Efficient simulation of prompt elasto-gravity signals (PEGS) based on a spherical self-gravitating earth model

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What are PEGS signals?
How can we measure and model them?
Are they useful for early warning?
What are PEGS signals?

Fault rupture (earthquake)

Elastic waves $u$

Mass redistribution

$\delta \rho = -\nabla \cdot (\rho u)$

Elastogravitational coupling

Secondary sources

$\delta g = \nabla \psi$

Gravity change

$\nabla^2 \psi = -4\pi G \delta \rho$

Propagates with speed of light!
What signals can arrive at receiver before $t = T_p$?

1. Gravity perturbation $\Delta g$ due to mass redistribution within Volume $S$.
2. Ground motion $\Delta a$ generated by secondary sources in Volume $R$. 

$$R = V_p \cdot T_p$$
How are they measured?

✓ Montagner et al. (2016) detected the PPEG signals of the 2011 Tohoku earthquake from records of a superconducting gravimeter (SG).

✓ Vallée et al. (2017) reported their more reliable detection of the same signals from low-noise records of 11 broadband seismometers (BB).

✓ Kimura et al. (2019) tried to verify the previous detections using array stacking of SG (failed), BB (successful) and tilmeters (failed).

✓ Vallée and Juhel (2019) presented new detections for 5 other large earthquakes (Mw ≥ 8.5) with different focal mechanisms.

✓ …

✓ Future: Use of GW detector measuring gravity gradient (expected resolution $10^{-13}/s^2$) based on the general relativity theory are being developed in Japan (Juhel et al. 2018).
Gravity gets into the earthquake game

Earthquakes generate large movements of mass, which slightly change the gravitational field. Unlike the elastic waves that propagate from the earthquake, the gravity perturbations travel at the speed of light. Vallée et al. have finally observed these gravity perturbations in seismometer records from the great Tohoku earthquake in Japan in 2011. The signal would have allowed an accurate magnitude estimation in minutes, rather than hours, for this catastrophic earthquake.

Science, this issue p. 1164

Abstract

After an earthquake, the earliest deformation signals are not expected to be carried by the fastest (P) elastic waves but by the speed-of-light changes of the gravitational field. However, these perturbations are weak and, so far, their detection has not been accurate enough to fully...
How are they modelled?

✓ Harms et al. (2015) calculated the transient PPEG gravity change based on a full-space model (only the gravity change).

✓ Harms et al. (2016) extended the full-space model to a half-space model (still only the gravity change).

✓ Heaton (2017) pointed out that all geophysical instruments as a spring-mass system response not only to the gravity change, but also to the inertial acceleration induced by it.

✓ Vallée et al. (2017) simulated the two effects successfully, but in a very primitive and time-consuming way:

\[
\text{Synthetic seismograms with reflectivity code} \rightarrow \text{Discrete spatial density variations in Volume S} \rightarrow \text{Transient gravity perturbation at station } (\Delta g) \text{ and at discrete elements in Volume R as secondary single-force sources} \rightarrow \text{Synthetic seismograms for all secondary sources at station } (\Delta a) \rightarrow \text{Million of synthetics are needed}
\]
Theory:

Equations of motion
(momentum equilibrium, Poisson’s equation)

\[
\begin{equation}
\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \mathbf{\sigma} + \rho \nabla (\psi - g u_r) + \rho g (\nabla \cdot \mathbf{u}) e_r + \mathbf{f}
\end{equation}
\]

\[
\nabla^2 \psi = 4\pi G \nabla \cdot (\rho \mathbf{u})
\]

Observables
(displacement, incremental gravity potential)

\[
\mathbf{u} = \mathbf{u}(r, \theta, \varphi, t)
\]

\[
\psi = \psi(r, \theta, \varphi, t)
\]

Source
(single forces, dislocations)

\[
\mathbf{f} = \mathbf{f}(r, \theta, \varphi, t)
\]

