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# Resolution and significant contributions of tidal forcing in flexible harmonic grouping computed using Singular Value Decomposition

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# Introduction

# Tidal Analysis

## Interest

- Our main interest of tidal analysis is the accurate and precise determination of tidal parameters, the quantities describing the Earth response to the tidal forcing.
- Widely used software, like ETERNA or Baytap-G, use an a-priori grouping of harmonics present in a tidal catalogues (e.g. Venedikov 1961, Hartmann & Wenzel 1995, Tamura 1987).
- Wave grouping of a harmonic development is a model parameterisation used to make the analysis problem overdetermined.

# Motivation

## Analysis defficiency

The common analysis methods minimize the data misfit only (e.g. `Eterna 3.4`).

- If model assumptions (e.g. credo of smoothness, known free-core resonance parameters, known ratio between response to degree 2 and degree 3 forcing) are incorrect, the analysis can lead to unnoticed artefacts
- Moving window tidal analyses of gravity recordings show temporal variations of tidal parameters for different stations (Meuers, 2004; Meuers et al. 2016; Schroth, 2018).
- We search for the causes of this phenomenon.

We abandoned the a-priori wave grouping and investigated the intrinsic nature of tidal harmonics using Singular Value Decomposition.

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SVD is a factorisation of a linear regression matrix. The regression matrix consists of tidal harmonics in-phase and quadrature signal for rigid Earth (tidal forcing to Earth surface, also described as "equilibrium tide").
That concept from signal processing allows us to linearize the problem such that gravity signal is linearly scaled to the forcing signal

$$A\cos(\omega t + \phi) = A \cdot \cos(\omega t) \cdot \cos(\phi) - A \cdot \sin(\omega t) \cdot \sin(\phi).$$

$$A\cos(\omega t + \phi) = A_C(t) \cdot \cos(\phi) + A_S(t) \cdot \sin(\phi).$$

And further to determine gravimetric parameters.
SVD allows us to study the significance of tidal harmonics, "natural" wave grouping, possible cross-talk between harmonics or groups and matrix null space.

# Tidal analysis

## Singular Value Decomposition

The tidal prediction problem:

$$\vec{s} = G\vec{m}$$

$\vec{s}$: synthetic gravity signal vector
$G$: rigid earth (equilibrium) tide signals matrix (forcing)[1]
$\vec{m}$: gravimetric factors (tidal parameters) vector (response)

SVD factorization:

$$G = U\Lambda V^T$$

**Diagonal $\Lambda$**: **singular values (SV)**. Large SV indicate essential components of $\vec{s}$.
**Orthonormal $V$**: a kind of **natural wave grouping**.
**Orthonormal $U$**: the corresponding **gravity variation**.

[1] Forward **operator G** may also include air pressure as a part of the forcing model.

# Singular Value Decomposition

## Matrix V

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It turns out in the investigation, that **Matrices V** indicate cross-talk.
On all the diagrams:

- Harmonic vectors are sorted with decreasing singular value.
- Absolute values of coefficients are plotted.
- Black points on displayed matrix V mean that absolute values of coefficients exceed $1/\sqrt{2}$.

# Singular Value Decomposition

## The cut-off

In the analysis, we would like to specify the level which is a limit of the significance of the model parameters.

$$k = \log_{10}\left(\frac{\lambda_1}{\lambda_p}\right)$$

Singular values $\lambda$ are sorted, so $\lambda_1$ is the largest. The ratio corresponds to the signal/noise range. We distinguish two different levels of cut-off $k$:

- $k = 3$, that corresponds to the usual range in superconducting gravimeter tidal observations
- $k = 7$, the level above the range present in tidal catalogues

Often we refer to them as the "Singular value threshold level". In most diagrams presented, we apply the cut-off level $k = 3$.

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$$R_m = V_p V_p^T$$

Model **resolution matrix** properties:

- Describes how the (generalised) inverse solution "smears" the original model into a recovered model.
- The true model "leaks" into adjacent model parameters and is reduced in its maximum amplitude in the recovered model.
- If we consider a model parameter for a harmonic, it may be partially parameter obtained for another harmonic.

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# Methods - RATA software

## Robust Approach to Tidal Analysis

The tidal forward operator **G** is a set of tidal harmonics that would be recorded on rigid Earth at a fixed place.

# Methods - RATA software

## Alternative tidal analysis with SVD

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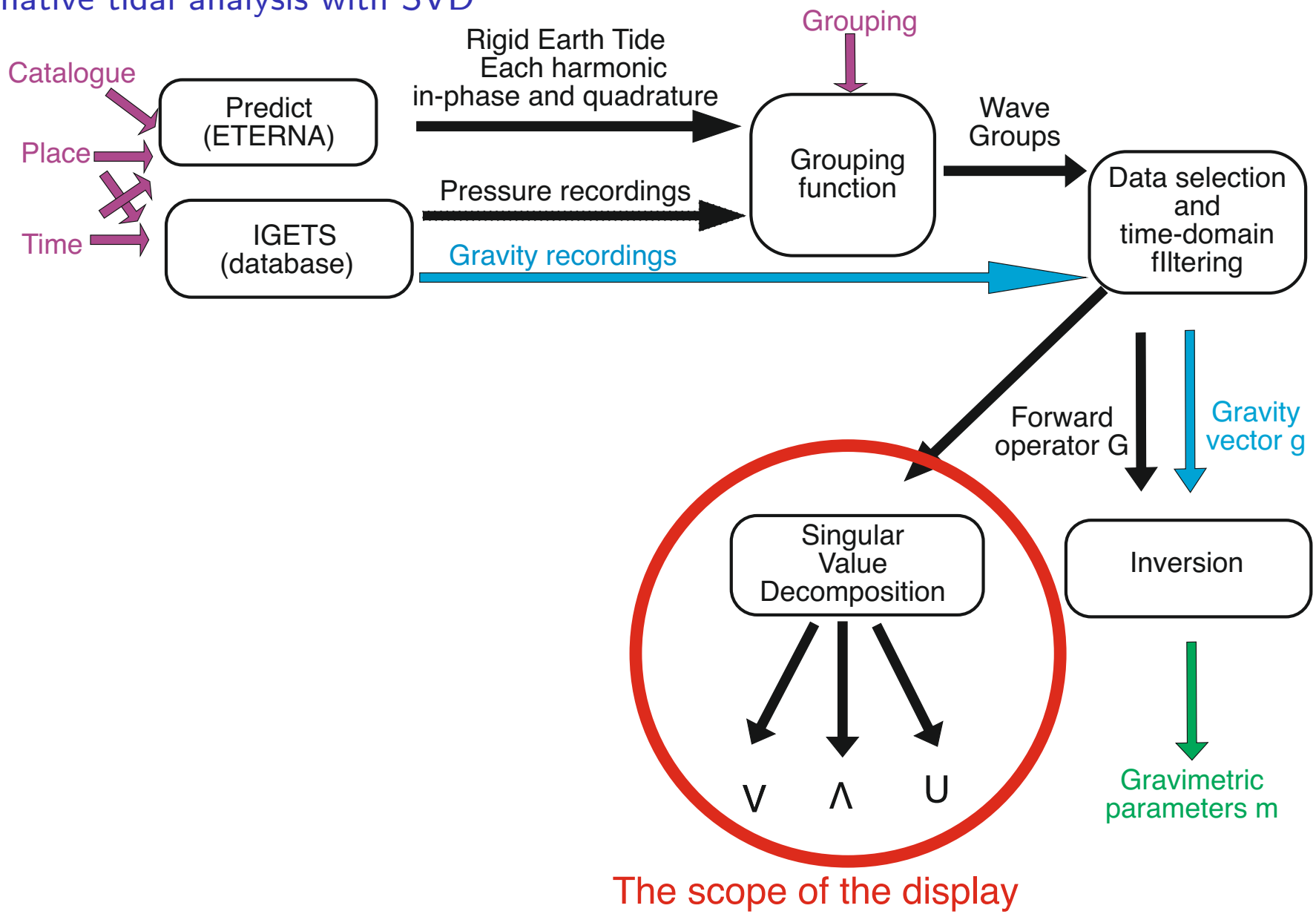
We compute time series for each harmonic present in the Tamura tidal catalogue by using a modified version of "Predict" (ETERNA package).

- Resulting **values can be, but do not need to be, grouped** prior to SVD analysis.
- Other than with conventional programs, wave groups can be **defined not only as frequency intervals**.
- One possibility is separation harmonics of degree 2 and 3
- We may investigate which singular vectors **do not significantly contribute** to the predicted tidal data or are **noise-sensitive**
- With proper time-domain signal filters, we may investigate the cross-talk between harmonics and **air pressure variations**.

