Drought Assessment in a Changing Climate with the Joint Deficit Index

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INTRODUCTION

- Drought is a major natural hazard with a serious impact on human societies and ecosystems. At least 11% of the European population and 17% of its territory have been affected by water shortage, and the total cost of droughts over the past thirty years is estimated at EUR 100 billion.
- The characterization of droughts is very dependent on the time scale that is involved. To obtain an overall drought assessment, the cumulative effects of water deficits over different times need to be examined together.
- We extend the empirical copula-based joint deficit index (JDI) of Kao & Govindaraju [KG10] to the Gaussian copula model.

DROUGHT DEFINITION

Standardised Precipitation Index

- Let D_i be the total precipitation of a certain month i.
- Let $x_w^{(m)}$ be the w-monthly precipitation with respect to month m:

$$x_w^{(m)} = \sum_{i=m-w+1}^{m} D_i \tag{1}$$

- Let F_{X_w} be the CDF:

$$F_{X_w}(x_w^{(m)}) = \Pr\{X_w \le x_w^{(m)}\} \tag{2}$$

- Given the input variable $x_w^{(m)}$, the **Standardised Precipitation Index (SPI)** at time scale w is [MDK93]:

$$\mathrm{SPI}_w = \phi^{-1}(u_w), \quad \text{with} \quad u_w = F_{X_w}(x_w^{(m)})\text{: uniformly distributed,} \tag{3}$$

and ϕ is the standard normal CDF. We get $\mathrm{SPI}_w \sim \mathcal{N}(0,1)$

Joint Deficit Index

- Examine various temporal scales (1-,...,12 months) together by means of the multivariate probability

$$\Pr\{X_1 \le x_1^{(m)}, \ldots, X_{12} \le x_{12}^{(m)}\} = \Pr\{U_1 \le u_1^{(m)}, \ldots, U_{12} \le u_{12}^{(m)}\}. \tag{4}$$

- Model multivariate probability with Copula function C:

$$\Pr\{U_1 \le u_1^{(m)}, \ldots, U_{12} \le u_{12}^{(m)}\} = C(u_1^{(m)}, \ldots, u_{12}^{(m)}) \tag{5}$$

- The Kendall distribution function

$$K_C(q) = \Pr\{C(U_1, \ldots, U_{12}) \le q\} \tag{6}$$

- Given the accumulations $(x_1^{(m)}, \ldots, x_{12}^{(m)})$ with respect to month m, the **Joint Deficit Index (JDI)** is defined as [KG10]:

$$\mathrm{JDI} = \phi^{-1}(K_C(q)) \sim \mathcal{N}(0,1), \quad \text{with} \quad q = C(u_1^{(m)}, \ldots, u_{12}^{(m)}). \tag{7}$$

- $\mathrm{JDI} > 0$, $\mathrm{JDI} < 0$, and $\mathrm{JDI} = 0 \Rightarrow$ wet, dry and normal conditions.

GAUSSIAN COPULA

The Gaussian copula C_G can be expressed as [dVdB18]:

$$C_G(u_1, \ldots, u_{12} \,|\, \Sigma) = \Phi_\Sigma\left(\phi^{-1}(u_1), \ldots, \phi^{-1}(u_{12})\right), \tag{8}$$

with Φ_Σ, the multivariate Gaussian CDF with:

- zero mean, and
- $\Sigma = \left(\mathrm{Cov}[\phi^{-1}(u_w), \phi^{-1}(u_{w'})]\right)$, a positive definite covariance matrix.

COVARIANCE MODELS

- **Main idea.** Make connection with spatial statistics: view $Y_w = \phi^{-1}(u_w)$ as a Gaussian random process at "location" w.
- Logarithmic distance between w and w': $h = |\log(w) - \log(w')|$,
- Covariance function:

$$\rho(h) = \mathrm{Cov}[Y_w, Y_{w'}]. \tag{9}$$

- Candidate models for $\rho(h)$: Matérn family, (powered) exponential family.

VARIOGRAM-BASED ESTIMATION

- The **variogram** of the stochastic process Y_w is

$$\gamma(w, w') = \frac{1}{2}\mathrm{Var}[Y_w - Y_{w'}]. \tag{10}$$

- Given n transformed data points $(y_{1,i}, \ldots, y_{12,i})$, $i = 1, \ldots, n$, the empirical variogram is given by

$$\hat{\gamma}(w, w') := \frac{1}{2n}\sum_{i=1}^{n}(y_{w,i} - y_{w',i})^2, \tag{11}$$

- Binned variogram:

$$\hat{\gamma}(h) = \frac{1}{|N_h|}\sum_{(w,w')\in N_h}\hat{\gamma}(w, w'), \tag{12}$$

where N_h denotes the set of pairs (w, w') such that the distance equals h, and $|N_h|$ is the number of pairs in the set N_h.

