Unstable Periodic Orbit Sampling in Climate Models

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MOTIVATION

Unstable Periodic Orbits can be used to **reconstruct the invariant measure** ν of a chaotic dynamical system.

Any chaotic trajectory can be approximated in terms of UPOs. In fact:

- Periodic orbits are **dense in the attractor**, it is always possible to find a UPO arbitrarily near a chaotic trajectory
- The trajectory can be thought as repelled by different UPOs neighborhoods

Once a suitable set of UPOs has been sampled, it is possible to calculate the **expectation value of a measurable observable** $\boldsymbol{\phi}$ with the following:

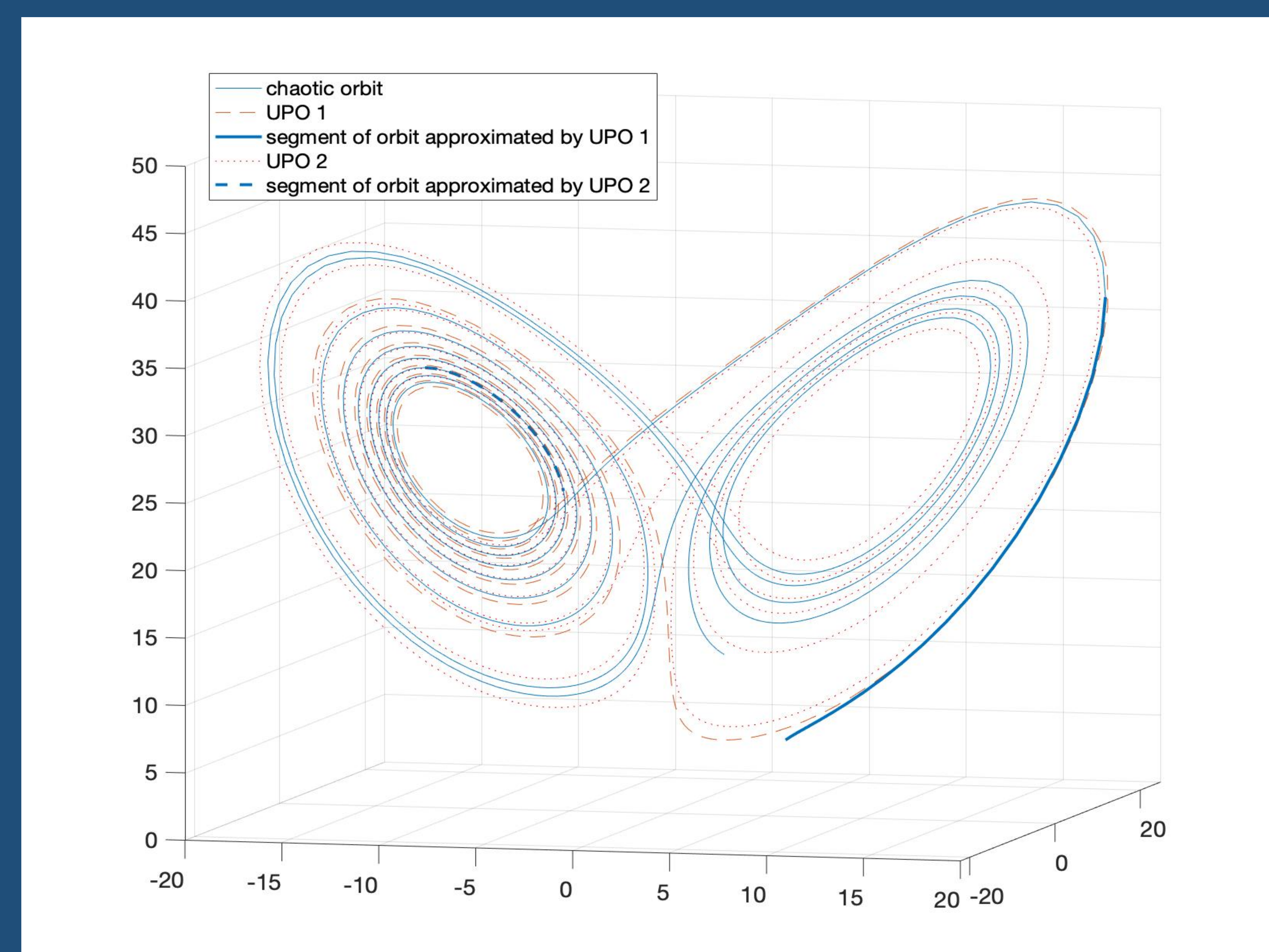
$$\nu(\phi) = \lim_{t\to\infty} \frac{\sum_{UP,p\le t} w^{UP} \bar{\phi}^{UP}}{\sum_{UP,p\le t} w^{UP}}$$

