Analysing Conceptual Climate Models with Monte Carlo Basin Bifurcation Analysis (MCBB)

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Monte Carlo basin bifurcation analysis

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Abstract
Many high-dimensional complex systems exhibit an enormously complex landscape of possible asymptotic states. Here, we present a numerical approach geared towards analyzing such systems. It is situated between the classical analysis with macroscopic order parameters and a more thorough,

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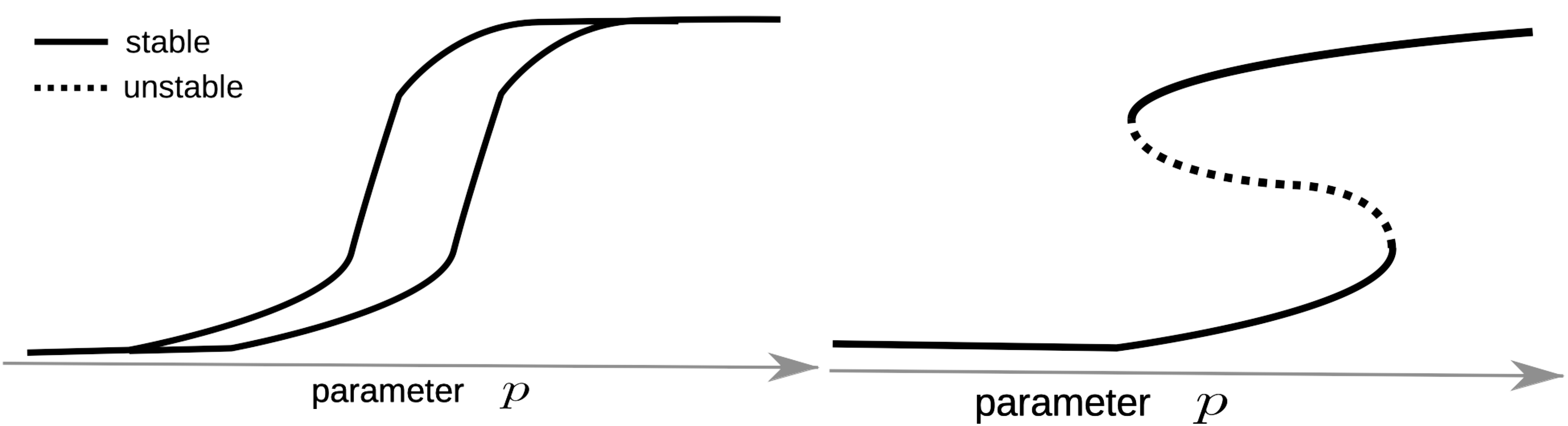
Maximilian Gelbrecht typo in URL Latest commit 7f13c83 23 days ago

docs	typo in URL	23 days ago
paper	uploaded scripts for paper	2 months ago
src	hiddenparvar now also with all IC gen options	2 months ago
test	added utility constrcutor for multi dim setups	3 months ago
.gitignore	more docs	16 months ago

https://github.com/maximilian-gelbrecht/MCBB.jl

Motivation

- **Multistability** is a universal phenomenon of complex systems
- Magnetism, human brain, gene expression networks, human perception, power grids, climate systems and many more exhibit multistable regimes
- Volume of the basin of attraction often interesting as well



Motivation

- For high-dimensional systems a traditional bifurcation analysis is often challenging
- Often one is also interested in classes of asymptotic states instead of every single possible asymptotic state
- **Aim:**
  - Fill gap between thorough bifurcation analysis and macroscopic order parameters
  - Learn classes of similar attractors that collectively have the largest basin of attraction
  - Understand how their *basin volumes change as a function of the parameters*
  - Get insights into the dynamics of these classes of asymptotic states
  - Apply it to climate dynamics

Method

- **Idea:**
  - Combine a sampling based approach with a clustering analysis
  - Don’t compare the high-dimensional trajectory tails with each other directly but with per-dimension measures

Algorithm

Given:

system $\dot{\mathbf{x}} = F(\mathbf{x}, t; \mathbf{p})$ or $\mathbf{x}_{n+1} = F(\mathbf{x}_n, \mathbf{x}_{n-1}, ...; \mathbf{p})$ with system dimension N_d	A set of N_m statistics $\{\mathcal{S}_i\}$ on the components $\mathbb{R}^{N_t} \to \mathbb{R}$ (e.g. mean and variance)	Distribution $\mathcal{U}_{IC}$ of the initial conditions and parameters $\mathcal{U}_p$

sample N initial conditions and N parameters from $\mathcal{U}_{IC}$ and $\mathcal{U}_p$

for every sample $i \in [1, N]$

- solve system for a long trajectory, only save the tail $\mathbf{x}(t; p)$
- for every system dimension $i_d \in [1, N_d]$ and for every statistic $i_m \in [1, N_m]$
  - compute matrix of statistics $S_{i,i_d,i_m} = \mathcal{S}_i(x_{i_d})$