Earth model
(spherical, self-gravitating, elastic)

\[
\rho = \rho(r), \quad g = g(r),
\]

\[
\lambda = \lambda(r), \quad \mu = \mu(r).
\]
```
# RECEIVER PARAMETERS
#
# 1. select output observables (1/0 = yes/no)
#    Note: the gravitation defined here is the space-based gravitational attraction force, and the gravity is
#    the response of ground-based gravimeter which includes the gravitational attraction, the inertial
#    force and the free-air gradient effect (all positive downwards).
# 2. output file name
# 3. output time window [sec] (<= Green's function time window)
# 4. selection of order of Butterworth bandpass filter (if <= 0, then no filtering), lower
#    and upper corner frequencies (<= cut-off frequency defined above)
# 5. lower and upper slowness cut-off [s/km] (slowness band-pass filter)
# 6. number of receiver
# 7. list of the station parameters
#    Format:
#    Lat  Lon  Name      Time_reduction
#    [deg] [deg]         [sec]
#    (Note: Time_reduction = start time of the time window)
#
# disp  velo  acce  strain  strn_rate  stress  strs_rate  rotation  rot_rate  gravitation  gravity
# 1      1      1      0       0          0       0         0        0        1           1

'tohoku'
500.0
3  0.00  0.20
0.00  0.40
1
36.43  137.31 'Kam'  0.0
```
QSSP output seismograms
\[ \Delta g \]
2011 Mw 9.0 Tohoku Earthquake - Station Kamioka (36.43°N, 137.31°E)

- **Dynamic solutions**
  - Upward
  - East
  - North

- **Static solutions**
  - Upward
  - East
  - North

Gravitational force [μGal]

Time since earthquake origin [s]

Onset of elastic P wave
2011 Mw 9.0 Tohoku Earthquake - Station Kamioka (36.43°N, 137.31°E)

- **Upward**: Blue line
- **East**: Cyan line
- **North**: Green line

Gravitational force [$\mu$Gal] vs. Time since earthquake origin [s]

GFZ Helmholz Centre Potsdam
QSSP output seismograms

\[ \Delta a = \ddot{\ddot{u}} \]
2011 Mw 9.0 Tohoku Earthquake - Station Kamioka (36.43°N, 137.31°E)

Vertical acceleration

Onset of elastic P wave
Vertical acceleration

2011 Mw 9.0 Tohoku Earthquake - Station Kamioka (36.43°N, 137.31°E)

Onset of P wave

Time since earthquake origin [s]

Ground acceleration [μGal]
Our new idea:

\[
\begin{align*}
\rho \frac{\partial^2 u}{\partial t^2} &= \nabla \cdot \sigma + \rho \nabla (\psi - g u_r) + \rho g (\nabla \cdot u) e_r + f \\
\nabla^2 \psi &= 4\pi G \nabla \cdot (\rho u)
\end{align*}
\]

\[
\begin{align*}
\rho \frac{\partial^2 u}{\partial t^2} &= \nabla \cdot \sigma + \rho \nabla (\psi - g u_r) + \rho g (\nabla \cdot u) e_r + f \\
\nabla^2 \psi &= \alpha \cdot 4\pi G \nabla \cdot (\rho u)
\end{align*}
\]

\(u_\alpha\) includes the same elastic waves and the same static gravity effects as \(u\), but the coupling effect by factor \(\alpha\) smaller/larger than \(u\).
\[ \Delta \alpha = \frac{\ddot{u} - \ddot{u}_\alpha}{1 - \alpha} \]

Independent of \( \alpha \)!
## Application to the 2011 Tohoku earthquake

<table>
<thead>
<tr>
<th>Source model:</th>
<th>Kinematic finite-fault model provided by Wei et al. (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth model:</td>
<td>AK135 modified with the local crust structure</td>
</tr>
<tr>
<td>QSSP synthetics</td>
<td></td>
</tr>
<tr>
<td>Cutoff frequency:</td>
<td>250 mHz</td>
</tr>
<tr>
<td>Cutoff harmonic degree:</td>
<td>2500</td>
</tr>
</tbody>
</table>
Synthetic PEGS time series at selected stations