- Estimate the set of parameters θ by minimizing the objective function

$$S(\theta) = \sum_{k=1}^{\tilde{n}}\left(\hat{\gamma}(h_k) - \gamma(h_k; \theta)\right)^2, \tag{13}$$

where $\gamma(h; \theta) = 1 - \rho(h; \theta)$ is the theoretical variogram, and $\tilde{n}$ the number of different h-values.

- Model selection with the Akaike Information Criterion (AIC), defined as:

$$\mathrm{AIC} = 2\,n_p + \tilde{n}\,\ln S(\hat{\theta}), \tag{14}$$

where n_p is the number of model parameters.

DATA

Place	Years	Provided by
Uccle	1898 – 2015	RMI
Marseille	1881 – 2004	ECA&D
Milan	1858 – 2008	ECA&D
St. Petersburg	1881 – 2013	ECA&D

Precipitation stations.

- **RMI:** Royal Meteorological Institute of Belgium.
- **ECA&D:** European Climate Assessment & Dataset [ea02a].

ESTIMATION RESULTS

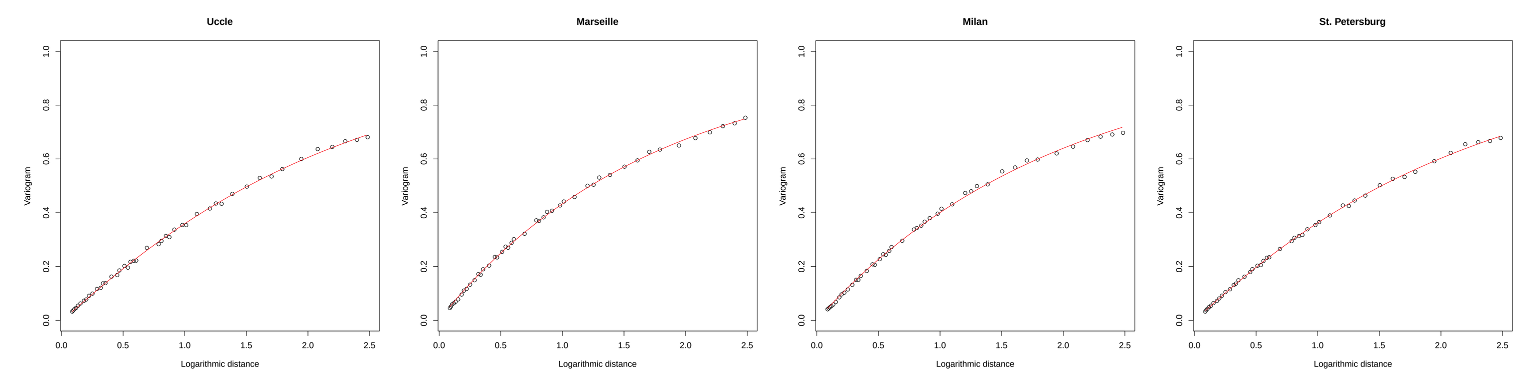

Figure: Variogram with standard normal marginals. *Dots:* empirical variogram. *Solid line:* theoretical variogram. (Powered exponential correlation function).

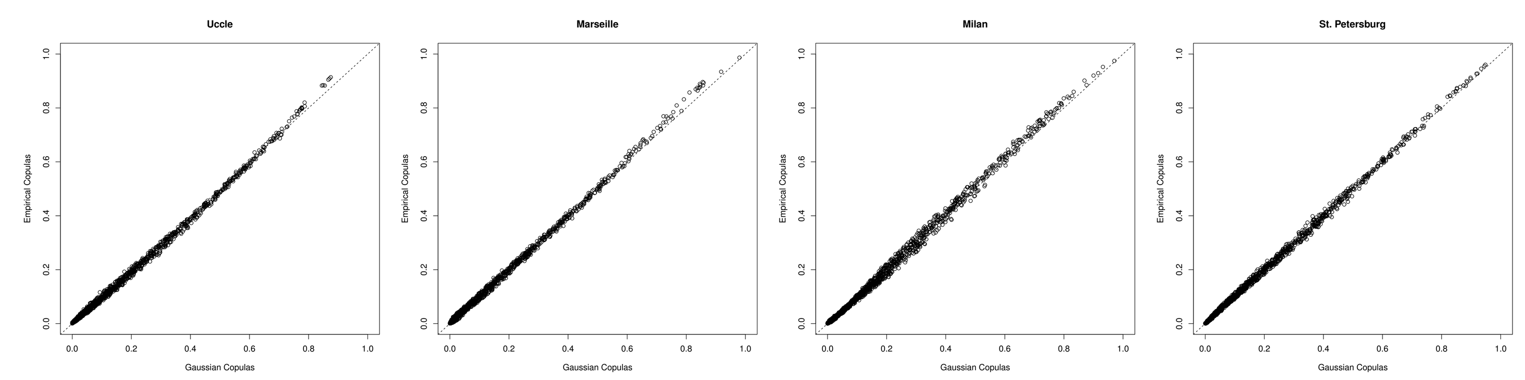

Figure: Goodness-of-fit plots. Empirical copulas versus Gaussian copulas. *Dash line:* leading diagonal.