# Methods - RATA software

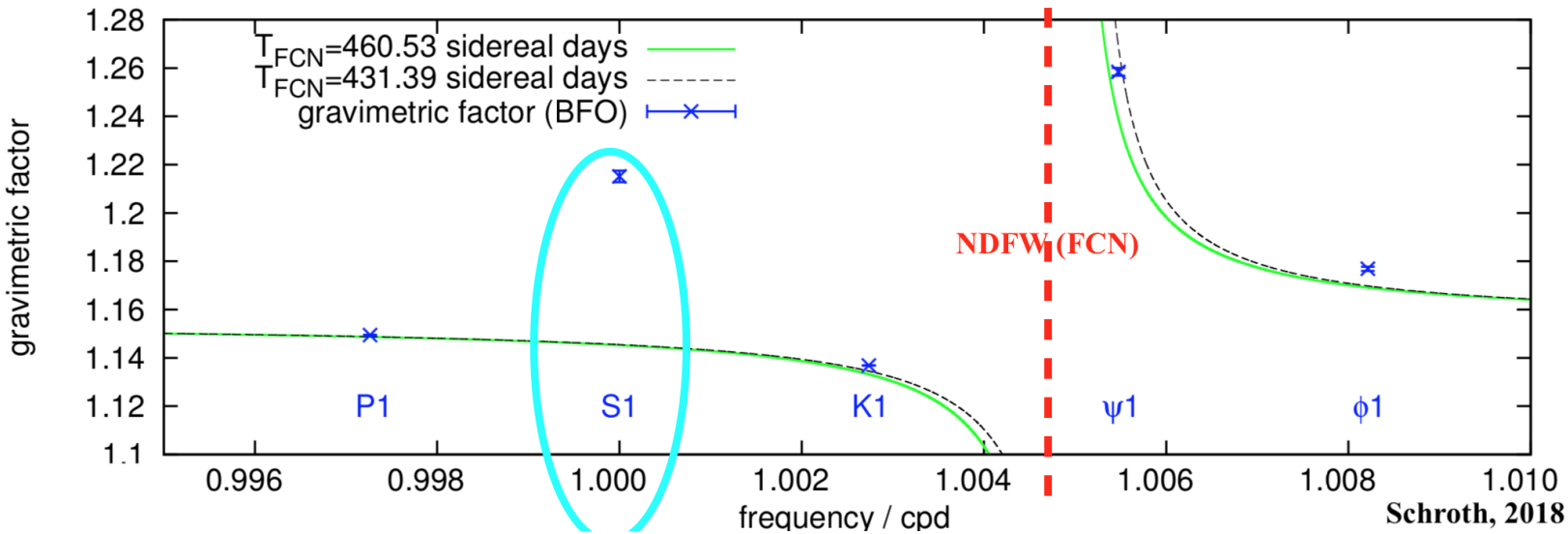
## Alternative tidal analysis with SVD



# 1. Atmospheric cross-talk

## Motivation

Frequency of S1 is exactly 1 CpD. Earth response is also affected by "radiation tides" which are induced by air pressure diurnal cycle.




That variation is included in the forcing model together with wave groups. We study the properties of S1 group and air pressure gravity signal.

# 1. Atmospheric cross-talk

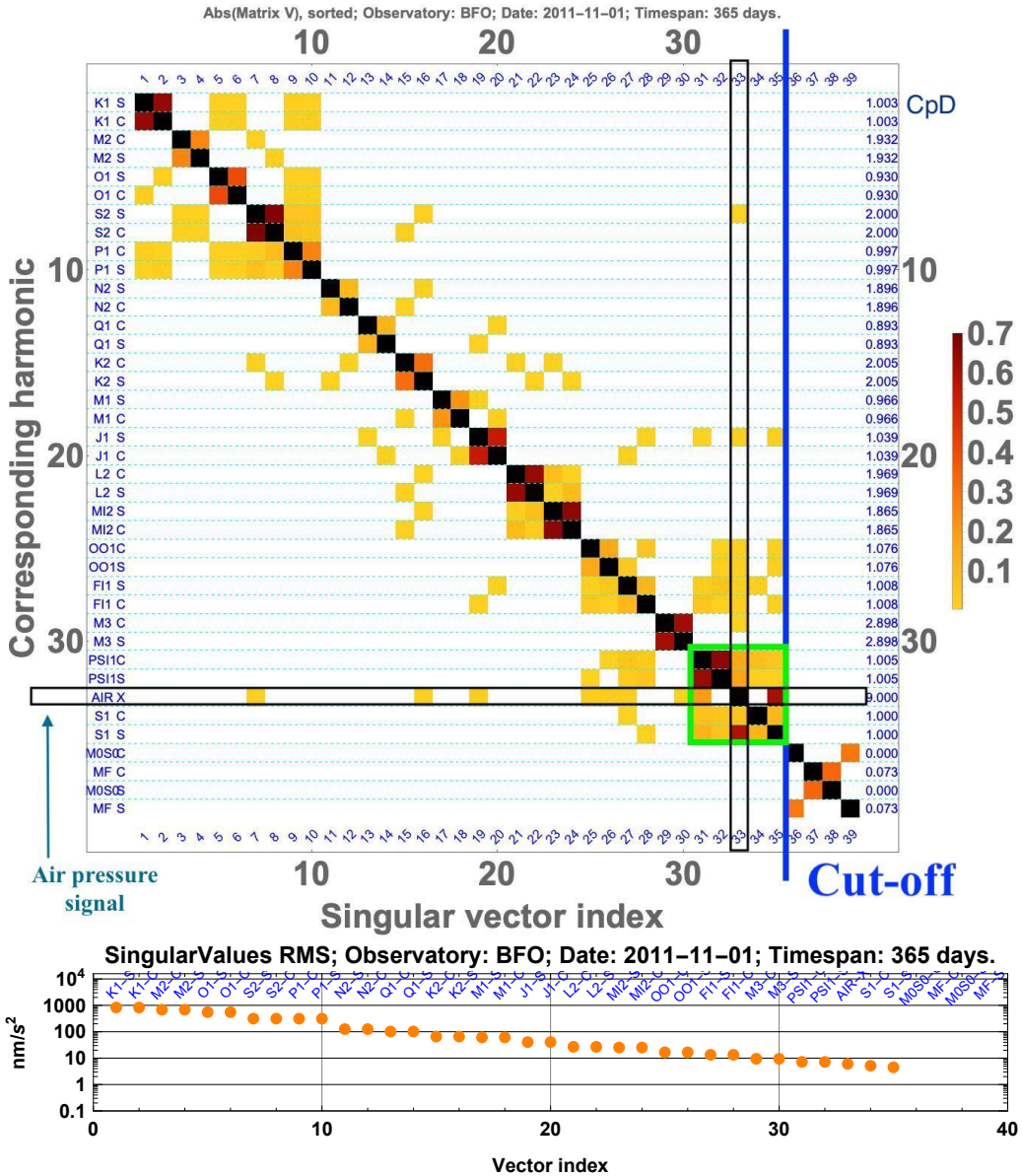
## Description

The next few slides present **investigation of atmospheric cross-talk** to tidal groups.

- We use 1 year length tidal forward operator with corresponding air pressure records at BFO.
- Standard grouping has been applied. All the data have been filtered.
- We expect to observe cross-talk between air pressure induced gravity and tidal harmonics from expected group K1 (S1)
- We verify if there is a cross-talk from the other harmonics, previously noticed for unfiltered data (Phi1, O1, MF).