<table>
<thead>
<tr>
<th>Depth</th>
<th>Component</th>
<th>Time Series</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>2276 km</td>
<td>East</td>
<td></td>
<td>nm/s^2</td>
</tr>
<tr>
<td>1284 km</td>
<td>North</td>
<td></td>
<td></td>
</tr>
<tr>
<td>427 km</td>
<td>Vertical</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- PPEG signal $\Delta s$
- Gravity change $\Delta g$
- Induced acceleration $\Delta a$

Time since earthquake origin [s]
Synthetics vs. data

Vallee et al. (2017)

Zhang et al. (2020)
Spatial distribution of peak PEGS signals

\[ \Delta s = \Delta \alpha - \Delta g \]

Observable with seismic instruments
On the potential use of PEGS

1. Most PEGS signals increase/decrease monotonically before the P wave arrival. For a Mw ~ 9.0 earthquake, their maximum is in the order of a few tenths of μGal, which is very small, but significantly over the noise level at several quiet stations.

2. It can be best detected at epicentral distances between 500-3000 km, but difficult without information of the P wave onset, implying a major limitation for earthquake early warning.

3. Far-field peak PEGS signals are proportional to the total seismic moment, providing strong constraints on the earthquake magnitude and therefore useful for tsunami early warning under certain ideal conditions.

4. The radiation pattern depends not only on the fastest P wave, but also on many other seismic wave phases (pP, sP, …), which can provide complementary constrains on the source mechanism.

5. …
How does the peak $\Delta S$ signal depend on the moment magnitude $M_w$ and rupture duration $T$?

**Two point source tests**

**Dip-slip**
strike/dip/rake = 0°/20°/90°, receiver azimuth 90°

**Strike-slip**
strike/dip/rake = 45°/90°/90°, receiver azimuth 90°

**Mw-T scaling law**

$$T = T_0 10^{M_w/2}$$
Zhang et al. (2020)

Dip-slip source

Strike-slip source

Zhang et al. (2020)
The first magnitude/duration inversion

Cost function:

\[ R = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \int_0^{t_i^p} [d_i(t) - s_i(t)]^2 dt \]

- \( d_i(t) \): data
- \( s_i(t) \): model
- \( \sigma_i^2 \): standard pre-seismic noise variance

Unknowns:

- Moment magnitude: \( M_w \)
- Rupture duration: \( T \)

Approach 1:

Assume \( M_w \)-\( T \) scaling law

\[ T = T_o \cdot 10^{M_w/2} \]

2D grid search \( M_w \) and \( T_o \)

Approach 2:

Fix

\[ T = 140 \text{ s (CMT)} \]

1D grid search for \( M_w \)

Confidence interval through Bootstrap tests
The 2011 Tohoku earthquake (11 best BB stations)
Confidence level via Bootstrap tests

Approach 1:
$$M_w = 9.00 \pm 0.05$$
Optimal $T_o = 0.00371\text{s}$
($T = 117\text{s}$)

Approach 2:
$$M_w = 9.06 \pm 0.03$$
Using fixed $T$ adopted from CMT ($9.08/140\text{s}$)
Seismic waveform inversion (Wie et al. 2012): Mw = 9.06
Geodetic inversion (Wang et al. 2013): Mw = 8.90
Conclusions

✓ A brand new approach, which is considerably simpler, faster and more accurate than previously used for simulating PEGS signals.

✓ A robust estimate of major source parameters of the 2011 Tohoku earthquake using the PEGS data recorded at 11 low-noise broadband stations.

Outlook

➢ Further theoretical investigations (it is just the beginning).
➢ Development of new measurement systems (e.g., gravity strainmeters).
➢ …
Thank you!