NEWTON METHOD

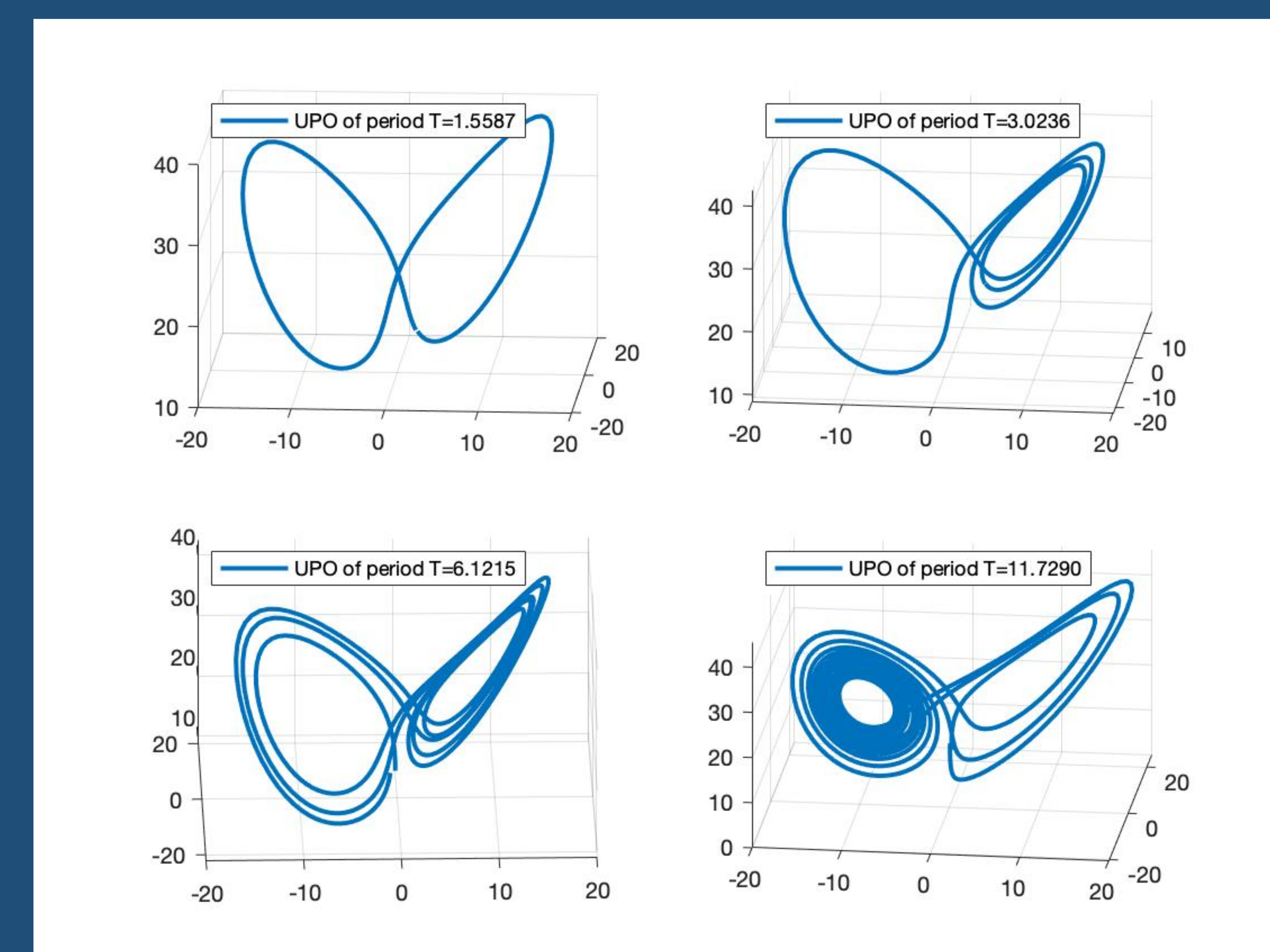
Newton method has been applied to create a **dataset of periodic orbits.**