# 1. Atmospheric cross-talk

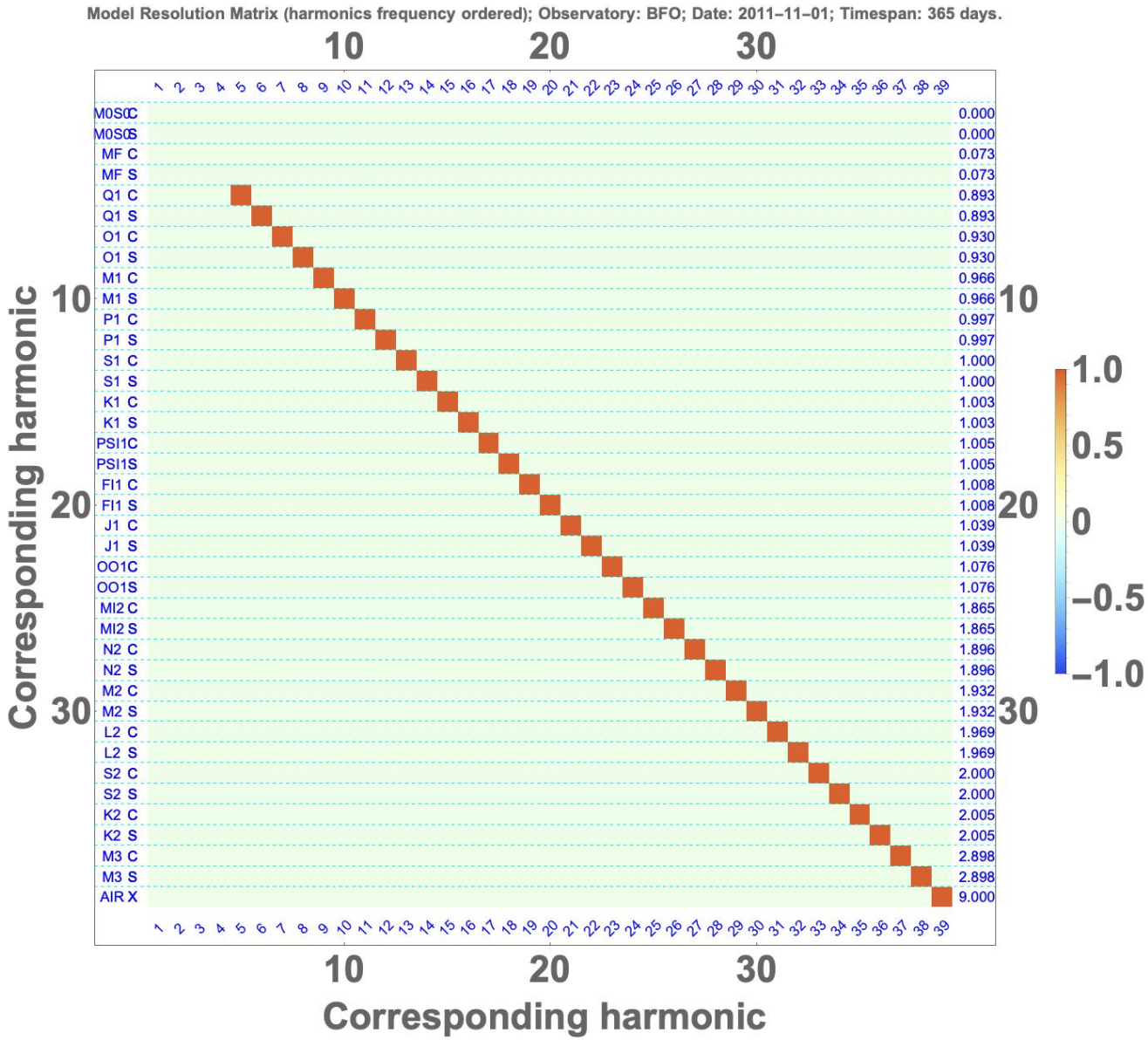
## Matrix V - absolute values




**C**: in-phase, **S**: quadrature signal

# 1. Atmospheric cross-talk

## Resolution matrix: 35/39 harmonics




**C**: in-phase, **S**: quadrature signal

# 1. Atmospheric cross-talk

## Conclusion

- Singular value decomposition indicates cross-talk to other harmonics in our dataset.
- While correlation with S1 group was expected, the reason for effect on other harmonics (e.g. Psi1, Phi1) remains unclear.
- The long-period trends have been correctly filtered, so they do not contribute to the signal.
- SVD indicates that all model parametres should be resolvable.

# 2. Separation of degree 2 and 3 tides

## Description

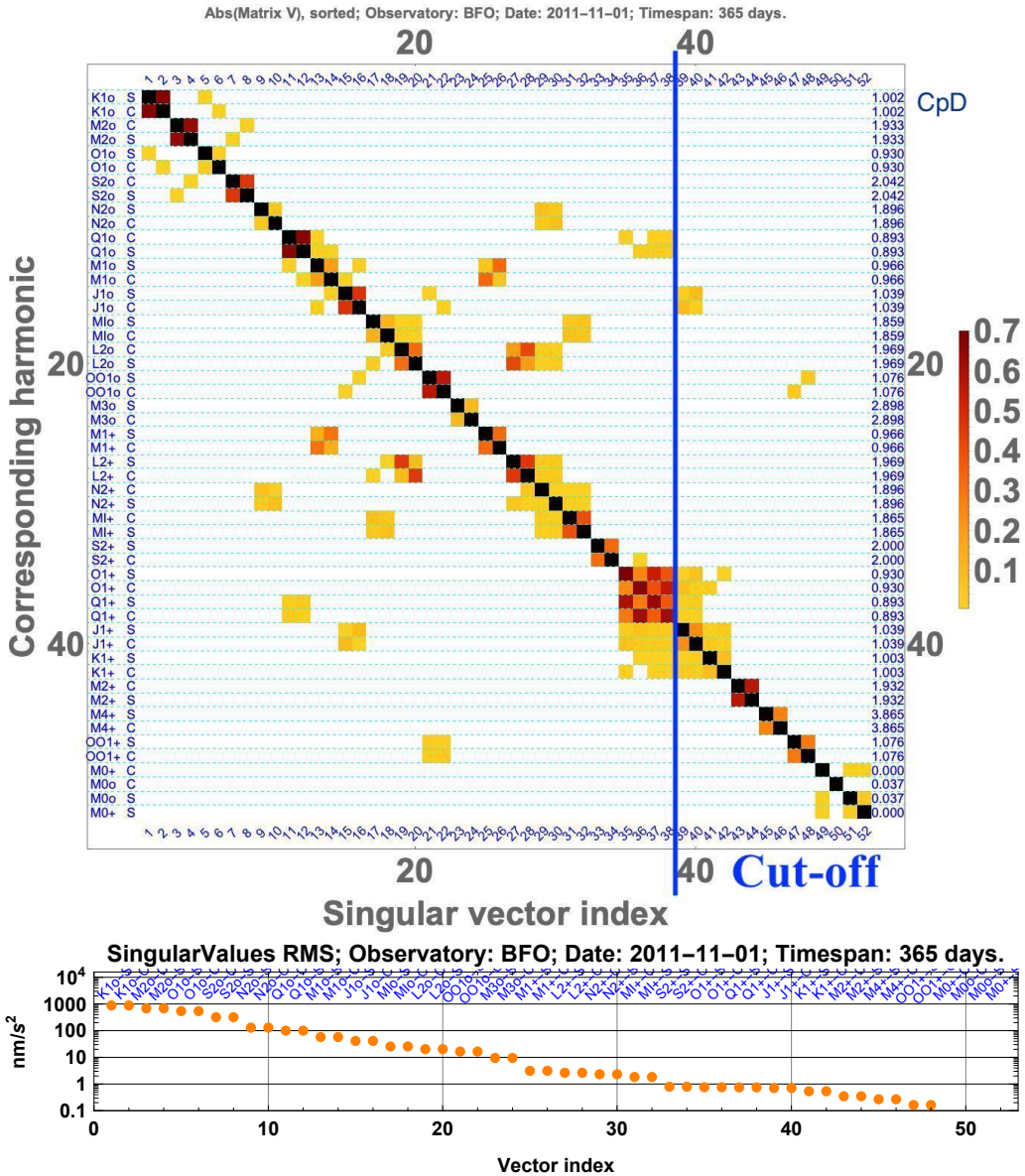
The next few slides present an investigation of the **possibility to determine the ratio of gravimetric factors of degree 2 and degree 3** tides. In this attempt, however, the groups combine of even or odd tides, respectively.

- We investigated properties of tidal forward operator for 1 year at BFO.
- Standard grouping has been applied, but harmonics of degree 2 and 3 have been separated.
- We suppose, that harmonics should be well resolvable.

We expect ratios to be close to the a-priori values, but this analysis goes beyond our current study.

