The inertial modes of a freely rotating planet

And their relation to nutation

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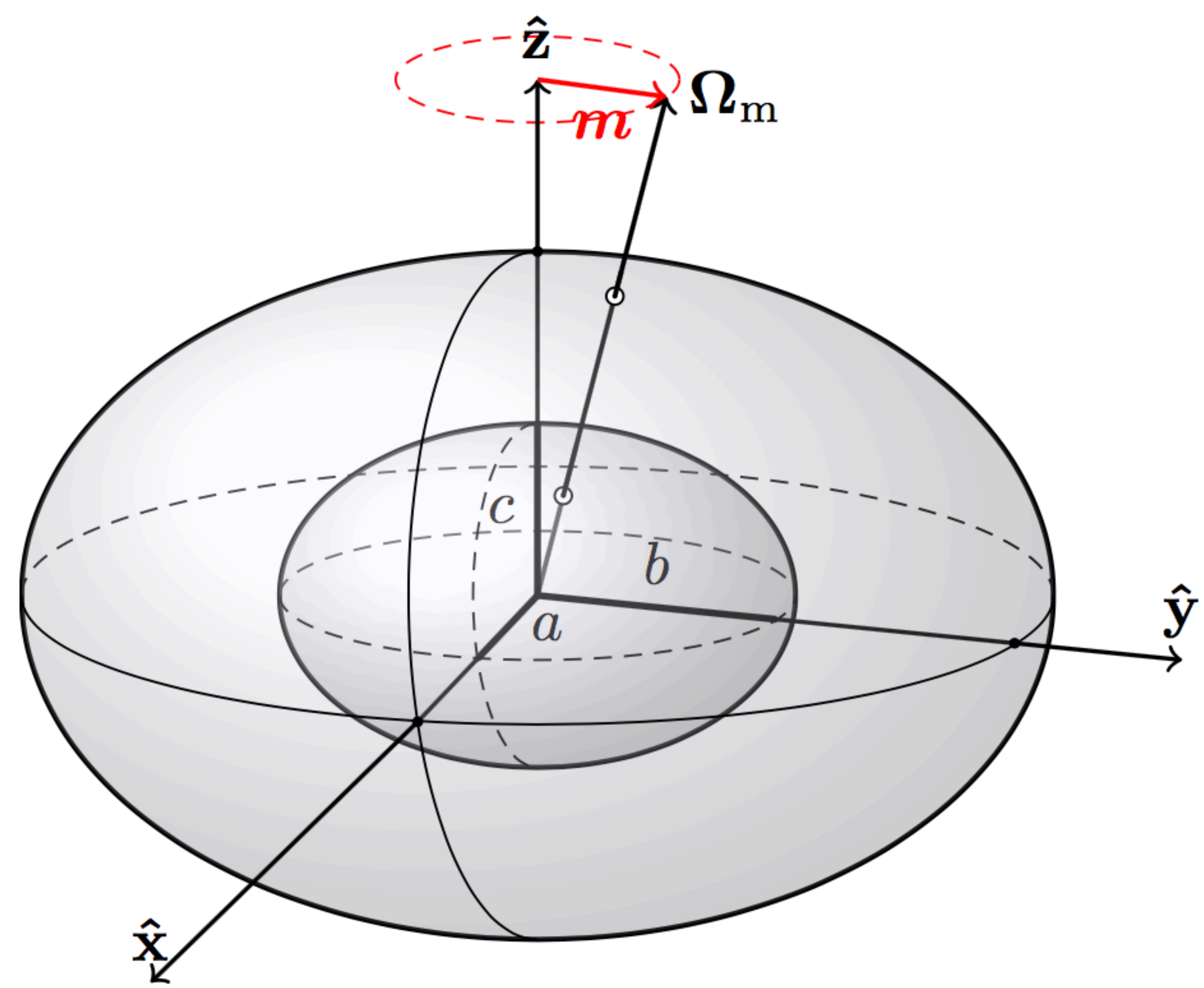
Motivation

- Inertial modes of a planet in a steady (or otherwise prescribed) state of rotation are well-known from **fluid dynamics studies**
- In **geodesy**, the theoretical description of the free nutations of the Earth is based on a simple inviscid flow in the core
- Can we make the two approach **talk to each other** ?



Dynamical flattening parameters

$$\alpha_f = \frac{a^2 + b^2 - 2c^2}{a^2 + b^2 + 2c^2}$$

$$\beta_f = \frac{a^2 - b^2}{a^2 + b^2 + 2c^2}$$

And analogous for the whole planet: α, β

J. Rekier et al. ROB (2020)

Coupled Core and Mantle

Analytical solution

- Simple two-layer model : **perfectly rigid mantle** + **inviscid fluid core**, both **ellipsoidal**
- **Liouville eq.** for the mantle (d.o.f: m^x, m^y) + **Eq. of vorticity** for the core (d.o.f: m_f^x, m_f^y)
- Correct **to all orders in the ellipticity**
- Different to traditional *inertial torque approximation* based on Liouville eq. for the fluid

Mantle only : Euler Free Wobble (EW)