Each UPO is found by iteration on the initial conditions $x(0) = y$ by applying the algorithm in order to obtain the root of the periodicity condition equation $x(T, y) - y = 0$

Numerical Sampling of Unstable Periodic Orbits



An example of approximation of two segments of a chaotic trajectory with UPOs



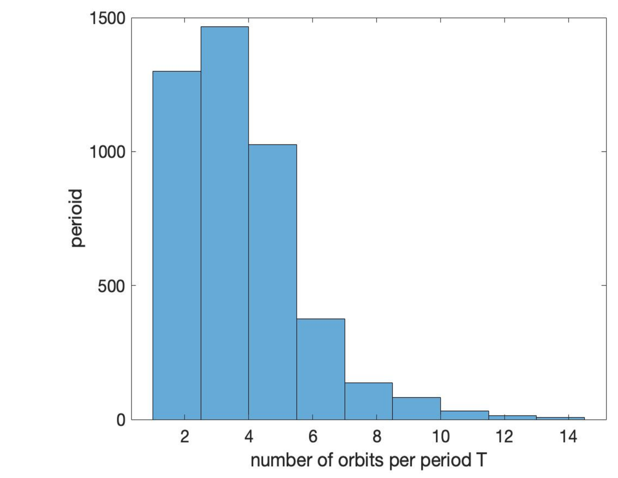
UPOs with different periods found with Newton Method

A mathematical tool in the study of Climate Science

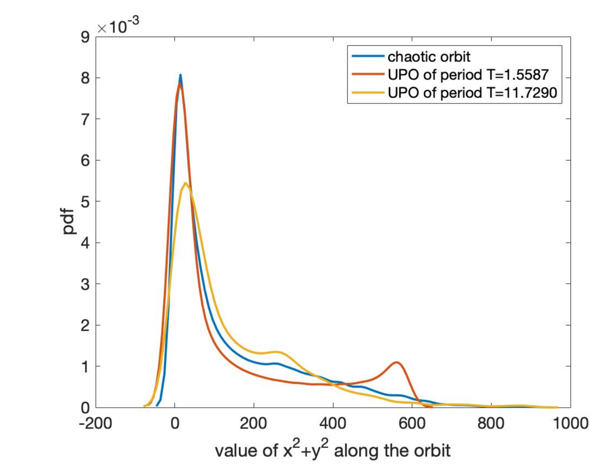
Lorenz-63

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = -xz + rx - y$$
$$\dot{z} = xy - bz$$

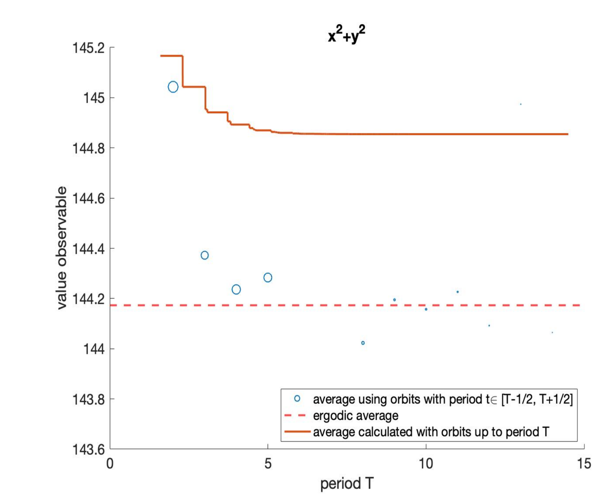
Frequency of UPOs periods in a database built with Newton Method



Pdf of the observable $\phi = x^2 + y^2$ along different orbits



Approximation of the expectation value of the observable $\phi = x^2 + y^2$ with trace formula



FUTURE WORK

- Investigate the dependence of the error in the approximation on the cut-off point in the trace formula
- Apply UPOs search algorithm in multiscale models

REFERENCES

- V. Lucarini and A. Gritsun, "A new mathematical framework for atmospheric blocking events," Climate Dynamics, vol. 54, no. 1-2, pp. 575– 598, 2020.
- Y. Saiki, "Numerical detection of unstable periodic orbits in continuous time dynamical systems with chaotic behaviors," 2007.