# 2. Separation of degree 2 and 3 tides

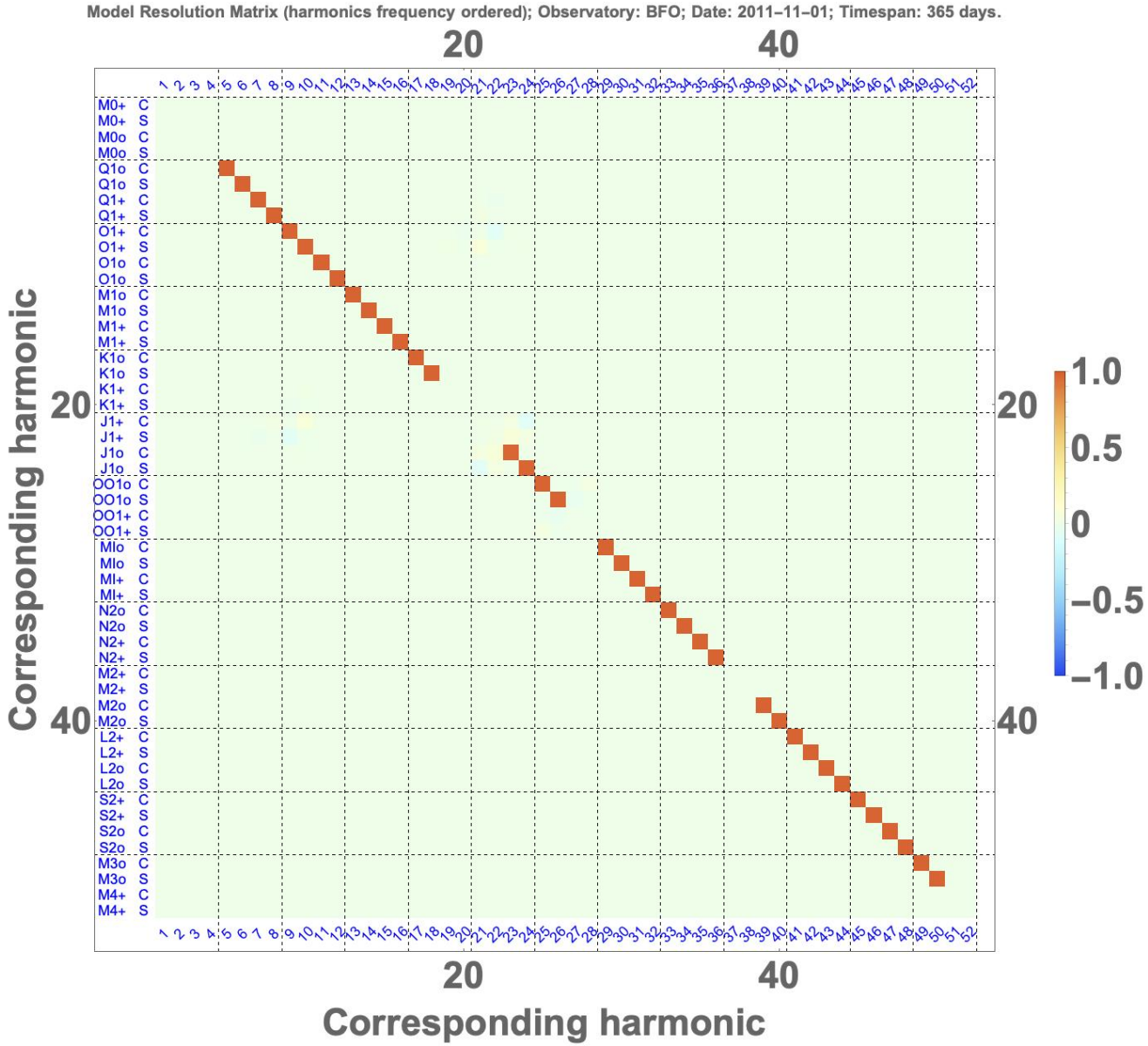
## Matrix V - absolute values




**C**: in-phase, **S**: quadrature signal; **o**: even (2), **+**: odd (3) harmonics

# 2. Separation of degree 2 and 3 tides

## Resolution matrix: 38/52 harmonics




**C**: in-phase, **S**: quadrature signal; **o**: even (2), **+**: odd (3) harmonics

# 2. Separation of degree 2 and 3 tides

## Conclusion

- As expected, SVD indicates the ability to determine the ration between harmonics of degree 2 and 3.
- However, parameters of some counterparts, such as odd K1 or M2, may not be determined.
- The results are reasonable since harmonics of the largest amplitudes are quite often of even degree.

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# Methods: synthetic harmonics

The tidal forward operator **G** is a set of harmonic signals with arbitrary frequencies.

# Methods - synthetic harmonic study

## Intrinsic wave grouping

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Number of harmonics 18

Recording length (hours) 12

Lower period in Hr 16

Center period in Hr 25.4001

Phase (time–length dependent) 0

Initial phase (constant) 0

Grouping factor 8

Amplitude 6.01

Number of groups 2

Group deviation 0.52

DisplayType: Frequency | Eterna

Precision: Best SG | Th. Astron. | Usual SG | Single prec.

Plots: Freq. CpD

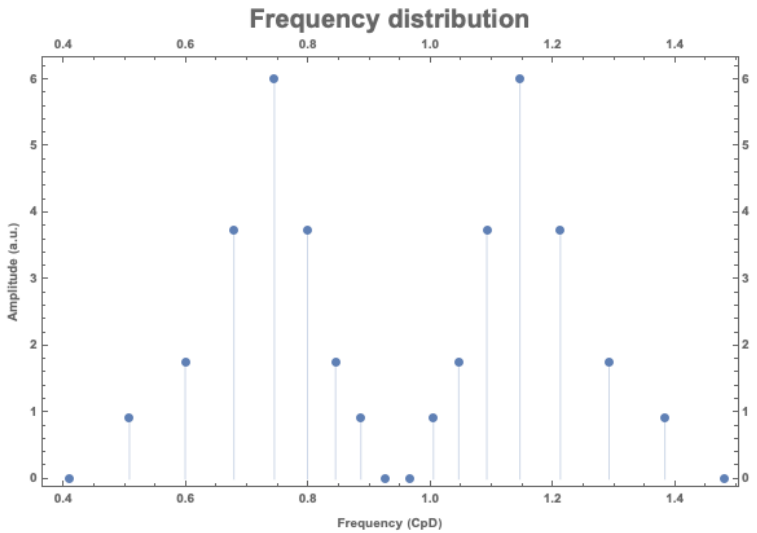
Plot values on Rd & Rm: No | Yes

***Vectors to show in U series***
Manipulator[1, {1, Sigmas[NumberOfVectors], 1}, AutoAction → False, ImageSize → Small, Appearance → Labeled]

Amplitude modulation: Constant | Gaussian | Bimodal

***Catalogue***: No | Doodson | Tamura

**WILL BE AVAILABLE ONLINE**




Difference in model resolution symmetry: 1.17.
Cutoff: 8, Sum of mod. res diag: 0.222, Sum of dat. res. diag: 0.667.
Vectors: 36; instances 12; Days: 0.5`.
Max Freq: 0.0625 CpH; Min Freq 0.0162 CpH;
Sep: 0.00407 CpH; Sep: 0.00165 CpH; Nyquist: 0.5 CpH.
Min Per: 16. Hr; Max Per 61.6 Hr; Sep: 246. Hr; Sep: 605. Hr; Nyquist: 2. Hr.
Max Freq: 1.5 CpD; Min Freq 0.39 CpD;
Sep: 0.0977 CpD; Sep: 0.0397 CpD; Nyquist: 12. CpD.
Amplitude: ; Function ;

**Est. release: early June**

# Methods - synthetic harmonic study

## Intrinsic wave grouping

- We also study synthetic sinusoidal harmonics and their intrinsic grouping behaviour.
- For that purpose, the Mathematica script is developed.
- It allows us to study the difference from uniformly distributed, equal harmonics, to the wave group that amplitudes are from the normal distribution.

Gaussian distribution is the first approach of the real harmonic distribution.

# 3. Resolution loss in central frequencies

## Description

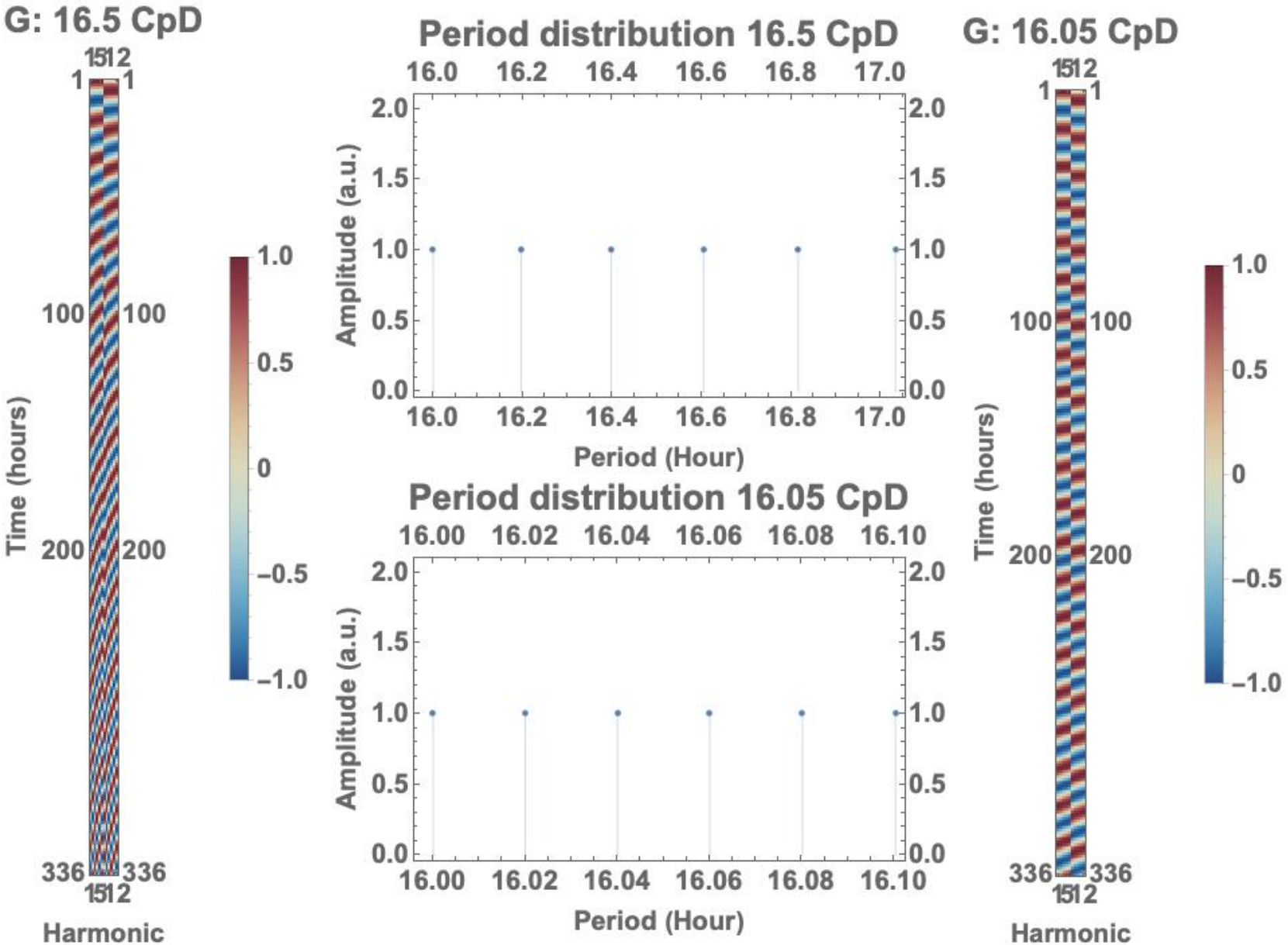
The next few slides present SVD analysis with fixed numbers of harmonics, fixed upper frequency and fixed length of time series. The sets differ by **the lower frequency in each set**, what modifies the separation between harmonics.

