Spatio-temporal decomposition of geophysical signals in North America

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Glossary of terms

- **BHM:** Bayesian hierarchical model.
- **GIA:** Glacial isostatic adjustment, denoted I.
- **GPS:** Global positioning system data, denoted G.
- **GRACE:** Gravity Recovery and Climate Experiment, denoted R.
- **INLA:** Integrated nested Laplace approximation.

Section 1

Context & modelling framework

GlobalMass

- Combine satellite and in-situ data related to different aspects of the sea level budget,
- Attribute global sea level rise to its component parts.

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Global sea level rise re-evaluation

The sea level budget *enigma*

$$\Delta\text{sea level}(t) = \Delta\text{barystatic}(t) + \Delta\text{steric}(t) + \text{GIA}$$

mass — density — ocean basins

- GIA: glacial isostatic adjustment
- inconsistencies between the discipline-specific estimates

GlobalMass Aims

- Simultaneous global estimates of all the components
- Close the sea level budget

Modelling framework

- Utilise **Bayesian hierarchical models** (BHM) as a flexible framework for statistical modelling of sea-level rise.
- Allows modelling of underlying latent processes and separation of sources.
- Can specify such models as

$$\begin{aligned} \textbf{Parameter:} &\quad \theta \sim p(\theta) \\ \textbf{Latent process:} &\quad x|\theta \sim p(x|\theta), \text{ where } x = \{x(\boldsymbol{u}), \boldsymbol{u} \in \Omega\} \\ \textbf{Observation:} &\quad y|x, \theta \sim p(y|x, \theta) \end{aligned}$$

for **observations** y, regions Ω, where the **underlying process** x is modelled using a zero-mean Gaussian with variance $Q(\theta)$, where θ is a vector of **hyperparameters**.

BHM for sea level rise

Observation layer (direct observations)

Observations, for example:

- Argo buoys
- Radar altimetry
- Tide gauges
- GRACE
- GPS
- ICESat/CryoSat
- River discharges
- in-situ data

Process layer (latent geophysical processes)

CHANGE IN SEA LEVEL = Change in water temperature + Change in water salinity + Change in land ice mass + Change in freshwater hydrology + Change in ocean floor

OCEANS (water temperature, water salinity) — LAND ICE (land ice mass) — HYDROLOGY (freshwater hydrology) — SOLID-EARTH (ocean floor)

Parameter layer (prior information)

Prior information about geophysical processes, for example:

- Outputs from models:
  - GIA forward models
  - Ocean/atmospheric general circulation models (GCMs)
  - Glacier mass balance models
  - Hydrology models
- Fundamental physical principles such as conservation of mass
- Known natural fluctuations such as orbital changes and polar wander

Section 2

Source separation of geophysical signals over North America

Geophysical signals over North America

Aim

To separate observations (GPS and GRACE data) over North America into the contributions provided by GIA and hydrology.

- **Observation layer:** GPS and GRACE data.
- **Latent process:** GIA and hydrology.
- **Parameter:** Prior information for GIA (forward models such as ICE-6G) and hydrology (basin information or forward models).

Process model

Propose that observations (GPS and GRACE) can be decomposed as

- Time-invariant process (GIA), and
- Time-evolving process (hydrology) which behaves like an AR(1).

Then we have

$$Y_t = A_t X_t + B_t Z + \omega_t, \;\; w_t \sim \mathcal{N}(0,\, v_t) \;\&\; B = [B_1, ..., B_T]'$$
$$X_t = \rho X_{t-1} + \varepsilon_t, \;\; \varepsilon_t \sim \mathcal{N}(0,\, Q^{-1})$$

for Y_t observations at time t, A_t an incidence matrix, X_t the hydrology process, ρ the AR(1) smoothing parameter, Z_t the GIA process with incidence matrix B_t and ω_t the error.

Modelling set-up

Approach

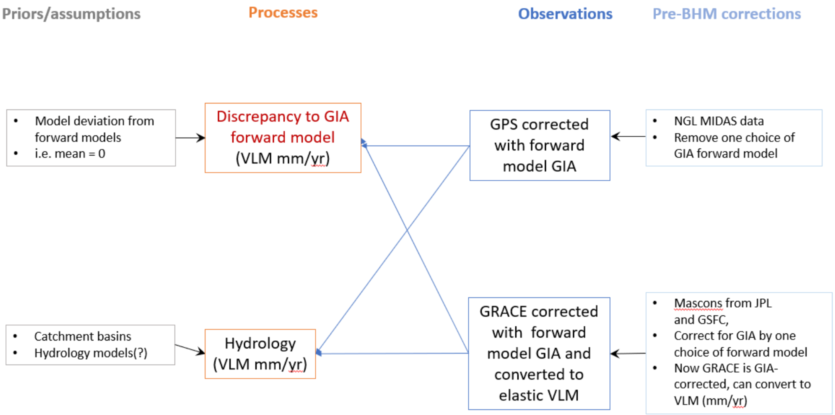
To separate observations (GPS and GRACE data) over North America into the contributions provided by GIA and hydrology.