Obtained: N $(N_d \times N_s)$ - matrices $\mathbf{S}_i$

compute $(N \times N)$ distance matrix $\mathbf{D}$ of all $\mathbf{S}_i$ to each other

Density-based clustering of $\mathbf{D}$ (e.g. DBSCAN)

analyse cluster memberships and measures for each cluster dependent on $\mathbf{p}$

Algorithm

Given:

system $\dot{\mathbf{x}} = F(\mathbf{x}, t; \mathbf{p})$
or $\mathbf{x}_{n+1} = F(\mathbf{x}_n, \mathbf{x}_{n-1}, \ldots; \mathbf{p})$
with system dimension N_d

A set of N_m statistics $\{\mathcal{S}_i\}$
on the components $\mathbb{R}^{N_i} \to \mathbb{R}$
(e.g. mean and variance)

Distribution $\mathcal{U}_{IC}$
of the initial conditions
and parameters $\mathcal{U}_p$

sample N initial conditions and N parameters from $\mathcal{U}_{IC}$ and $\mathcal{U}_p$

for every sample $i \in [1, N]$

solve system for a long trajectory, only s

for every system dimension $i_d \in [1, N$ … statistic $i_m \in [1, N_m]$

compute matrix of statistics S_{i,i_d,i_m}

Obtained: N $(N_d \times N_s)$ - matrices $\mathbf{S}_i$

compute $(N \times N)$ distance matrix $\mathbf{D}$ of all $\mathbf{S}_i$ to each other

Density-based clustering of $\mathbf{D}$ (e.g. DBSCAN)

analyse cluster memberships and measures for each cluster dependent on $\mathbf{p}$

more detailed information in the paper
https://doi.org/10.1088/1367-2630/ab7a05

Software Implementation: MCBB.jl

- Open Source software package MCBB.jl available
- Julia lang, easy to read and write, fast programming language
- Excellent state-of-the-art differential equations solvers (thanks to DifferentialEquations.jl)

GitHub repository

https://github.com/maximilian-gelbrecht/MCBB.jl

Documentation

https://maximilian-gelbrecht.github.io/MCBB.jl/dev/

Applications

- MCBB is a modular, flexible method suitable for many different kinds of mid- to high-dimensional complex systems
- Maps, ODEs, …
- **Examples**
  - **Dodds-Watts model of social and biological contagion**
  - Kuramoto network
  - Stuart-Landau oscillator network

  } **paper**

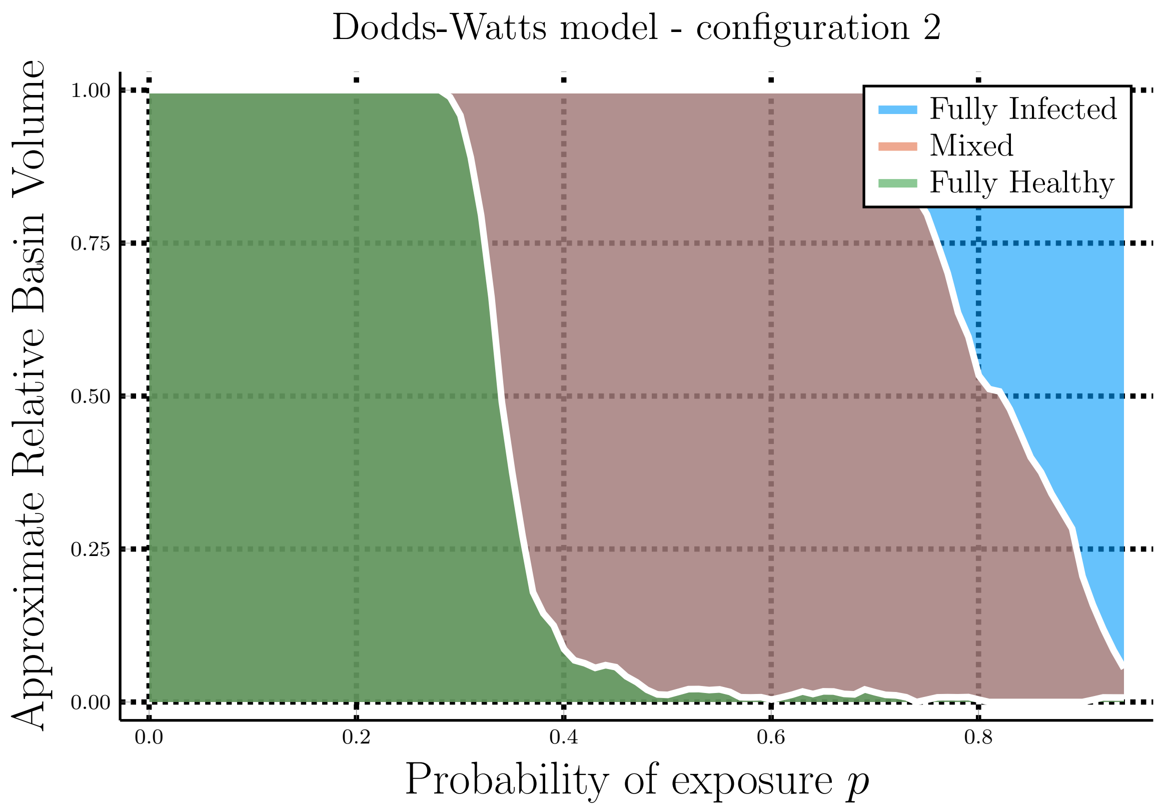
  - conceptual climate models (work in progress)
  - **modified Lorenz 96 model** (here)