- Harmonics are distributed equidistantly (uniformly) within the specified frequency range.
- All harmonics have the same amplitude.
- We present results for two different cut-off conditions.

We expect the smaller the frequency interval between harmonics, the larger the off-diagonal values of the resolution matrix. The central frequencies are primarily affected.

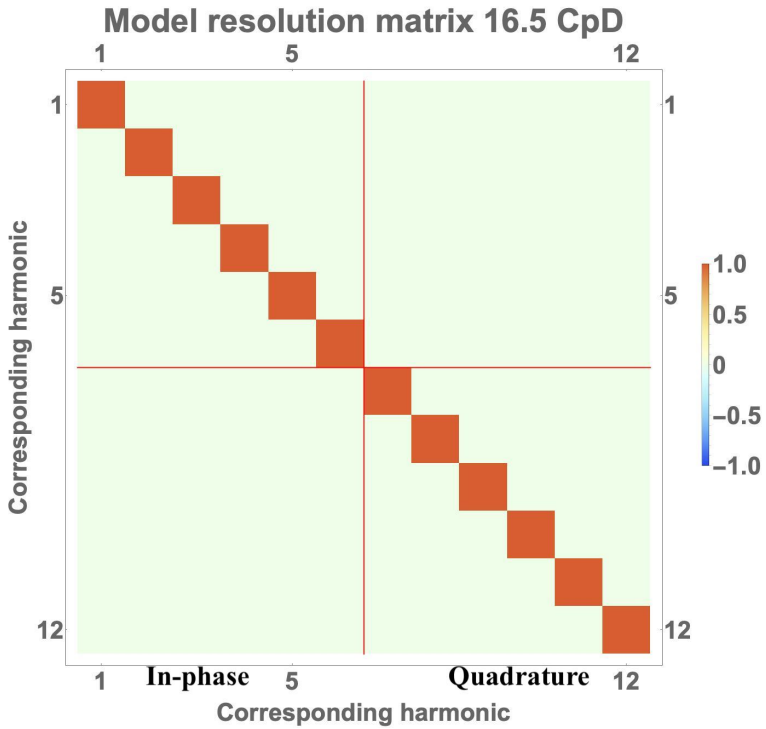
# 3. Resolution loss in central frequencies

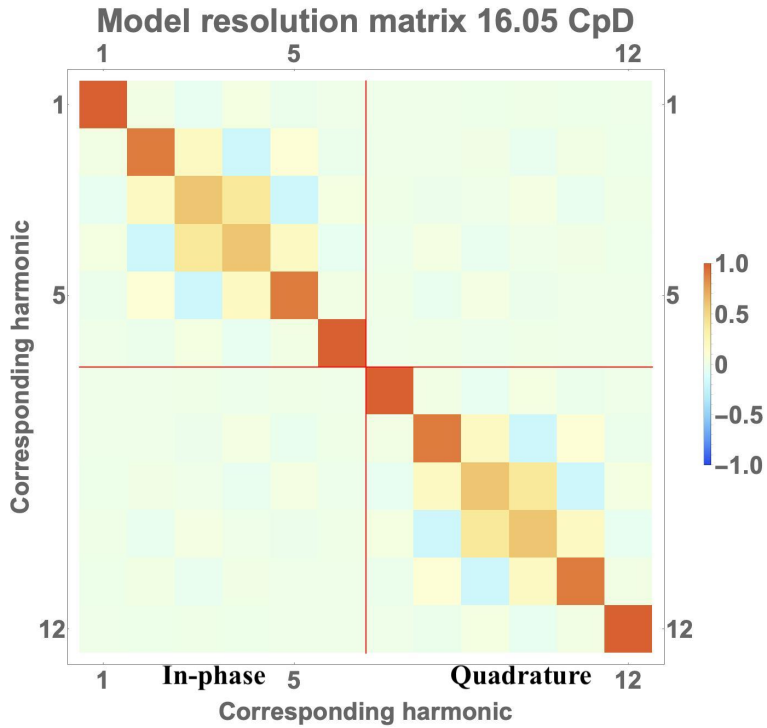
## Different frequency distribution

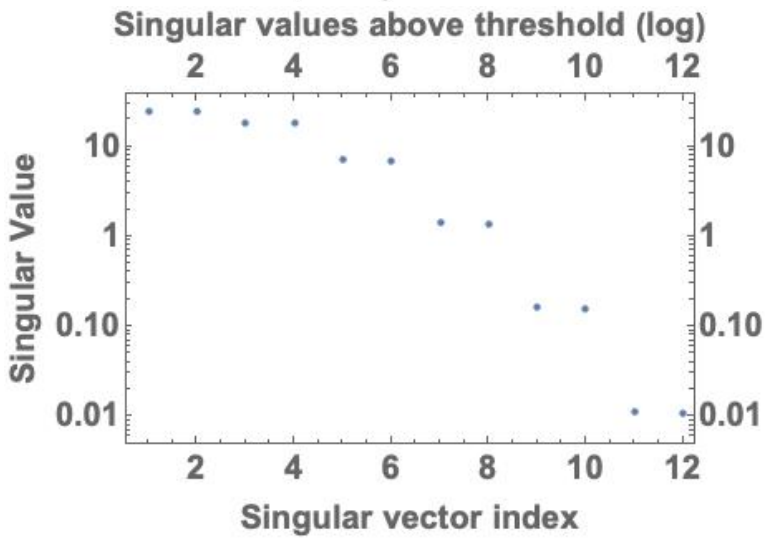


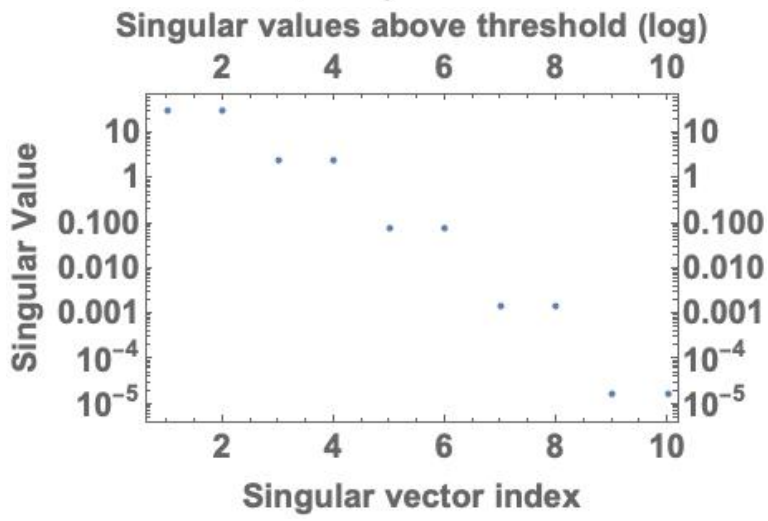
# 3. Resolution loss in central frequencies

