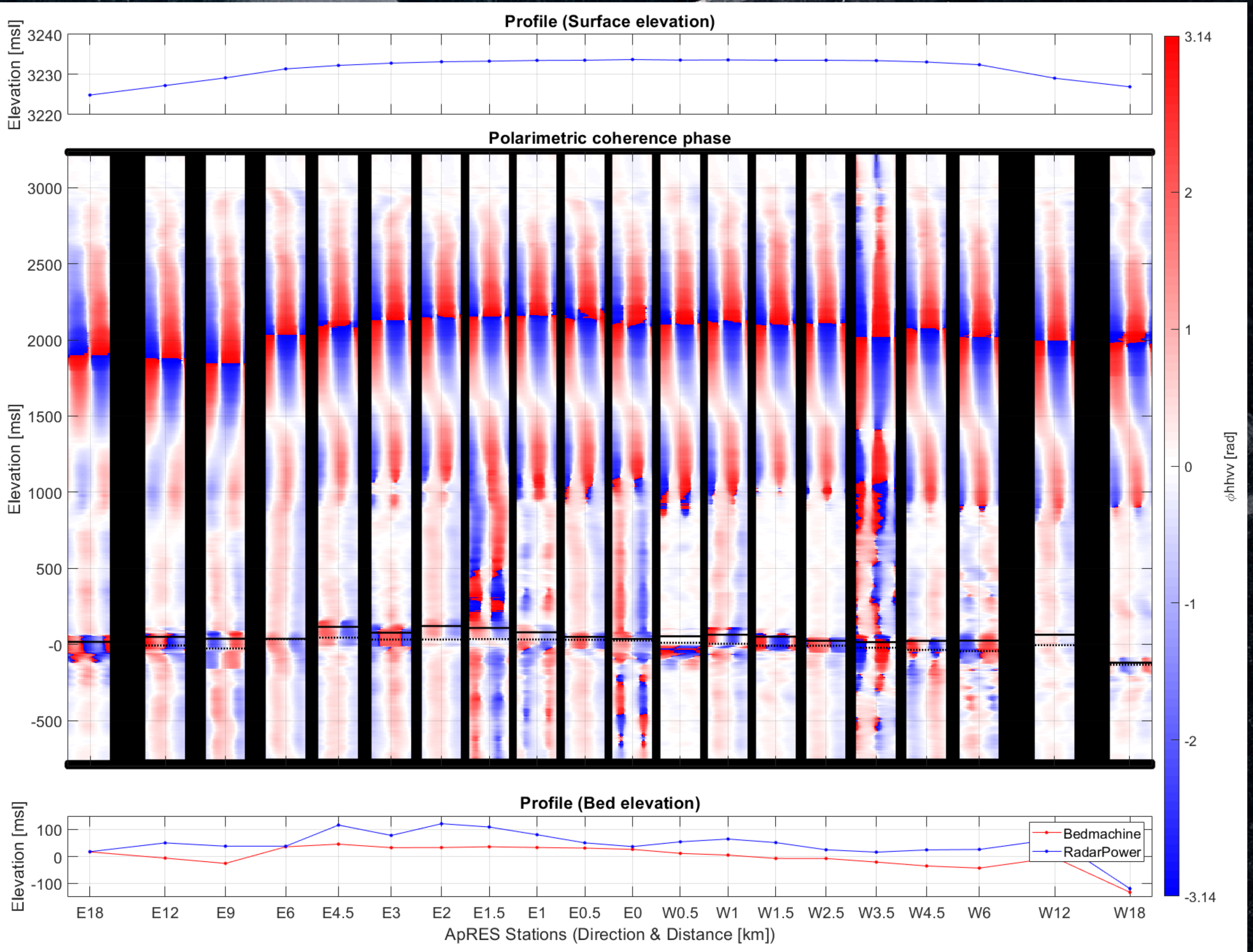
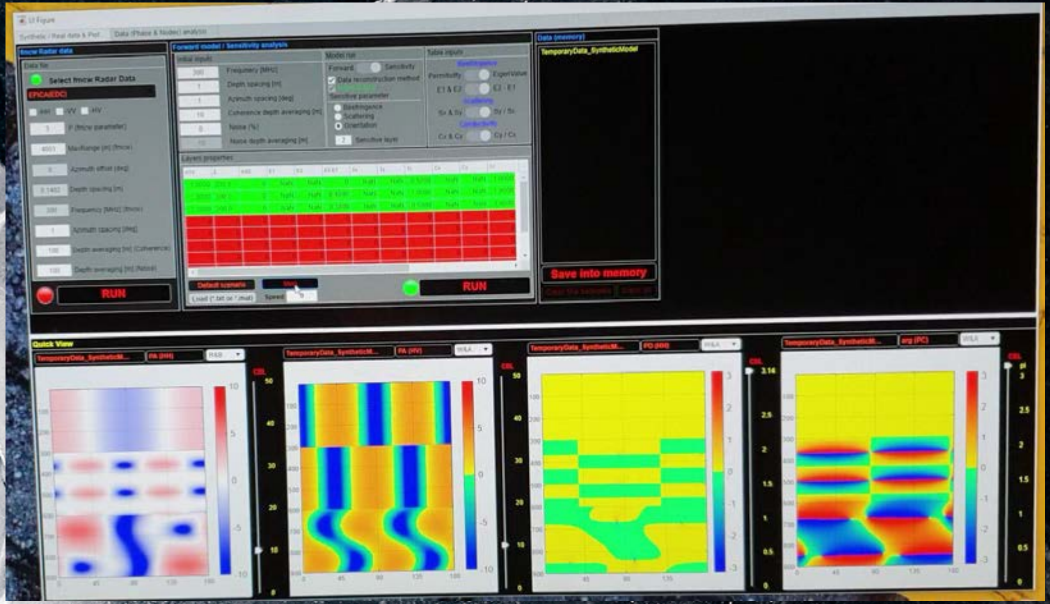
Results: Polarimetric coherence phase difference (the whole profile)