Coupled : Free Core Nutation (FCN) & Chandler Wobble (CW)

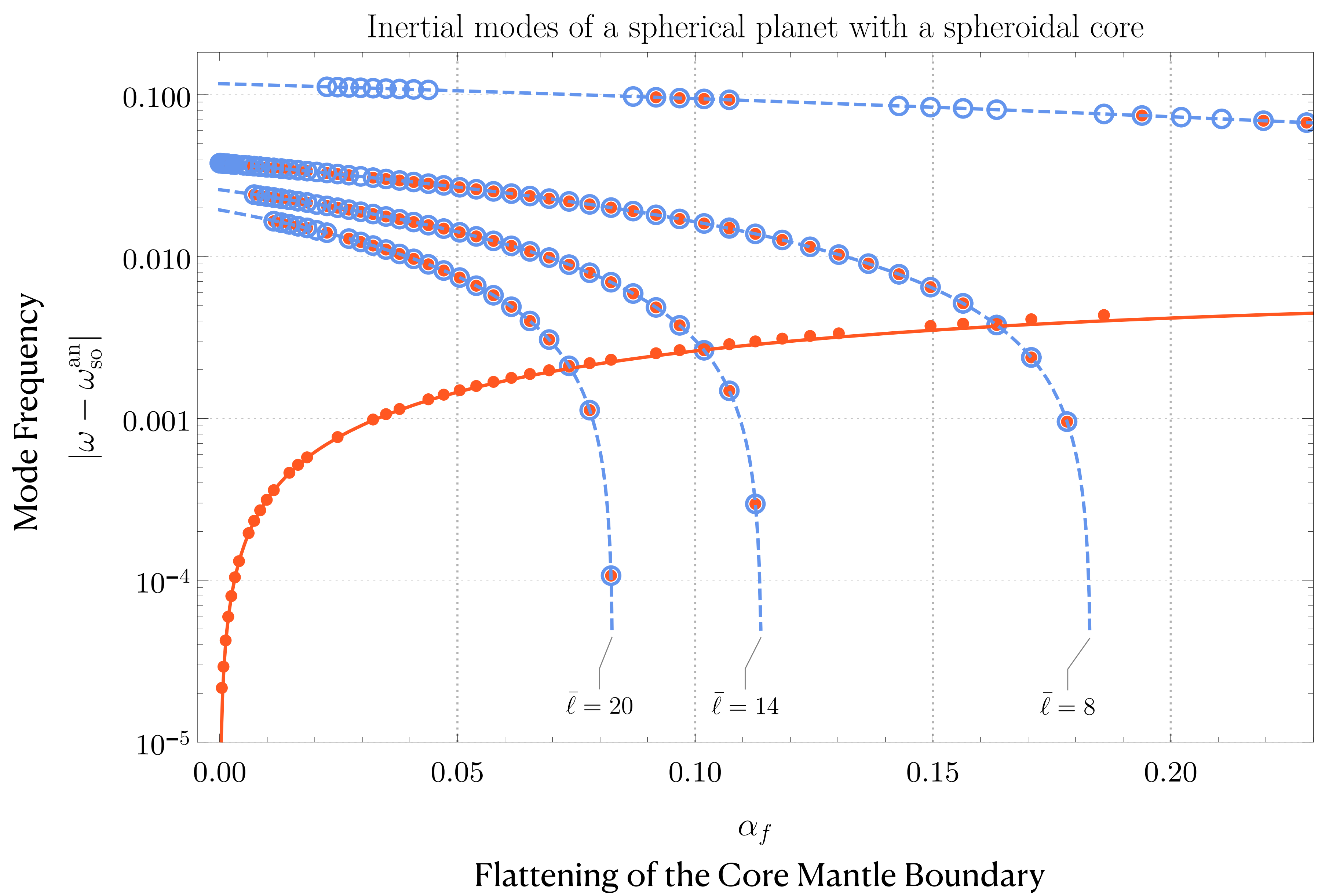
$$\begin{pmatrix} \omega & -\frac{i(\alpha-\beta)}{(1-\beta)} & \frac{A_f(1-\alpha_f)(1+\alpha_f-2\beta_f)}{A(1-\beta_f)^2}\,\omega & \frac{iA_f(1-\alpha_f)(1+\alpha_f+2\beta_f)}{A\left(1-\beta_f^2\right)} \\ \frac{i(\alpha+\beta)}{(1+\beta)} & \omega & -\frac{iA_f(1-\beta)(1-\alpha_f)(1+\alpha_f-2\beta_f)}{A(1+\beta)(1-\beta_f)^2} & \frac{A_f(1-\beta)(1-\alpha_f)(1+\alpha_f+2\beta_f)}{A(1+\beta)\left(1-\beta_f^2\right)}\,\omega \\ \omega & 0 & \omega & \frac{i(1+\alpha_f+2\beta_f)}{(1+\beta_f)} \\ 0 & \omega & -\frac{i(1+\alpha_f-2\beta_f)}{(1-\beta_f)} & \omega \end{pmatrix} \begin{pmatrix} m^x \\ m^y \\ m_f^x \\ m_f^y \end{pmatrix} = 0$$