1. Convert data into appropriate units (mm of water equivalent).
2. Model discrepancy (mean-zero) between
   - the simulation m (for example, ICE-6G model)
   - and the true process X (see Sha et al. (2019) for justification).

Modelling set-up

3. Calculate annual averages of adjusted GPS and GRACE data and yearly differences (subtract values for year $t-1$ from year t).
4. Set up observation equations relating data to processes.
5. Fit spatio-temporal model with annual time-step using R-INLA (see implementation slides), including appropriate priors.
6. Updated discrepancy field is then mapped back to simulation (North America) grid & GIA reconstructed by adding back m.
7. Hydrology field reconstructed by mapping to North America grid.

Modelling set-up chart



Observation equations

Let $Y_t := (G^t, R^t)$, we may write the ICE-6G observations $\tilde{Y}_t$ as

$$\tilde{Y}_t := Y_t - \bar{y}_t = \begin{bmatrix} Y_t^G - \bar{y}_t \\ Y_t^R - \bar{y}_t \end{bmatrix} = \begin{bmatrix} A^t & B^t \\ C^t & D^t \end{bmatrix} \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix},$$

Y_t are the **observations**, α and β the **latent processes** for GIA and hydrology, $\tilde{Y}_t$ is the ICE-6G adjusted observation at time t, $\bar{y}_t$ is this adjustment of observations Y_t and A_t, B_t, C_t, D_t the **incidence matrices**.

Implementation: R-INLA

- Bayesian inference based upon making a series of Laplace approximations and numerical integrations.
- Advantageous over MCMC when it comes to large-scale (spatial) data.
- **Lindgren et al (2011):** Gaussian fields can be expressed as solution of an SPDE, which may be approximated using finite elements whose elements are triangles over field's domain.
- Map observations to points on this finite element mesh using incidence matrix.
- **Benefit:** The precision matrix for the field has sparse approximation $\boldsymbol{Q}$, with $\boldsymbol{Q}^{-1}$ close to Σ. Q is quick to compute using this approach ($\mathcal{O}(n^{3/2})$ vs. $\mathcal{O}(n^3)$ for the corresponding dense Σ).

Key challenges

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1. **Integration of area-level data within the INLA framework:**
   - The matrix A specified in the SPDE approach is designed to deal with point-referenced data.
   - When Y_i a point observation at location s_i, then

     $$Y_i = \sum_j \alpha_j \phi_j(s_i) = \phi(s_i)\alpha,$$

     hence $A_{ij} = \phi_j(s_i)$ for point-level observations such as the GPS data presented.
   - Not the case with area-level data such as GRACE.
   - **Idea:** Modify the incidence matrix A to allow for areal data using integral approximations.

Key challenges

2. Challenges in capturing time-varying hydrology signal appropriately:

- Initial approach: AR(1) parameter tends towards 1.
- Investigation into AR(1) points to bi-modal distribution (near 1 and away from 1).
- This implies a considerable time-invariant signal present in hydrology as well.
- This signal is absorbed into time-invariant GIA process!
- **Idea:** Use idea of partition models (see Sha et al. (2019) to allow signal to vary from one catchment to the next.

Key challenges

3.Loss of sparsity which arises when combining time-invariant and time-evolving processes:

- Addition of the time-invariant field to time-varying process may erase sparsity in the all-at-once calculation of the likelihood.
- May be sufficient sparsity in the two-process problem over North America, but may not hold for more processes on a global scale.
- **Alternative approach:** Model through a Kalman filter (sequential likelihood).

Section 3

Further work

Further work

1. Adapt model to account for these challenges.
2. Implementing BHM for synthetic data to check how well model captures the signal.
3. Extend approach used here to global sea level rise (slide 7). This involves:
   - Inclusion of further processes and datasets.
   - Handling of large-scale global datasets.
   - Maintaining sparsity where possible to ensure computational complexity is reasonable.
   - Implementing alternative approaches where this becomes infeasible.

Contact me

Results not included as currently in progress, but for more information/further discussion:

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References

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