## Small threshold (SV ratio: $10^7$)



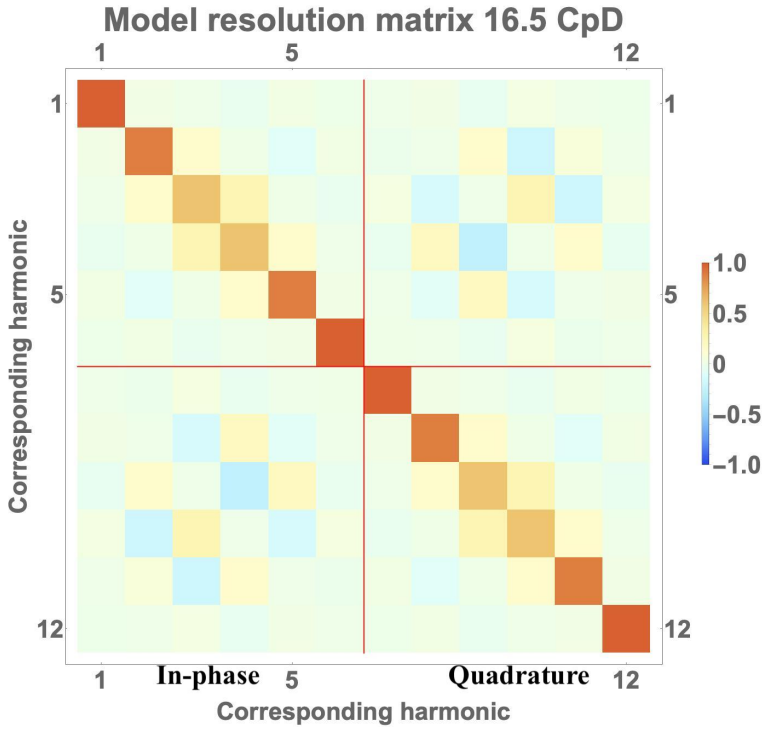



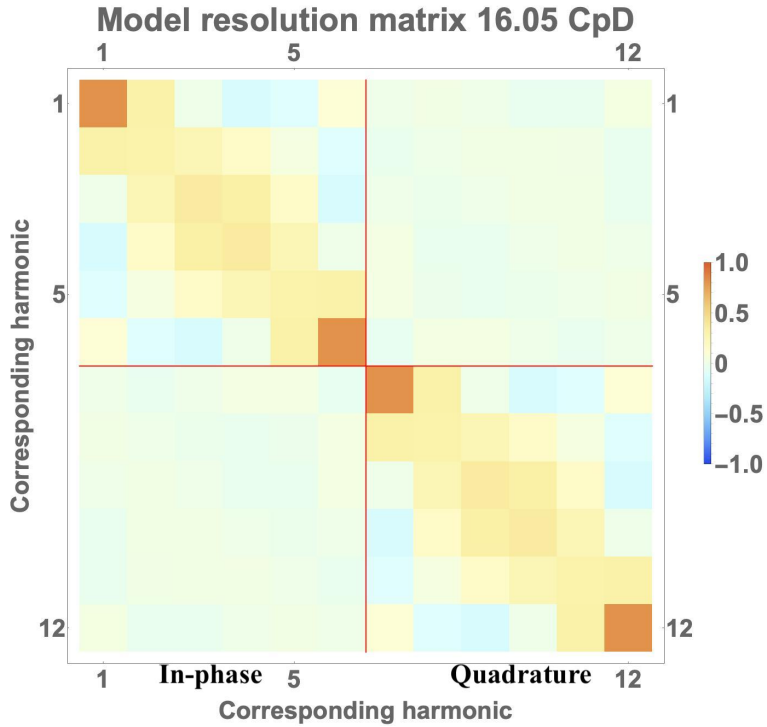



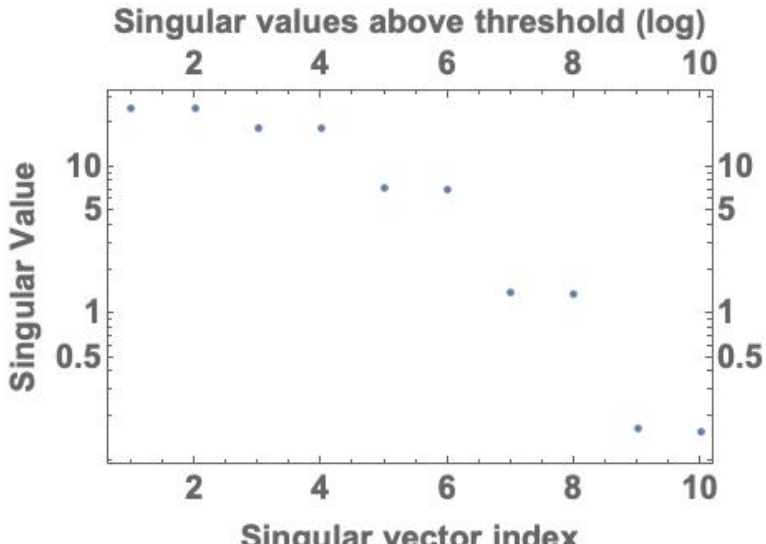



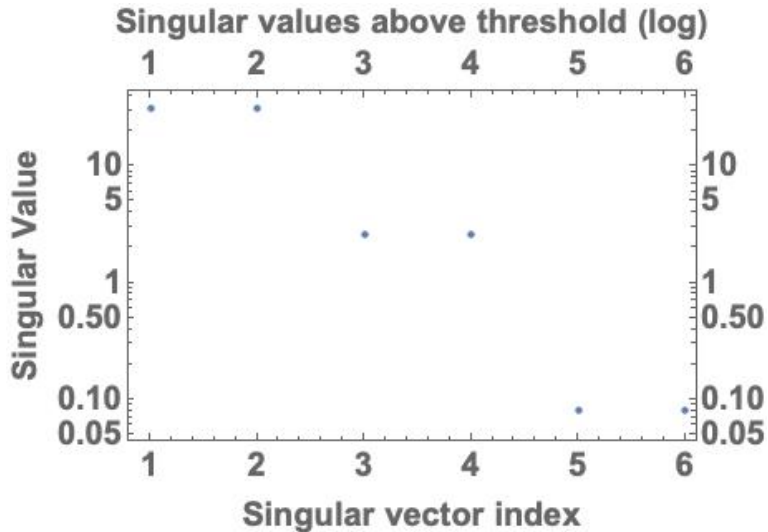
# 3. Resolution loss in central frequencies

## Large threshold (SV ratio: $10^3$)



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Considering constant time series length, the modified Rayleigh criterion gives fixed separation condition (Munk & Hasselmann, 1964, Godin, 1970).

- The smaller the frequency separation, the fewer harmonics meet the criterion.
- The corner frequencies become indistinguishable from only one close frequency
- The central frequencies cannot be discriminated between two neighbouring harmonics.

# 4. Intrinsic wave grouping

## Description

The last few slides present two different kinds of (intrinsic) wave grouping based on Singular Value Decomposition. Sets **differ by the amplitude distribution**.

- Harmonics are equally distributed in the frequency domain.
- The left plots display harmonics with equal amplitudes, the right plots represent harmonics have amplitudes from Gaussian distribution.
- The first slide displays two tidal operators and harmonic frequency distribution.
- Further slides present resolution matrices with corresponding singular value spectra.

We expect that the questionable wave grouping apparent in the analysis of harmonics from uniform distribution clarifies for more realistic, normally distributed harmonics.