Core only : Spin-over mode (SO)

Coupled Core and Mantle

Numerical solution

- Direct resolution to **Poincaré eq.** (fluid) coupled to **Liouville eq.** (mantle)
- Spheroidal core with **arbitrary ellipticity**
- Frequency of SO subtracted from the plot for readability



The frequencies of the inertial modes of the freely rotating planet (red dots) **are the same** as those of a steadily rotating planet (blue circles) **except for the Spin-over/FCN**

Conclusion

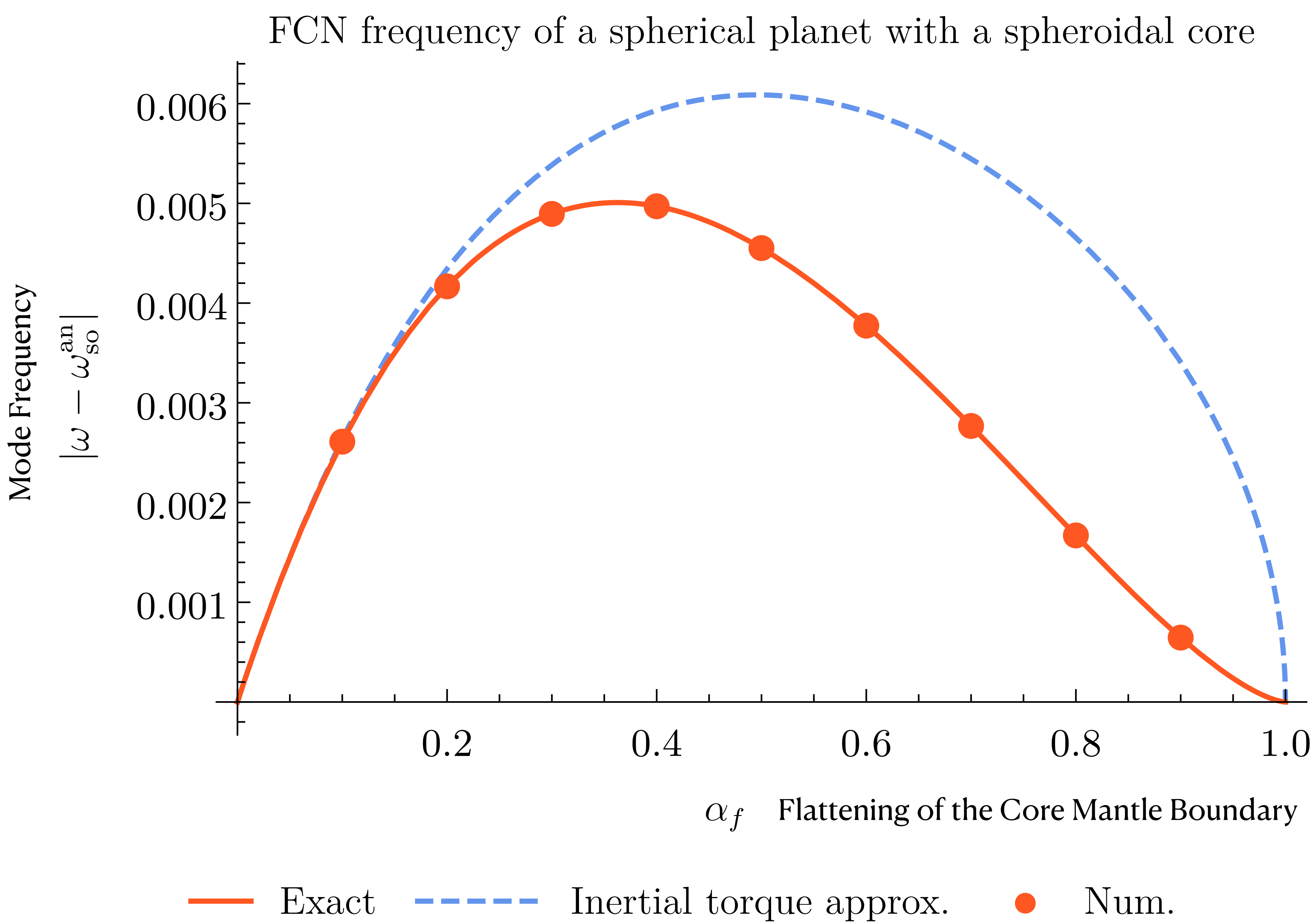
& Future work

- Inertial modes of a freely rotating two-layer planet are **the same** as for the steadily rotating planet **except for the SO which gives rise to the FCN**
- *Free Core Nutation is to the Spin-Over what the Chandler Wobble is to the Eulerian Free Wobble*
- Viscosity and density stratification are expected to induce **further couplings between rotational modes and inertial modes** other than SO (see Triana et al. 2019)

Preprint available from: https://arxiv.org/abs/2004.08218

Supplementary material

Comparison to the inertial torque approximation



The inertial torque approximation is only valid to **first order in the ellipticity** while the solution based on the eq. of vorticity agrees with the numerical solution **to all orders**.