Application: Dodds Watts model

- Dodds-Watts model of social and biological contagion
- Generalisation of SI(RS) models from epidemiology
- Population with N Individuals that can be either **S**usceptible, **I**nfected or **R**ecovered
- Dodds-Watts model introduces a dosage memory into these models (all details see Dodds, Watts arXiv:1705.10783)

Application: Dodds Watts model - MCBB results

- Area in the plot correspondents to the basin volume



- Additional tools to identify the dynamics of the individual classes of the asymptotic states in the paper / library
- Here, coexistence of states where the population is fully healthy (green), only some individuals are infected (red) and fully infected (blue)

Application: Lorenz 96

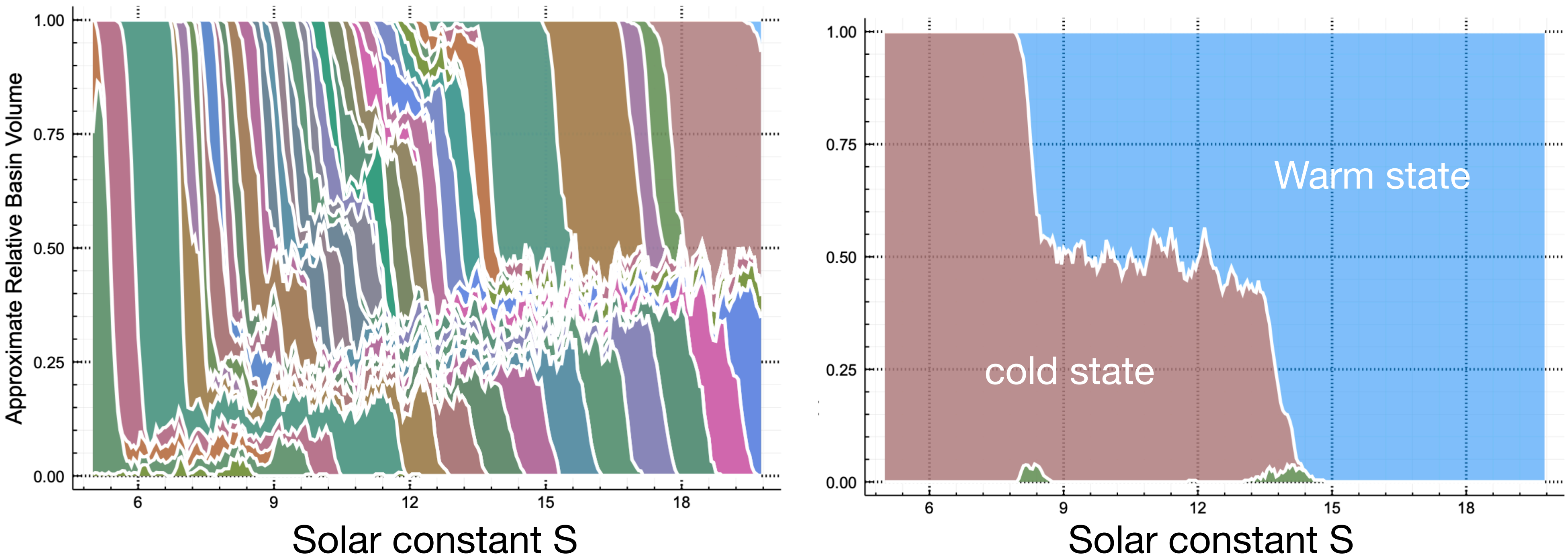
- 1-Layer Lorenz 96 model coupled to a simple EBM
- add an additional “wiggle” to the EBM to invoke more stable states than the regular cold / warm state
- add noise -> SDE

$$dX_k = (-X_{k-2}X_{k-1} + \underbrace{X_{k+1}X_{k+1}}_{\text{Lorenz 96}} - X_k + F)dt + \underbrace{\sigma_X dW}_{\text{Noise}}$$

$$dF = (EBM(\mathbf{X}, F) + A \cdot \sin\underbrace{(\omega(F - F_0))}_{\text{Additional Wiggle}})dt + \underbrace{\sigma_{EBM} dW}_{\text{Noise}}$$

Application: Lorenz 96

$\sigma_{EBM} = 0$ σ_{EBM} large



- Sinus wiggle introduces many additional stable states
- For large noise amplitudes only the “deepest” states in the EBM are relevant and the sinus-wiggle is not important anymore
- Further analysis with MCBB possible (and also experiments with two parameters)