Conclusions

- AIC-based model selection shows that the powered exponential family is the most suited covariance function.
- Excellent fit of the Gaussian copula.

DROUGHT MONITORING

- Drought category estimation according to the probability of occurrence of JDI.

Table: Drought monitor classification of Svoboda *et al.* [ea02b].

Category	Drought condition	Probability of occurrence (%)	Normal quantiles
D0	Abnormally dry	20 – 30	−0.84 to −0.52
D1	Moderate drought	10 – 20	−1.28 to −0.84
D2	Severe drought	5 – 10	−1.64 to −1.28
D3	Extreme drought	2 – 5	−2.05 to −1.64
D4	Exceptional drought	2	−2.05

CLIMATE SIMULATIONS

- **EURO-CORDEX Data:** we consider a multi-model ensemble of 15 RCP's.
- Spatial resolution: EUR11.
- Select gridpoint closest to Uccle (Belgium).
- Historical runs (1950–2005).
- Future runs. Two emission scenario's: RCP4.5 & RCP8.5 (2006-2100).

CLIMATE CHANGE

- Fit the Gaussian copula to the historical runs.
- Compute future JDI-values by applying the Gaussian copula to the future runs.

Table: Change of the occurrence frequency of future JDI-values per drought category (%).

Condition	RCP4.5	RCP8.5
Abnormally dry	60	30
Moderate drought	70	50
Severe drought	70	40
Extreme drought	130	190
Exceptional drought	390	420

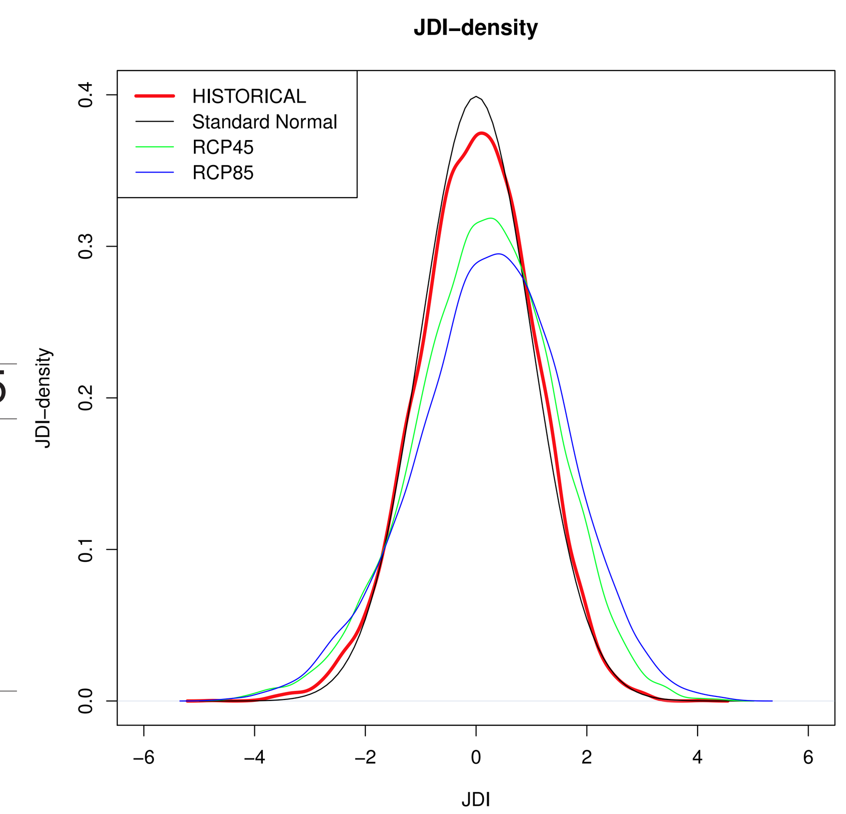

Figure: Probability density of JDI (History vs. Future)

Conclusions

- Our work confirms the "Dry gets drier, wet gets wetter" paradigm.
- There is a pronounced increase of future extreme and exceptional droughts.
- The drought scenarios based on RCP4.5 & RCP8.5 are not very different.

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