# 4. Intrinsic wave grouping

## Example groups

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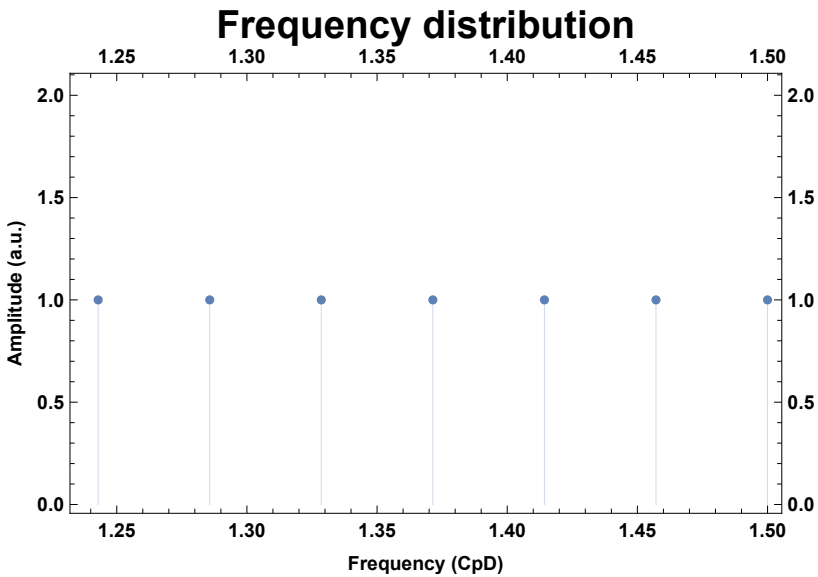
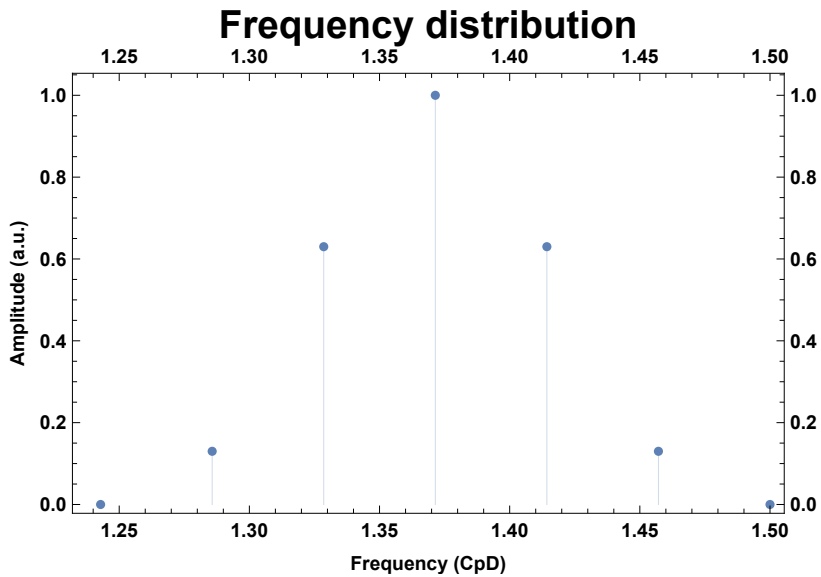
### Constant amplitude