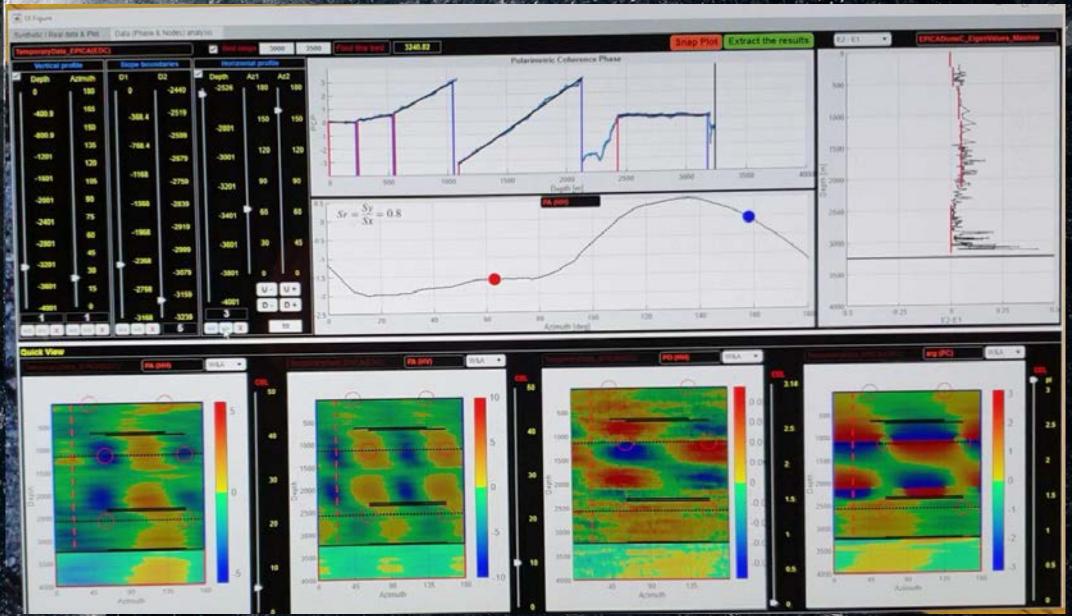
- It seems the horizontal fabric anisotropy in the whole area is very low
- The degree of horizontal anisotropy slightly decreases by getting away from the dome at both sides



Stand-alone application

- We designed a stand-alone application for this method.
- It is written in Matlab™ and can be installed on any computer even without having Matlab™ license.
- It generates any kind of forward model and sensitivity analysis for different parameters.
- The parameter estimation part is helping the user to estimate all the fabric parameters in a fast way.
- You can easily load any kind of ApRES data into the program by only copy them into the data folder.
- After adjusting the azimuths and selecting the horizontal boundaries, the program estimates all the parameters and displays real time forward model results for compression.
- All the data can be saved as image and text file
- We are looking forward to making this application available for public access.





Conclusion

- It is possible to continuously estimate fabric parameters using polarimetric radar:
  - In a slow flow velocity area like a Dome.
  - Under the assumption of the greatest eigen vector (E3) is vertical.
  - Using an optimization method where:
    - The initial values are very important.
    - The initial fabric orientation is obtained from the HV power anomaly.
    - The initial E2-E1 is obtained by the depth gradient of polarimetric coherence phase shift at the selected orientation (along the principal axes).
    - HH power anomaly is the optimization cost function.

- Analyzing EPICA ApRES data shows:
  - Estimated horizontal anisotropy (E2-E1) is very close to the ice core analysis
  - The area has a very low horizontal fabric anisotropy.
  - The principal axes of the fabric below 200 m is rotated around 40° counterclockwise relative to the top 200 m and it is parallel to the ice divide (perpendicular to the ice flow).
  - The reflection coefficient ratio is very small and constant with depth.

Outlook

- ❑ Estimating the fabric parameters for more sites
- ❑ Interpreting the internal ice deformation using the estimated parameters
- ❑ Testing the validity of this method on a fast flow velocity area
- ❑ Debugging the optimization application and make it available in GitHub for public access