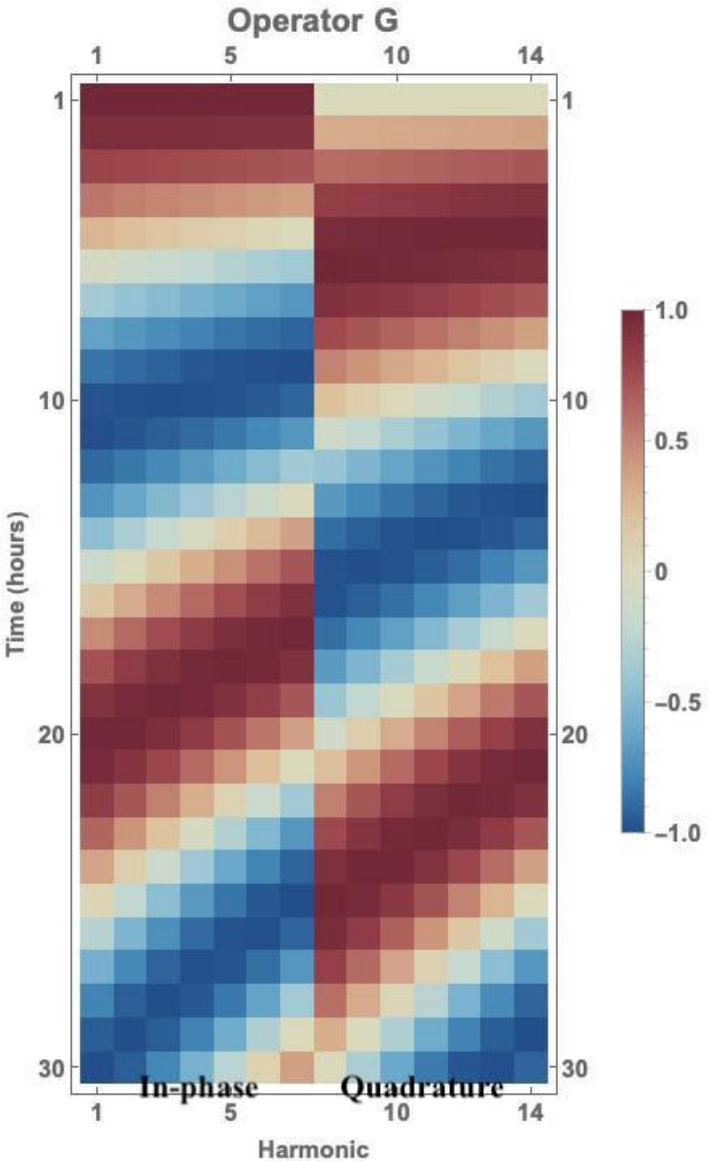



### Gaussian amplitude

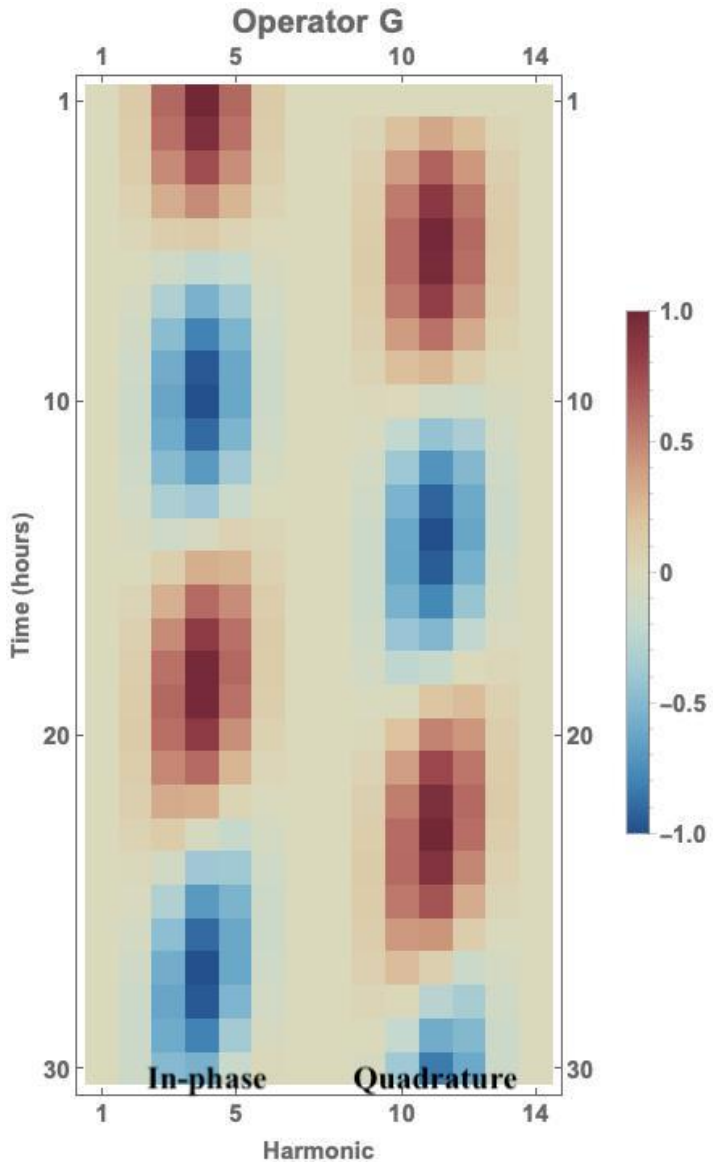


# 4. Intrinsic wave grouping

## Operator G (harmonic signal)



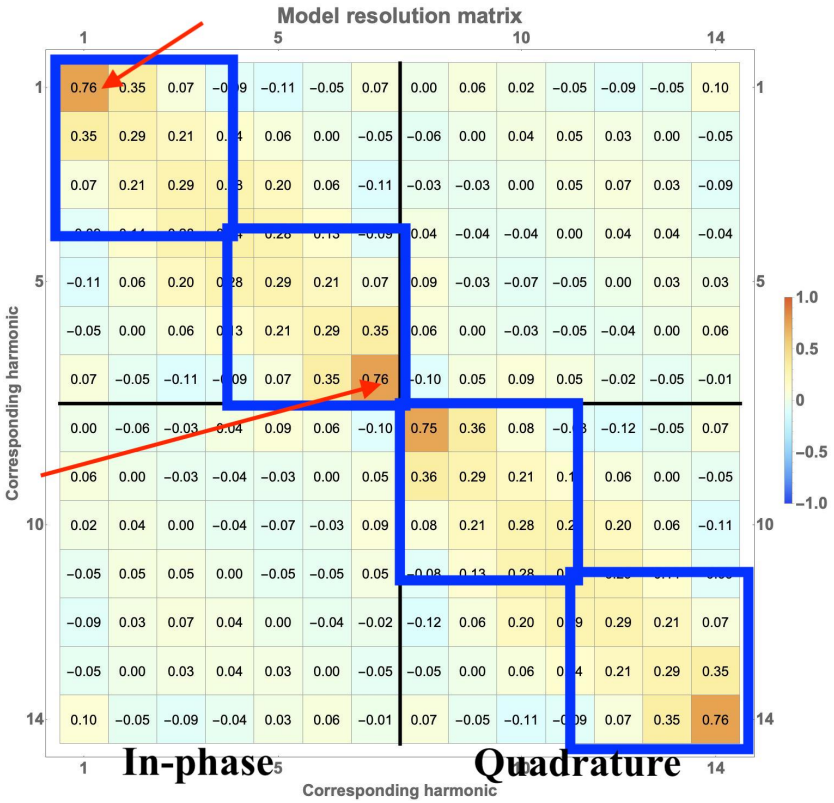

Constant amplitude



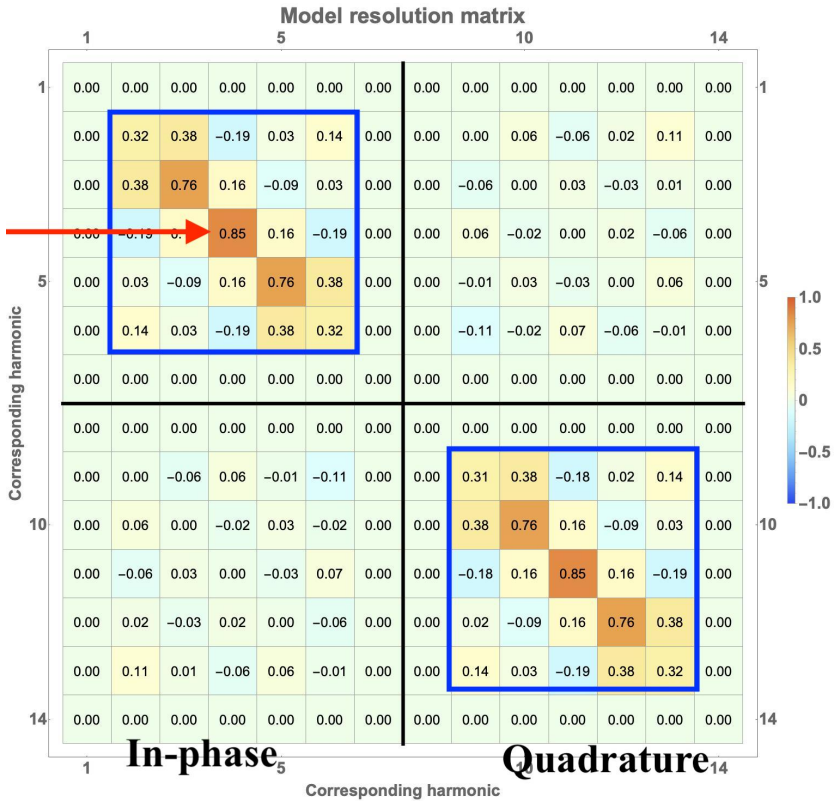

Gaussian amplitude

# 4. Intrinsic wave grouping

## Standard grouping




Constant amplitude



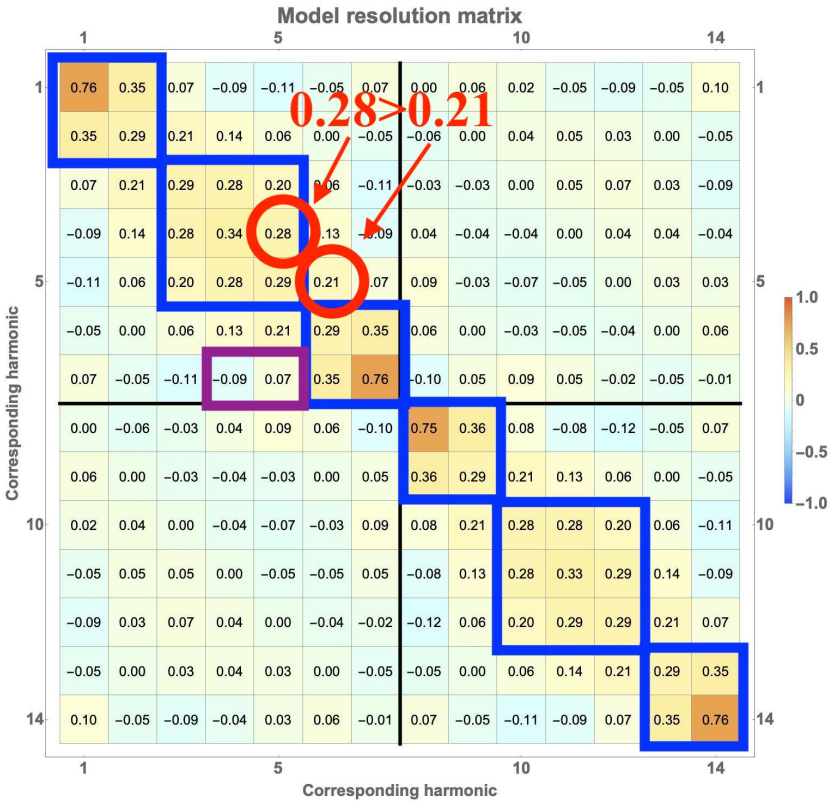

Gaussian amplitude

In Tidal Analysis we group harmonics with the most resolvable (largest) harmonics.

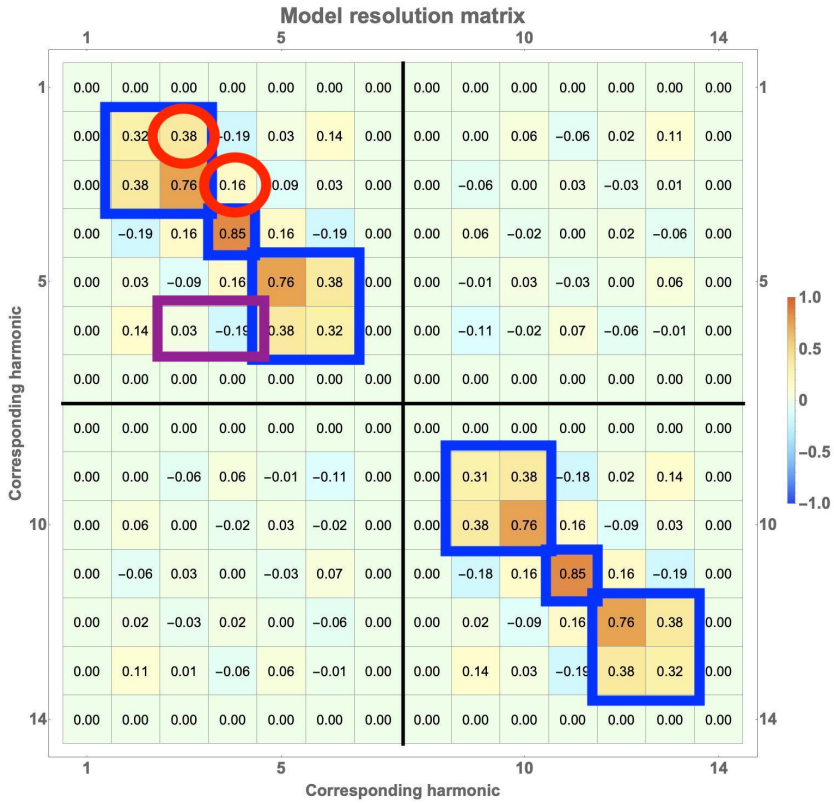
- Blue boxes indicate such groups on both resolution matrices.
- On the right-hand side, the neighbouring frequencies around central frequency are combined together in one group.
- On the left-hand side, they form two such groups.

# 4. Intrinsic wave grouping

## Resolution grouping




Constant amplitude




Gaussian amplitude

# 4. Intrinsic wave grouping

## Resolution grouping: Conclusions

The new blue boxes indicate apparent groups from resolution matrices.

- The off-diagonal coefficients between harmonic 5 and 6 is smaller than between 4 and 5.
- Therefore model parameter obtained for harmonic 5 would be partially harmonic 5 (0.29), harmonic 4 (0.28) and harmonic 6 in (0.21).
- Smearing between harmonic 4 and 6 is negligible. Thus, this might indicate another kind of wave grouping.

Similar indications we may notice on the second plot. However,

- They do not differ that much from the "standard" approach since the other harmonics are resolvable too.
- Probably the results for more realistic values of harmonics would be consistent with the common wave grouping.

# Summary

- 1. Atmospheric pressure may be properly distinguished from other forcings, but some cross-talk is unclear
- 2. Model parameters of degree even and odd tides should be resolvable (in general)
- 3. There is apparent consistency with Rayleigh criterion
- 4. Resolution matrix may be a tool to investigate (or confirm) the wave grouping

# Future plans

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- Detailed investigation of demonstrated problems
- Add penalty terms to the inversion problem
- Introduce model constraints (e.g. “credo of smoothness”)
- Allow for data residuals in inversion (e.g. mentioned air-pressure “radiation tides”)

# References


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